TEXT-BOOK

OF

THEORETICAL NAVAL ARCHITECTURE

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BY

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PREFACE

This book has been prepared in order to provide students and draughtsmen engaged in Shipbuilders' and Naval Architects' drawing offices with a text-book which should explain the calculations which continually have to be performed. It is intended, also, that the work, and more especially its later portions, shall serve as a text-book for the theoretical portion of the examinations of the Science and Art Department in Naval Architecture. It has not been found possible to include all the subjects given in the Honours portion of the syllabus, such as advanced stability work, the rolling of ships, the vibration of ships, etc. These subjects will be found fully treated in one or other of the books given in the list on page 292.

A special feature of the book is the large number of examples given in the text and at the ends of the chapters. By means of these examples, the student is able to test his grasp of the principles and processes given in the text. It is hoped that these examples, many of which have been taken from actual drawing office calculations, will form a valuable feature of the book.

Particulars are given throughout the work and on page 292 as to the books that should be consulted for fuller treatment of the subjects dealt with.

In the Appendix are given the syllabus and specimen questions of the examination in Naval Architecture conducted by the Science and Art Department. These are given by the permission of the Controller of Her Majesty's Stationery Office.

I have to thank Mr. A. W. Johns, Instructor in Naval Architecture at the Royal Naval College, Greenwich, for reading through the proofs and for sundry suggestions. I also wish to express my indebtedness to Sir W. H. White, K.C.B., F.R.S., Assistant Controller and Director of Naval Construction of the Royal Navy, for the interest he has shown and the encouragement he has given me during the progress of the book.

E. L. ATTWOOD.

LONDON,
February, 1899.

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CHAPTER I.

AREAS, VOLUMES, WEIGHTS, DISPLACEMENT, ETC.

Areas of Plane Figures.

A Rectangle.—This is a four-sided figure having its opposite sides parallel to one another and all its angles right angles. Such a figure is shown in Fig. 1. Its area is the product of the length and the breadth, or AB × BC. Thus a rectangular plate 6 feet long and 3 feet broad will contain—

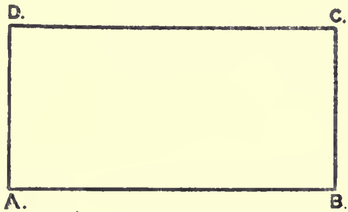

Fig. 1.

$$6 \times 3 = 18 \text{ square feet}$$

and if of such a thickness as to weigh $12\frac{1}{2}$ lbs. per square foot, will weigh—

$$18 \times 12\frac{1}{2} = 225 \text{ lbs.}$$

A Square.—This is a particular case of the above, the length being equal to the breadth. Thus a square hatch of $3\frac{1}{2}$ feet side will have an area of—

$$3\frac{1}{2} \times 3\frac{1}{2} = \frac{7}{2} \times \frac{7}{2} = \frac{49}{4}$$
$$= 12\frac{1}{4} \text{ square feet}$$

A Triangle.—This is a figure contained by three straight lines, as ABC in Fig. 2. From the vertex C drop a perpendicular on to the base AB (or AB produced, if necessary). Then the area is given by half the product of the base into the height, or—

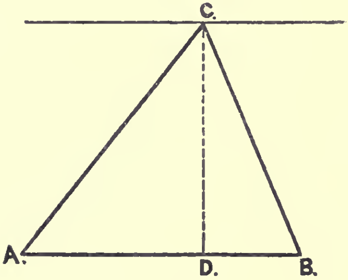

FIG. 2.

$$\frac{1}{2}(\mathrm{AB} \times \mathrm{CD})$$

If we draw through the apex C a line parallel to the base AB, any triangle having its apex on this line, and having AB for its base, will be equal in area to the triangle ABC. If more convenient, we can consider either A or B as the apex, and BC or AC accordingly as the base.

Thus a triangle of base $5\frac{1}{2}$ feet and perpendicular drawn from the apex $2\frac{1}{4}$ feet, will have for its area—

$$\frac{1}{2} \times 5\frac{1}{2} \times 2\frac{1}{4} = \frac{1}{2} \times \frac{11}{2} \times \frac{9}{4} = \frac{99}{16}$$
$$= 6\tfrac{3}{16} \text{ square feet}$$

If this triangle be the form of a plate weighing 20 lbs. to the square foot, the weight of the plate will be—

$$\frac{99}{16} \times 20 = 123\tfrac{3}{4} \text{ lbs.}$$

A Trapezoid.—This is a figure formed of four straight lines, of which two only are parallel. Fig. 3 gives such a figure, ABCD.

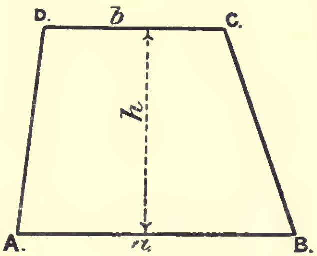

FIG. 3.

If the lengths of the parallel sides AB and CD are a and b respectively, and h is the perpendicular distance between them, the area of the trapezoid is given by—

$$\frac{1}{2}(a + b) \times h$$

or one-half the sum of the parallel sides multiplied by the perpendicular distance between them.

Example.—An armour plate is of the form of a trapezoid with parallel sides 8′ 3″ and 8′ 9″ long, and their distance apart 12 feet. Find its weight if 6 inches thick, the material of the armour plate weighing 490 lbs. per cubic foot.

First we must find the area, which is given by—

$$\left(\frac{8' \, 3'' + 8' \, 9''}{2}\right) \times 12 \text{ square feet} = \tfrac{17}{2} \times 12$$
$$= 102 \text{ square feet}$$

The plate being 6 inches thick = $\frac{1}{2}$ foot, the cubical contents of the plate will be—

$$102 \times \tfrac{1}{2} = 51 \text{ cubic feet}$$

The weight will therefore be—

$$51 \times 490 \text{ lbs.} = \frac{51 \times 490}{2240}$$
$$= 11{\cdot}15 \text{ tons}$$

A **Trapezium** is a quadrilateral or four-sided figure of which no two sides are parallel.

Such a figure is ABCD (Fig. 4). Its area may be found by drawing a diagonal BD and adding together the areas of the triangles ABD, BDC. These both have the same base, BD. Therefore from A and C drop perpendiculars AE and CF on to BD. Then the area of the trapezium is given by—

$$\tfrac{1}{2}(AE + CF) \times BD$$

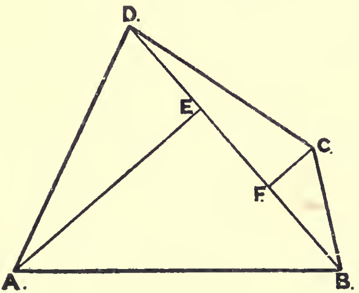

Fig. 4.

Example.—Draw a trapezium on scale $\frac{1}{2}$ inch = 1 foot, where four sides taken in order are 6, 5, 6, and 10 feet respectively, and the diagonal from the starting-point 10 feet. Find its area in square feet.

Ans. 40 sq. feet.

A Circle.—This is a figure all points of whose boundary are equally distant from a fixed point within it called the *centre*. The boundary is called its *circumference*, and any line from the centre to the circumference is called a *radius*. Any line passing through the centre and with its ends on the circumference is called a *diameter*.

The ratio between the circumference of a circle and its diameter is called π,[1] and $\pi = 3{\cdot}1416$, or nearly $\frac{22}{7}$

Thus the length of a thin wire forming the circumference of a circle of diameter 5 feet is given by—

$$\begin{aligned} \pi \times 5 &= 5 \times 3{\cdot}1416 \text{ feet} \\ &= 15{\cdot}7080 \text{ feet} \end{aligned}$$

$$\text{or using } \pi = \tfrac{22}{7}, \text{ the circumference} = 5 \times \tfrac{22}{7} = \tfrac{110}{7} = 15\tfrac{5}{7} \text{ feet nearly}$$

The circumference of a mast 2′ 6″ in diameter is given by—

$$2\tfrac{1}{2} \times \pi \text{ feet} = \tfrac{5}{2} \times \tfrac{22}{7} = \tfrac{55}{7} = 7\tfrac{6}{7} \text{ feet nearly}$$

The *area of a circle* of diameter d is given by—

$$\frac{\pi \times d^2}{4} \qquad (d^2 = d \times d)$$

Thus a solid pillar 4 inches in diameter has a sectional area of—

$$\begin{aligned} \frac{\pi \times 4^2}{4} &= \tfrac{22}{7} \times 4 \\ &= 12\tfrac{4}{7} \text{ square inches nearly} \end{aligned}$$

A hollow pillar 5 inches external diameter and $\frac{1}{4}$ inch thick will have a sectional area obtained by subtracting the area of a circle $4\frac{1}{2}$ inches diameter from the area of a circle 5 inches diameter

$$\begin{aligned} &= \left(\frac{\pi(5)^2}{4}\right) - \left(\frac{\pi(4\frac{1}{2})^2}{4}\right) \\ &= 3{\cdot}73 \text{ square inches} \end{aligned}$$

The same result may be obtained by taking a mean diameter of the ring, finding its circumference, and multiplying by the breadth of the ring.

$$\begin{aligned} \text{Mean diameter} &= 4\tfrac{3}{4} \text{ inches} \\ \text{Circumference} &= \tfrac{19}{4} \times \tfrac{22}{7} \text{ inches} \\ \text{Area} &= (\tfrac{19}{4} \times \tfrac{22}{7}) \times \tfrac{1}{4} \text{ square inches} \\ &= 3{\cdot}73 \text{ square inches as before} \end{aligned}$$

[1] This is the Greek letter *pi*, and is always used to denote 3·1416, or $\frac{22}{7}$ nearly; that is, the ratio borne by the circumference of a circle to its diameter.

Trapezoidal Rule.—We have already seen (p. 2) that the area of a trapezoid, as ABCD, Fig. 5, is given by $\frac{1}{2}(AD + BC)AB$, or calling AD, BC, and AB y_1, y_2, and h respectively the area is given by—

$$\tfrac{1}{2}(y_1 + y_2)h$$

If, now, we have two trapezoids joined together, as in

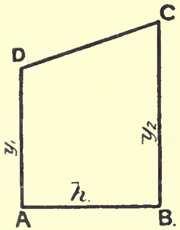

FIG. 5.

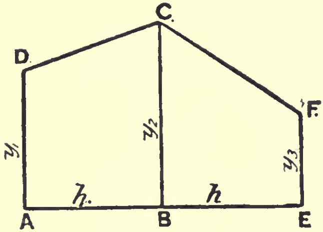

FIG. 6.

Fig. 6, having BE = AB, the area of the added part will be given by—

$$\tfrac{1}{2}(y_2 + y_3)h$$

The area of the whole figure is given by—

$$\tfrac{1}{2}(y_1 + y_2)h + \tfrac{1}{2}(y_2 + y_3)h = \tfrac{1}{2}h(y_1 + 2y_2 + y_3)$$

If we took a third trapezoid and joined on in a similar manner, the area of the whole figure would be given by

$$\tfrac{1}{2}h(y_1 + 2y_2 + 2y_3 + y_4) = h\left(\frac{y_1 + y_4}{2} + y_2 + y_3\right)$$

Trapezoidal rule for finding the area of a curvilinear figure, as ABCD, Fig. 7.

Divide the base AB into a convenient number of equal parts, as AE, EG, etc., each of length equal to h, say. Set up perpendiculars to the base, as EF, GH, etc. If we join DF, FH, etc., by straight lines, shown dotted, the area required will very nearly equal the sum of the areas of the trapezoids ADFE, EFHG, etc. Or using the lengths y_1, y_2, etc., as indicated in the figure—

$$\text{Area} = h\left(\frac{y_1 + y_7}{2} + y_2 + y_3 + y_4 + y_5 + y_6\right)$$

In the case of the area shown in Fig. 7, the area will be somewhat greater than that given by this rule. If the curve, however, bent towards the base line, the actual area would be somewhat less than that given by this rule. In any case, the closer the perpendiculars are taken together the less will be the error involved by using this rule. Putting this rule into words, we have—

To find the area of a curvilinear figure, as ABCD, Fig. 7, *by means of the trapezoidal rule, divide the base into any convenient number of equal parts, and erect perpendiculars to the base meeting the curve; then to the half-sum of the first and last of these add the sum of all the intermediate ones; the result multiplied by the common distance apart will give the area required.*

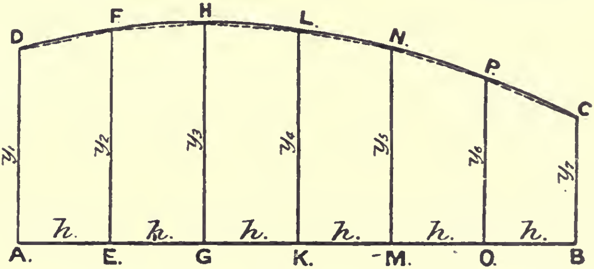

FIG. 7.

The perpendiculars to the base AB, as AD, EF, are termed "*ordinates*," and any measurement along the base from a given starting-point is termed an "*abscissa*." Thus the point P on the curve has an ordinate OP and an abscissa AO when referred to the point A as origin.

Simpson's First Rule.[1]—This rule assumes that the curved line DC, forming one boundary of the curvilinear area ABCD, Fig. 8, is a portion of a curve known as *a parabola of the second order.*[2] In practice it is found that the results given by its application to ordinary curves are very accurate,

[1] It is usual to call these rules Simpson's rules, but the first rule was given before Simpson's time by James Stirling, in his "Methodus Differentialis," published in 1730.

[2] A "parabola of the second order" is one whose equation referred to co-ordinate axes is of the form $y = a_0 + a_1x + a_2x^2$, where a_0, a_1, a_2 are constants.

and it is this rule that is most extensively used in finding the areas of curvilinear figures required in ship calculations.

Let ABCD, Fig. 8, be a figure bounded on one side by the curved line DC, which, as stated above, is assumed to be a parabola of the second order. AB is the base, and AD and BC are end ordinates perpendicular to the base.

Bisect AB in E, and draw EF perpendicular to AB, meeting the curve in F. Then the area is given by—

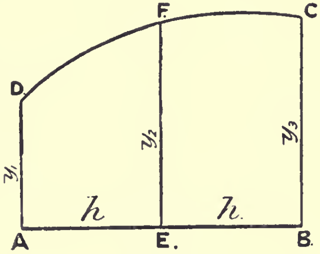

Fig. 8.

$$\tfrac{1}{3}\text{AE}(\text{AD} + 4\text{EF} + \text{BC})$$

or using y_1, y_2, y_3 to represent the ordinates, h the common interval between them—

$$\text{Area} = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

Now, a long curvilinear area[1] may be divided up into a number of portions similar to the above, to each of which the above rule will apply. Thus the area of the portion GHNM of the area Fig. 7 will be given by—

$$\frac{h}{3}(y_3 + 4y_4 + y_5)$$

and the portion MNCB will have an area given by—

$$\frac{h}{3}(y_5 + 4y_6 + y_7)$$

Therefore the total area will be, supposing all the ordinates are a common distance h apart—

$$\frac{h}{3}(y_1 + 4y_2 + 2y_3 + 4y_4 + 2y_5 + 4y_6 + y_7)$$

Ordinates, as GH, MN, which divide the figure into the elementary areas are termed "*dividing ordinates.*"

Ordinates between these, as EF, KL, OP, are termed "*intermediate ordinates.*"

[1] The curve is supposed continuous. If the curvature changes abruptly at any point, this point must be at a dividing ordinate.

Notice that the area must have an *even* number of *intervals*, or, what is the same thing, an *odd* number of *ordinates*, for Simpson's first rule to be applicable.

Therefore, putting Simpson's first rule into words, we have—

Divide the base into a convenient even number of equal parts, and erect ordinates meeting the curve. Then to the sum of the end ordinates add four times the even ordinates and twice the odd ordinates. The sum thus obtained, multiplied by one-third the common distance apart of the ordinates, will give the area.

Approximate Proof of Simpson's First Rule.—The truth of Simpson's first rule may be understood by the following approximate proof: [1]—

Let DFC, Fig. 9, be a curved line on the base AB, and with end ordinates AD, BC perpendicular to AB. Divide AB equally in E, and draw the ordinate EF perpendicular to AB. Then with the ordinary notation—

$$\text{Area} = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

by Simpson's first rule. Now divide AB into three equal parts by the points G and H. Draw perpendiculars GJ and HK to the base AB. At F draw a tangent to the curve, meeting GJ and HK in J and K. Join DJ and KC. Now, it is evident that the area we want is very nearly equal to the area ADJKCB. This will be found by adding together the areas of the trapezoids ADJG, GJKH, HKCB.

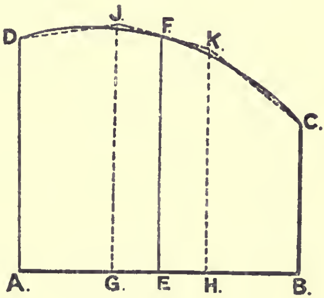

Fig. 9.

$$\text{Area of ADJG} = \tfrac{1}{2}(\text{AD} + \text{GJ})\text{AG}$$
$$\text{,, GJKH} = \tfrac{1}{2}(\text{GJ} + \text{HK})\text{GH}$$
$$\text{,, HKCB} = \tfrac{1}{2}(\text{HK} + \text{BC})\text{HB}$$

[1] Another proof will be found on p. 73. The mathematical proof will be found in appendix A, p. 245.

Now, AG = GH = HB = $\frac{1}{3}$AB = $\frac{2}{3}$AE, therefore the total area is—

$$\frac{1}{2}\left(\frac{2AE}{3}\right)(AD + 2GJ + 2HK + BC)$$

Now, AE = h, and GJ + HK = 2EF (this may be seen at once by measuring with a strip of paper), therefore the total area is—

$$\frac{h}{3}(AD + 4EF + BC) = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

which is the same as that given by Simpson's first rule.

Application of Simpson's First Rule.—*Example.*—A curvilinear area has ordinates at a common distance apart of 2 feet, the lengths being 1·45, 2·65, 4·35, 6·45, 8·50, 10·40, and 11·85 feet respectively. Find the area of the figure in square feet.

In finding the area of such a curvilinear figure by means of Simpson's first rule, the work is arranged as follows:—

Number of ordinate.	Length of ordinate.	Simpson's multipliers.[1]	Functions of ordinates.
1	1·45	1	1·45
2	2·65	4	10·60
3	4·35	2	8·70
4	6·45	4	25·80
5	8·50	2	17·00
6	10·40	4	41·60
7	11·85	1	11·85

117·00 sum of functions

Common interval = 2 feet

$\frac{1}{3}$ common interval = $\frac{2}{3}$ feet

area = 117 × $\frac{2}{3}$ = 78 square feet

The length of this curvilinear figure is 12 feet, and it has been divided into an *even* number of intervals, viz. 6, 2 feet apart, giving an *odd* number of ordinates, viz. 7. We are consequently able to apply Simpson's first rule to finding its area. Four columns are used. In the first column are placed the numbers of the ordinates, starting from one end of the figure. In the second column are placed, in the proper order, the lengths of the ordinates corresponding to the numbers in the first column. These lengths are expressed in feet and

[1] Sometimes the multipliers used are half these, viz. $\frac{1}{2}$, 2, 1, 2, 1, 2, $\frac{1}{2}$, and the result at the end is multiplied by two-thirds the common interval.

decimals of a foot, and are best measured off with a decimal scale. If a scale showing feet and inches is used, then the inches should be converted into decimals of a foot; thus, 6′ 9″ = 6·75′, and 6′ $3\frac{1}{2}$″ = 6·3′. In the next column are placed Simpson's multipliers in their proper order and opposite their corresponding ordinates. The order may be remembered by combining together the multipliers for the elementary area first considered—

	1	4	1				
			1	4	1		
					1	4	1
or	1	4	2	4	2	4	1

The last column contains the product of the length of the ordinate and its multiplier given in the third column. These are termed the "*functions of ordinates.*" The sum of the figures in the last column is termed the "*sum of functions of ordinates.*" This has to be multiplied by one-third the common interval, or in this case $\frac{2}{3}$. The area then is given by—

$$117 \times \frac{2}{3} = 78 \text{ square feet}$$

Simpson's Second Rule.—This rule assumes that the curved line DC, forming one boundary of the curvilinear area

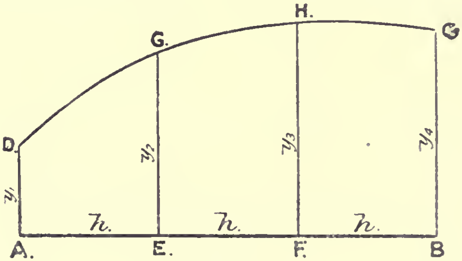

Fig. 10.

ABCD, Fig. 10, is a portion of a curve known as "*a parabola of the third order.*" [1]

Let ABCD, Fig. 10, be a figure bounded on one side by the curved line DC, which, as stated above, is assumed to be

[1] A "parabola of the third order" is one whose equation referred to co-ordinate axes is of the form $y = a_0 + a_1x + a_2x^2 + a_3x^3$, where a_0, a_1, a_2, a_3 are constants.

"*a parabola of the third order.*" AB is the base, and AD and BC are end ordinates perpendicular to the base. Divide the base AB into three equal parts by points E and F, and draw EG, FH perpendicular to AB, meeting the curve in G and H respectively. Then the area is given by—

$$\tfrac{3}{8}\text{AE}(\text{AD} + 3\text{EG} + 3\text{FH} + \text{BC})$$

or, using y_1, y_2, y_3, y_4 to represent the ordinates, and h the common interval between them—

$$\text{Area} = \tfrac{3}{8}h(y_1 + 3y_2 + 3y_3 + y_4)$$

Now, a long curvilinear area[1] may be divided into a number of portions similar to the above, to each of which the above rule will apply. Thus the area of the portion KLCB in Fig. 7 will be given by—

$$\tfrac{3}{8}h(y_4 + 3y_5 + 3y_6 + y_7)$$

Consequently the total area of ABCD, Fig. 7, will be, supposing all the ordinates are a common distance h apart—

$$\tfrac{3}{8}h(y_1 + 3y_2 + 3y_3 + 2y_4 + 3y_5 + 3y_6 + y_7)$$

The ordinate KL is termed a "*dividing ordinate*," and the others, EF, GH, MN, OP, are termed "*intermediate ordinates.*" This rule may be approximately proved by a process similar to that adopted on p. 8 for the first rule.

Application of Simpson's Second Rule.—*Example.*—A curvilinear area has ordinates at a common distance apart of 2 feet, the lengths being 1·45, 2·65, 4·35, 6·45, 8·50, 10·40, and 11·85 feet respectively. Find the area of the figure in square feet by the use of Simpson's second rule.

In finding the area of such a curvilinear figure by means of Simpson's second rule, the work is arranged as follows :—

Number of ordinate.	Length of ordinate.	Simpson's multipliers.	Functions of ordinates.
1	1·45	1	1·45
2	2·65	3	7·95
3	4·35	3	13·05
4	6·45	2	12·90
5	8·50	3	25·50
6	10·40	3	31·20
7	11·85	1	11·85

103·90 sum of functions

Common interval = 2 feet

$\tfrac{3}{8}$ common interval = $\tfrac{6}{8} = \tfrac{3}{4}$

$103·9 \times \tfrac{3}{4} = 77·925$ square feet

[1] See footnote on p. 7.

This curvilinear area is the same as already taken for an example of the application of Simpson's first rule. It will be noticed that the number of intervals is 6 or a *multiple of* 3. We are consequently able to apply Simpson's second rule to finding the area. The columns are arranged as in the previous case, the multipliers used being those for the second rule. The order may be remembered by combining together the multipliers for the elementary area with three intervals first considered—

	1	3	3	1			
				1	3	3	1
or	1	3	3	2	3	3	1

For nine intervals the multipliers would be 1, 3, 3, 2, 3, 3, 2, 3, 3, 1.

The sum of the functions of ordinates has in this case to be multiplied by $\frac{3}{8}$ the common interval, or $\frac{3}{8} \times 2 = \frac{3}{4}$, and consequently the area is—

$$103 \cdot 9 \times \frac{3}{4} = 77 \cdot 925 \text{ square feet}$$

It will be noticed how nearly the areas as obtained by the two rules agree. In practice the first rule is used in nearly all cases, because it is much simpler than the second rule and quite as accurate. It sometimes happens, however, that we only have four ordinates to deal with, and in this case Simpson's second rule must be used.

To find the Area of a Portion of a Curvilinear Area contained between Two Consecutive Ordinates.—Such a portion is AEFD, Fig. 8. In order to obtain this area, we require the three ordinates to the curve $y_1 y_2 y_3$. The curve DFC is assumed to be, as in Simpson's first rule, a parabola of the second order. Using the ordinary notation, we have—

$$\text{Area of ADFE} = \frac{1}{12}h(5y_1 + 8y_2 - y_3)$$

or, putting this into words—

To eight times the middle ordinate add five times the near end ordinate and subtract the far end ordinate; multiply the remainder by $\frac{1}{12}$ the common interval: the result will be the area.

Thus, if the ordinates of the curve in Fig. 8 be 8·5, 10·4, 11·85 feet, and 2 feet apart, the area of AEFD will be given by—

$$\tfrac{1}{12} \times 2(\overline{5 \times 8{\cdot}5} + \overline{8 \times 10{\cdot}4} - 11{\cdot}85) = 18{\cdot}97 \text{ square feet}$$

Similarly the area of EBCF will be given by—

$$\tfrac{1}{12} \times 2(\overline{5 \times 11{\cdot}85} + \overline{8 \times 10{\cdot}4} - 8{\cdot}5) = 22{\cdot}32 \text{ square feet}$$

giving a total area of the whole figure as 41·29 square feet.

Obtaining this area by means of Simpson's first rule, we should obtain 41·3 square feet.

This rule is sometimes known as the "*five-eight*" rule.

Subdivided Intervals.—When the curvature of a line forming a boundary of an area, as Fig. 11, is very sharp, it is found that the distance apart of ordinates, as used for the straighter part of the curve, does not give a sufficiently accurate result. In such a case, ordinates are drawn at a sub-multiple of the ordinary distance apart of the main ordinates.

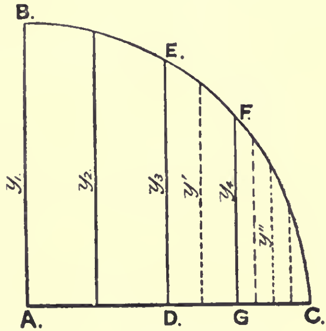

Fig. 11.

Take ABC, a quadrant of a circle (Fig. 11), and draw the three ordinates y_2, y_3, y_4 a distance h apart. Then we should get the area approximately by putting the ordinates through Simpson's first rule. Now, the curve EFC is very sharp, and the result obtained is very far from being an accurate one. Now put in the intermediate ordinates y', y''. Then the area of the portion DEC will be given by—

$$\tfrac{1}{3}\left(\frac{h}{2}\right)(y_3 + 4y' + 2y_4 + 4y'' + y_5)$$

or we may write this— ($y_5 = 0$ at end)

$$\tfrac{1}{3}h(\tfrac{1}{2}y_3 + 2y' + y_4 + 2y'' + \tfrac{1}{2}y_5)$$

The area of the portion ABED is given by—

$$\tfrac{1}{3}h(y_1 + 4y_2 + y_3)$$

or the area of the whole figure—

$$\frac{1}{3}h(y_1 + 4y_2 + 1\frac{1}{2}y_3 + 2y' + y_4 + 2y'' + \frac{1}{2}y_5)$$

Thus the multipliers for ordinates one-half the ordinary distance apart are $\frac{1}{2}$, 2, $\frac{1}{2}$, and for ordinates one-quarter the ordinary distance apart are $\frac{1}{4}$, 1, $\frac{1}{2}$, 1, $\frac{1}{4}$. Thus we diminish the multiplier of each ordinate of a set of subdivided intervals in the same proportion as the intervals are subdivided. Each ordinate is then multiplied by its proper multiplier found in this way, and the sum of the products multiplied by $\frac{1}{3}$ or $\frac{3}{8}$ the whole interval according as the first or second rule is used. An exercise on the use and necessity for subdivided intervals will be found on p. 41.

Algebraic Expression for the Area of a Figure bounded by a Plane Curve.—It is often convenient to be able to express in a short form the area of a plane curvilinear figure.

In Fig. 12, let ABCD be a strip cut off by the ordinates AB, CD, a distance Δx apart, Δx being supposed small. Then the area of this strip is very nearly—

$$y \times \Delta x$$

where y is the length of the ordinate AB. If now we imagine the strip to become indefinitely narrow, the small triangularpiece BDE will disappear, and calling dx the breadth of the strip, its area will be—

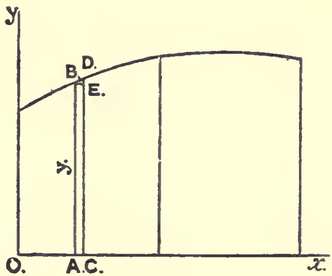

FIG. 12.

$$y \times dx$$

The area of the whole curvilinear figure would be found if we added together the areas of all such strips, and this could be written—

$$\int y \,.\, dx$$

where the symbol $\int$ may be regarded as indicating the sum of all such strips as $y \,.\, dx$. We have already found that

Simpson's rules enable us to find the areas of such figures, so we may look upon the expression for the area—

$$\int y \cdot dx$$

as meaning that, to find the area of a figure, we take the length of the ordinate y at convenient intervals, and put them through Simpson's multipliers. The result, multiplied by $\frac{1}{3}$ or $\frac{3}{8}$ the common interval, as the case may be, will give the area. A proper understanding of the above will be found of great service in dealing with moments in the next chapter.

To find the Area of a Figure bounded by a Plane Curve and Two Radii.—Let OAB, Fig. 13, be such a figure, OA, OB being the bounding radii.

Take two points very close together on the curve PP′; join OP, OP′, and let OP $= r$ and the small angle POP′ $= \Delta\theta$ in circular measure.[1] Then OP $=$ OP′ $= r$ very nearly, and the area of the elementary portion OPP′ $= \frac{1}{2}r(r \cdot \Delta\theta)$, $r \cdot \Delta\theta$ being the length of PP′, and regarding OPP′ as a triangle. If now we consider OP, OP′ to become indefinitely close together, and consequently the angle POP′ indefinitely small $= d\theta$ say, any error in regarding POP′ as a triangle will disappear, and we shall have—

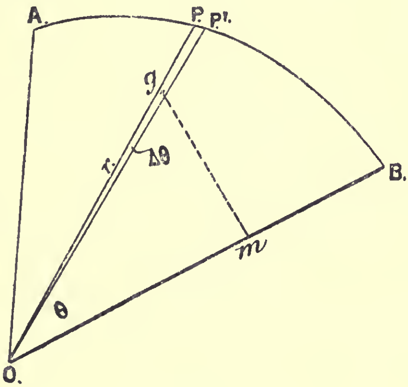

FIG. 13.

$$\text{Area POP}' = \frac{r^2}{2} \cdot d\theta$$

and the whole area AOB is the sum of all such areas which can be drawn between OA and OB, or—

$$\int \frac{r^2}{2} \cdot d\theta$$

[1] See p. 86.

Now, this exactly corresponds to the algebraic expression for the area of an ordinary plane curvilinear area, viz.—

$$\int y \, . \, dx \text{ (see p. 15)}$$

y corresponding to $\frac{r^2}{2}$ and dx corresponding to $d\theta$. Therefore divide the angle between the bounding radii into an even number of equal angular intervals by means of radii. Measure these radii, and treat their half-squares as ordinates of a curve by Simpson's first rule, multiplying the addition by $\frac{1}{3}$ the common angular interval *in circular measure*. Simpson's second rule may be used in a similar manner.

The *circular measure of an angle*[1] is the number of degrees it contains multiplied by $\frac{\pi}{180}$, or 0·01745. Thus the circular measure of—

$$90^\circ = \frac{\pi}{2} = \frac{3·1416}{2} = 1·5708$$

and the circular measure of 15° is 0·26175.

Example.—To find the area of a figure bounded by a plane curve and two radii 90° apart, the lengths of radii 15° apart being 0, 2·6, 5·2, 7·8, 10·5, 13·1, 15·7.

Angle from first radius.	Length of radius.	Square of length.	Simpson's multipliers.	Functions of squares.
0°	0·0	0·0	1	0·0
15°	2·6	6·8	4	27·2
30°	5·2	27·0	2	54·0
45°	7·8	60·8	4	243·2
60°	10·5	110·2	2	220·4
75°	13·1	171·6	4	686·4
90°	15·7	246·5	1	246·5

1477·7 sum of functions

Circular measure of $15^\circ = 0·26175$

$$\therefore \text{area} = 1477·7 \times \tfrac{1}{3} \times 0·26175 \times \tfrac{1}{2}$$
$$= 64\tfrac{1}{2} \text{ square feet nearly}$$

The process is exactly the same as in Simpson's rule for a plane area with equidistant ordinates. To save labour, the squares of the radii are put through the proper multipliers, the multiplication by $\frac{1}{2}$ being performed at the end.

[1] See also p. 86.

Measurement of Volumes.

The Capacity or Volume of a Rectangular Block is the product of the length, breadth, and depth, or, in other words, the area of one face multiplied by the thickness. All these dimensions must be expressed in the same units. Thus the volume of an armour plate 12 feet long, $8\frac{1}{4}$ feet wide, and 18 inches thick, is given by—

$$12 \times 8\tfrac{1}{4} \times 1\tfrac{1}{2} = 12 \times \tfrac{33}{4} \times \tfrac{3}{2} = \tfrac{297}{2} = 148\tfrac{1}{2} \text{ cubic feet}$$

The Volume of a Solid of Constant Section is the area of its section multiplied by its length. Thus a pipe 2 feet in diameter and 100 feet long has a section of $\frac{\pi 4}{4} = \frac{22}{7}$ square feet, and a volume of $\frac{22}{7} \times 100 = \frac{2200}{7} = 314\frac{2}{7}$ cubic feet.

A hollow pillar 7′ 6″ long, 5 inches external diameter, and $\frac{1}{4}$ inch thick, has a sectional area of—

$$3{\cdot}73 \text{ square inches}$$
$$\text{or } \frac{3{\cdot}73}{144} \text{ square feet}$$

and the volume of material of which it is composed is—

$$\left(\frac{3{\cdot}73}{144}\right) \times \frac{15}{2} = \frac{18{\cdot}65}{96}$$
$$= 0{\cdot}195 \text{ cubic foot}$$

Volume of a Sphere.—This is given by $\frac{\pi}{6} . d^3$, where d is the diameter. Thus the volume of a ball 3 inches in diameter is given by—

$$\frac{\pi}{6} . 27 = \frac{22 \times 27}{42}$$
$$= 14\tfrac{1}{7} \text{ cubic inches}$$

Volume of a Pyramid.—This is a solid having a base in the shape of a polygon, and a point called its vertex not in the same plane as the base. The vertex is joined by straight lines to all points on the boundary of the base. Its volume is given by the product of the area of the base and one-third the perpendicular distance of the vertex from the base. A *cone* is a particular case of the pyramid having for its base a figure with a continuous curve, and a *right circular cone* is a cone having for its base a circle and its vertex immediately over the centre of the base.

To find the Volume of a Solid bounded by a Curved Surface.—The volumes of such bodies as this are continually required in ship calculation work, the most important case being the volume of the under-water portion of a vessel. In this case, the volume is bounded on one side by a plane surface, the water-plane of the vessel. Volumes of compartments are frequently required, such as those for containing fresh water or coal-bunkers. The body is divided by a series of planes spaced equally apart. The area of each section is obtained by means of one of the rules already explained. These areas are treated as the ordinates of a new curve, which may be run in, with ordinates the spacing of the planes apart. It is often desirable to draw this curve with areas as ordinates as in Fig. 14, because, if the surface is a fair

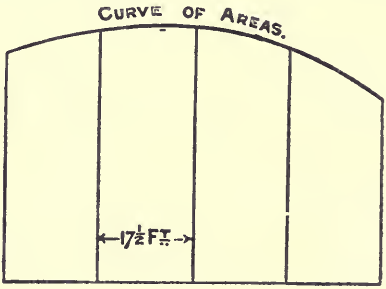

FIG. 14.

surface, the curve of areas should be a fair curve, and should run evenly through all the spots; any inaccuracy may then be detected. The area of the curve of areas is then obtained by one of Simpson's rules as convenient, and this area will represent the cubical contents of the body.

Example.—A coal-bunker has sections 17′ 6″ apart, and the areas of these sections are 98, 123, 137, 135, 122 square feet respectively. Find the volume of the bunker and the number of tons of coal it will hold, taking 44 cubic feet of coal to weigh 1 ton.

Areas.	Simpson's multipliers.	Functions of areas.
98	1	98
123	4	492
137	2	274
135	4	540
122	1	122

1526 sum of functions

$$\tfrac{1}{3} \text{ common interval} = \tfrac{1}{3} \times 17\tfrac{1}{2} = \tfrac{35}{6}$$
$$\therefore \text{volume} = 1526 \times \tfrac{35}{6} \text{ cubic feet}$$
$$= 8902 \text{ cubic feet.}$$

and the bunker will hold $\frac{8902}{44} = 202$ tons

The under-water portion of a ship is symmetrical about the fore-and-aft middle line plane, so that only one-half need be calculated. We may divide the volume in two ways—

1. By equidistant planes parallel to the load water-plane.

2. By equidistant planes perpendicular to the middle-line plane and to the load water-plane.

The volume as obtained by both methods should be the same, and they are used to check each other.

Examples.—1. The under-water portion of a vessel is divided by vertical sections 10 feet apart of the following areas: 0·3, 22·7, 48·8, 73·2, 88·4, 82·8, 58·7, 26·2, 3·9 square feet. Find the volume in cubic feet. (The curve of sectional areas is given in Fig. 15.)

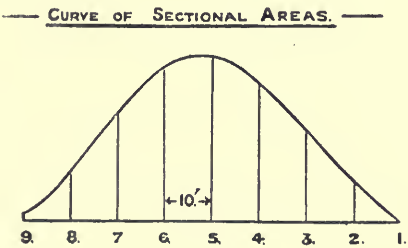

FIG. 15.

The number of ordinates being odd, Simpson's first rule can be applied as indicated in the following calculation :—

Number of section.	Area of section.	Simpson's multipliers.	Function of area.
1	0·3	1	0·3
2	22·7	4	90·8
3	48·8	2	97·6
4	73·2	4	292·8
5	88·4	2	176·8
6	82·8	4	331·2
7	58·7	2	117·4
8	26·2	4	104·8
9	3·9	1	3·9

1215·6 sum of functions

$$\tfrac{1}{3}\text{ common interval} = \tfrac{10}{3}$$
$$\therefore \text{volume} = 1215{\cdot}6 \times \tfrac{10}{3}$$
$$= 4052 \text{ cubic feet}$$

2. The under-water portion of the above vessel is divided by planes parallel to the load water-plane and $1\frac{1}{2}$ feet apart of the following areas: 944, 795, 605, 396, 231, 120, 68, 25, 8 square feet. Find the volume in cubic feet.

The number of areas being odd, Simpson's first rule can be applied, as indicated in the following calculation:—

Number of water-line.	Area of water-plane.	Simpson's multipliers.	Function of area.
1	944	1	944
2	795	4	3180
3	605	2	1210
4	396	4	1584
5	231	2	462
6	120	4	480
7	68	2	136
8	25	4	100
9	8	1	8

8104 sum of functions

$$\tfrac{1}{3}\text{ common interval} = \tfrac{1}{3} \times \tfrac{3}{2}$$
$$\therefore \text{volume} = 8104 \times \tfrac{1}{2}$$
$$= 4052 \text{ cubic feet}$$

which is the same result as was obtained above by taking the areas of vertical sections and putting them through Simpson's rule.

In practice this volume is found by means of a "displacement sheet," or by the "planimeter." An explanation of these will be found in Chapter II.

Displacement.—The amount of water displaced or put aside by a vessel afloat is termed her "*displacement*." This may be reckoned as a volume, when it is expressed in cubic feet, or as a weight, when it is expressed in tons. It is usual to take salt water to weigh 64 lbs. per cubic foot, and consequently $\frac{2240}{64} = 35$ cubic feet of salt water will weigh one ton. Fresh water, on the other hand, is regarded as weighing $62\frac{1}{4}$ lbs. per cubic foot, or 36 cubic feet to the ton. The volume displacement is therefore 35 or 36 times the weight displacement, according as we are dealing with salt or fresh water.

If a vessel is floating in equilibrium in still water, the weight of water she displaces must exactly equal the weight of the vessel herself with everything she has on board.

That this must be true may be understood from the following illustrations—

1. Take a large basin and stand it in a dish (see Fig. 16).

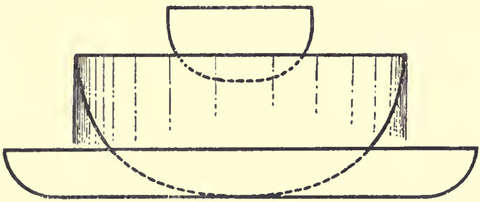

Fig. 16.

Just fill the basin to the brim with water. Now carefully place a smaller basin into the water. It will be found that some of the water in the large basin will be displaced, and water will spill over the edge of the large basin into the dish below. It is evident that the water displaced by the basin is equal in amount to the water that has been caught by the dish, and if this water be weighed it will be found, if the experiment be conducted accurately, that the small basin is equal in weight to the water in the dish, that is, to the water it has displaced.

2. Consider a vessel floating in equilibrium in still water, and imagine, if it were possible, that the water is solidified, maintaining the same level, and therefore the same density. If now we lift the vessel out, we shall have a cavity left behind which

will be exactly of the form of the under-water portion of the ship, as Fig. 17. Now let the cavity be filled up with water. The amount of water we pour in will evidently be equal to the volume of displacement of the vessel. Now suppose that the solidified water outside again becomes water. The water we have poured in will remain where it is, and will be supported by the water surrounding it. The support given, first to the vessel and now to the water we have poured in, by the sur-

FIG. 17.

rounding water must be the same, since the condition of the outside water is the same. Consequently, it follows that the weight of the vessel must equal the weight of water poured in to fill the cavity, or, in other words, the weight of the vessel is equal to the weight of water displaced.

If the vessel whose displacement has been calculated on P is floating at her L.W.P. in salt water, her total weight will be—

$$4052 \div 35 = 115 \cdot 8 \text{ tons}$$

If she floated at the same L.W.P. in fresh water, her total weight would be—

$$4052 \div 36 = 112\tfrac{1}{2} \text{ tons}$$

It will be at once seen that this property of floating bodies is of very great assistance to us in dealing with ships. For, to find the weight of a ship floating at a given line, we do not need to estimate the weight of the ship, but we calculate out from the drawings the displacement in tons up to the given line, and this must equal the total weight of the ship.

Curve of Displacement.—The calculation given on p. 20 gives the displacement of the vessel up to the load-water plane, but the draught of a ship continually varies owing to different weights of cargo, coal, stores, etc., on board, and it is desirable to have a means of determining quickly the displacement at any given draught. From the rules we have already investigated, the displacement in tons can be calculated up to each water-plane in succession. If we set down a scale of mean draughts, and set off perpendiculars to this scale at the places where each water-plane comes, and on these set off on a convenient scale the displacement we have found up to that water-plane, then we should have a number of spots through which we shall be able to pass a fair curve if the calculations are correct.

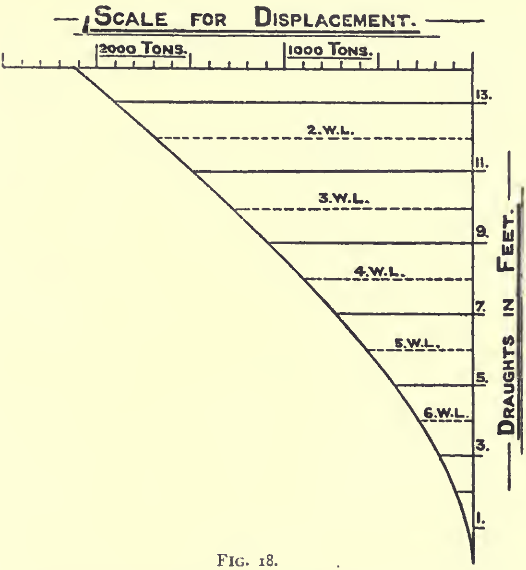

FIG. 18.

A curve obtained in this way is termed a "*curve of displacement*," and at any given mean draught we can measure the displacement of the vessel at that draught, and consequently know at once the total weight of the vessel with everything she has on board. This will not be quite accurate if the vessel is floating at a water-plane not parallel to the designed load water-plane. Fig. 18 gives a "curve of displacement" for a vessel, and the following calculation shows in detail the method of obtaining the information necessary to construct it.

The areas of a vessel's water-planes, two feet apart, are as follows :—

L.W.L.	... ...	7800 square feet.
2 W.L.	... ...	7450 ,,
3 W.L.	... ...	6960 ,,
4 W.L.	... ...	6290 ,,
5 W.L.	... ...	5460 ,,
6 W.L.	... ...	4320 ,,
7 W.L.	... ...	2610 ,,

The mean draught to the L.W.L. is 14′ 0″, and the displacement below the lowest W.L. is 71 tons.

To find the displacement to the L.W.L.

Number of W.L.	Area of water-plane.	Simpson's multipliers.	Function of area.
1	7800	1	7,800
2	7450	4	29,800
3	6960	2	13,920
4	6290	4	25,160
5	5460	2	10,920
6	4320	4	17,280
7	2610	1	2,610
			107,490

$\frac{1}{3}$ common interval $= \frac{1}{3} \times 2$

$\therefore$ displacement in cubic feet $= 107{,}490 \times \frac{2}{3}$

and displacement in tons, salt water $= 107{,}490 \times \frac{2}{105}$

$= 2047$ tons without the appendage

Next we require the displacement up to No. 2 W.L., and we subtract from the total the displacement of the layer between 1 and 2, which is found by using the *five-eight* rule as follows :—

Number of W.L.	Area of water-plane.	Simpson's multipliers.	Function of area.
1	7800	5	39,000
2	7450	8	59,600
3	6960	−1	−6,960
			91,640

Displacement in tons between No. 1 and No. 2 W.L.'s $= 91{,}640 \times \frac{2}{12} \times \frac{1}{35}$

$= 436$ tons nearly

∴ the displacement up to No. 2 W.L. is $2047 - 436$ $= 1611$ tons without the appendage

The displacement between 1 and 3 W.L.'s can be found by putting the areas of 1, 2 and 3 W.L.'s through Simpson's first rule, the result being 848 tons nearly.

∴ the displacement up to No. 3 W.L. is $2047 - 848$ $= 1199$ tons without the appendage

The displacement up to No. 4 W.L. can be obtained by putting the areas of 4, 5, 6, and 7 W.L.'s through Simpson's second rule, the result being—

819 tons without the appendage

The displacement up to No. 5 W.L. can be obtained by putting the areas of 5, 6, and 7 W.L.'s through Simpson's first rule, the result being—

482 tons without the appendage

The displacement up to No. 6 W.L. can be obtained by means of the five-eight rule, the result being—

201 tons without the appendage

Collecting the above results together, and adding in the appendage below No. 7 W.L., we have—

Displacement up to	L.W.L.	... ...	2118 tons.
,, ,,	2 W.L.	... ...	1682 ,,
,, ,,	3 W.L.	... ...	1270 ,,
,, ,,	4 W.L.	... ...	890 ,,
,, ,,	5 W.L.	... ...	553 ,,
,, ,,	6 W.L.	... ...	272 ,,
,, ,,	7 W.L.	... ...	71 ,,

These displacements, set out at the corresponding draughts, are shown in Fig. 18, and the fair curve drawn through forms the "*curve of displacement*" of the vessel. It is usual to complete the curve as indicated right down to the keel, although the ship could never float at a less draught than that given by the weight of her structure alone, or when she was launched.

Tons per Inch Immersion.—It is frequently necessary to know how much a vessel will sink, when floating at a given water-line, if certain known weights are placed on board, or how much she will rise if certain known weights are removed. Since the total displacement of the vessel must equal the weight of the vessel herself, the extra displacement caused by putting a weight on board must equal this weight. If A is the area

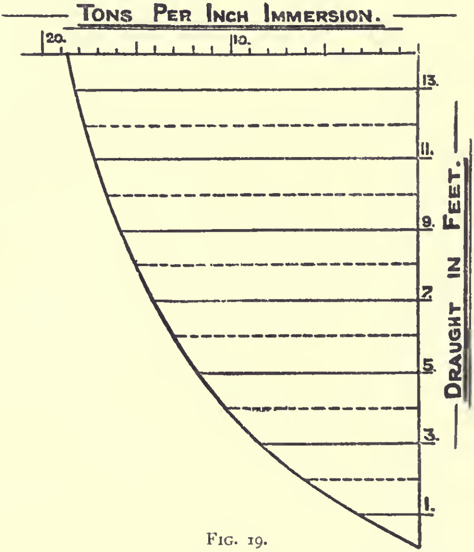

Fig. 19.

of a given water-plane in square feet, then the displacement of a layer 1 foot thick at this water-plane, supposing the vessel parallel-sided in its neighbourhood, is—

$$A \text{ cubic feet}$$

$$\text{or } \frac{A}{35} \text{ tons in salt water}$$

For a layer 1 inch thick only, the displacement is—

$$\frac{A}{35 \times 12} \text{ tons}$$

and this must be the number of tons we must place on board in order to sink the vessel 1 inch, or the number of tons we must take out in order to lighten the vessel 1 inch. This is termed the "*tons per inch immersion*" at the given water-line. This assumes that the vessel is parallel-sided at the water-line for the depth of 1 inch up and 1 inch down, which may, for all practical purposes, be taken as the case. If, then, we obtain the tons per inch immersion at successive water-planes parallel to the load water-plane, we shall be able to construct a "*curve of tons per inch immersion*" in the same way in which the curve of displacement was constructed. Such a curve is shown in Fig. 19, constructed for the same vessel for which the displacement curve was calculated. By setting up any mean draught, say 11 feet, we can measure off the "tons per inch immersion," supposing the vessel is floating parallel to the load water-plane; in this case it is $17\frac{1}{2}$ tons. Suppose this ship is floating at a mean draught of 11 feet, and we wish to know how much she will lighten by burning 100 tons of coal. We find, as above, the tons per inch to be $17\frac{1}{2}$, and the decrease in draught is therefore—

$$100 \div 17\tfrac{1}{2} = 5\tfrac{3}{4} \text{ inches nearly}$$

Curve of Areas of Midship Section.—This curve is usually plotted off on the same drawing as the displacement curve and the curve of tons per inch immersion. The ordinates of the immersed part of the midship section being known, we can calculate its area up to each of the water-planes in exactly the same way as the displacement has been calculated. These areas are set out on a convenient scale at the respective mean draughts, and a line drawn through the points thus obtained. If the calculations are correct, this should be a fair curve, and is known as "*the curve of areas of midship section.*" By means of this curve we are able to determine the area of the midship section up to any given mean draught.

Fig. 20 gives *the curve of areas of midship section* for the vessel for which we have already determined the displacement curve and the curve of tons per inch immersion.

Coefficient of Fineness of Midship Section.—If we draw a rectangle with depth equal to the draught of water at the midship section to top of keel, and breadth equal to the

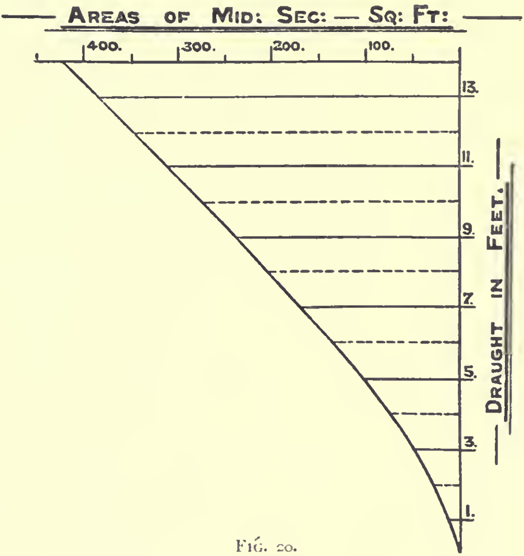

FIG. 20.

extreme breadth at the midship section, we shall obtain what may be termed the circumscribing rectangle of the immersed midship section. The area of the immersed midship section will be less than the area of this rectangle, and the ratio—

$$\frac{\text{area of immersed midship section}}{\text{area of its circumscribing rectangle}}$$

is termed the *coefficient of fineness of midship section.*

Example.—The midship section of a vessel is 68 feet broad at its broadest part, and the draught of water is 26 feet. The area of the immersed midship section is 1584 square feet. Find the coefficient of fineness of the midship section.

$$\begin{aligned} \text{Area of circumscribing rectangle} &= 68 \times 26 \\ &= 1768 \text{ square feet} \\ \therefore \text{coefficient} &= \tfrac{1584}{1768} = 0{\cdot}895 \end{aligned}$$

If a vessel of similar form to the above has a breadth at the midship section of 59′ 6″ and a draught of 22′ 9″, the area of its immersed midship section will be—

$$59\tfrac{1}{2} \times 22\tfrac{3}{4} \times 0{\cdot}895 = 1213 \text{ square feet}$$

The value of the midship section coefficient varies in ordinary ships from about 0·85 to 0·95, the latter value being for a section with very flat bottom.

Coefficient of Fineness of Water-plane.—This is the ratio between the area of the water-plane and its circumscribing rectangle.

The value of this coefficient for the load water-plane may be taken as follows :—

For ships with fine ends	0·7
For ships of ordinary form	0·75
For ships with bluff ends	0·85

Block Coefficient of Fineness of Displacement.—This is the ratio of the volume of displacement to the volume of a block having the same length between perpendiculars, extreme breadth, and mean draught as the vessel. The draught should be taken from the top of keel.

Thus a vessel is 380 feet long, 75 feet broad, with 27′ 6″ mean draught, and 14,150 tons displacement. What is its block coefficient of fineness or displacement?

$$\text{Volume of displacement} = 14{,}150 \times 35 \text{ cubic feet}$$

$$\text{Volume of circumscribing solid} = 380 \times 75 \times 27\tfrac{1}{2} \text{ cubic feet}$$

$$\therefore \left.\begin{array}{l}\text{coefficient of fineness of}\\ \text{displacement}\end{array}\right\} = \frac{14150 \times 35}{380 \times 75 \times 27\frac{1}{2}} = 0{\cdot}63$$

This coefficient gives a very good indication of the fineness of the underwater portion of a vessel, and can be calculated and tabulated for vessels with known speeds. Then, if in the early stages of a design we have the desired dimensions given, with the speed required, we can select the coefficient of fineness which appears most suitable for the vessel, and so determine very quickly the displacement that can be obtained under the conditions given.

Example.—A vessel has to be 400 feet long, 42 feet beam, 17 feet draught, and 13½ knots speed. What would be the probable displacement?

From available data, it would appear that a block coefficient of fineness of 0·625 would be desirable. Consequently the displacement would be—

$$(400 \times 42 \times 17 \times 0{\cdot}625) \div 35 \text{ tons} = 5100 \text{ tons about}$$

The following may be taken as average values of the block coefficient of fineness of displacement in various types of ships:—

Recent battleships	·60–·65
Recent fast cruisers	·50–·55
Fast mail steamers	·50–·55
Ordinary steamships	·55–·65
Cargo steamers	·65–·80
Sailing vessels	·65–·75
Steam-yachts	·35–·45

Prismatic Coefficient of Fineness of Displacement.—This coefficient is often used as a criterion of the fineness of the underwater portion of a vessel. It is the ratio between the volume of displacement and the volume of a prismatic solid the same length between perpendiculars as the vessel, and having a constant cross-section equal in area to the immersed midship section.

Example.—A vessel is 300 feet long, 2100 tons displacement, and has the area of her immersed midship section 425 square feet. What is her prismatic coefficient of fineness?

$$\text{Volume of displacement} = 2100 \times 35 \text{ cubic feet}$$
$$\text{Volume of prismatic solid} = 300 \times 425 \quad ,,$$
$$\therefore \text{coefficient} = \frac{2100 \times 35}{300 \times 425}$$
$$= 0{\cdot}577$$

Difference in Draught of Water when floating in Sea Water and when floating in River Water.—Sea water is denser than river water; that is to say, a given volume of sea water—say a cubic foot—weighs more than the same volume of river water. In consequence of this, a vessel, on passing from the river to the sea, if she maintains the same weight, will rise in the water, and have a greater freeboard than when she started. Sea water weighs 64 lbs. to the cubic foot, and the water in a river such as the Thames may be taken as weighing 63 lbs. to the cubic foot. In Fig. 21, let the right-hand portion represent the ship floating in river water, and the left-hand portion represent the ship floating in salt water. The difference between the two water-planes will be the amount the ship will rise on passing into sea water.

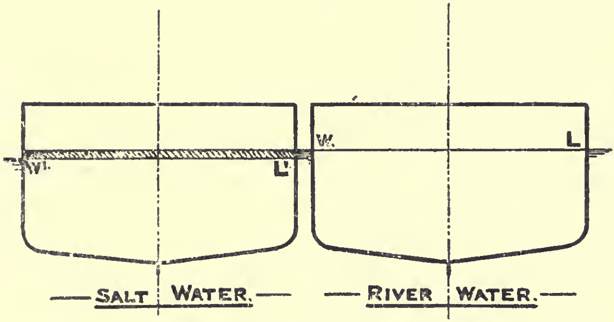

FIG. 21.

Let W = the weight of the ship in tons;

T = the tons per inch immersion at the water-line W′L′ in salt water;

x = the difference in draught between the water-lines WL, W′L′ in inches.

Then the volume of displacement—

$$\text{in river water} = \frac{W \times 2240}{63}$$

$$\text{in sea water} = \frac{W \times 2240}{64}$$

$$\therefore \text{ the volume of the layer} = \frac{W \times 2240}{63} - \frac{W \times 2240}{64}$$

$$= \frac{W \times 2240}{63 \times 64}$$

Now, the volume of the layer also $= x \times T \times \frac{2240}{64}$; therefore we have—

$$x \times T \times \frac{2240}{64} = \frac{W \times 2240}{63 \times 64}$$

$$\text{or } x = \frac{W}{63T} \text{ inches}$$

This may be put in another way. A ship, if floating in salt water, will weigh $\frac{1}{64}$ less than if floating to the same water-line in river water. Thus, if W is the weight of the ship floating at a given line in salt water, her weight if floating at the same line in fresh water is—

$$\frac{1}{64}\text{W less}$$

and this must be the weight of the layer of displacement between the salt-water line and the river-water line for a given weight W of the ship. If T be the tons per inch for salt water, the tons per inch for fresh water will be $\frac{63}{64}$T. Therefore the difference in draught will be—

$$\frac{1}{64}\text{W} \div \frac{63}{64}\text{T} = \frac{\text{W}}{63\text{T}} \text{ inches, as above}$$

Sinkage caused by a Central Compartment of a Vessel being open to the Sea.—Take the simple case of a box-shaped vessel, ABCD, Fig. 22, floating at the water-line WL.

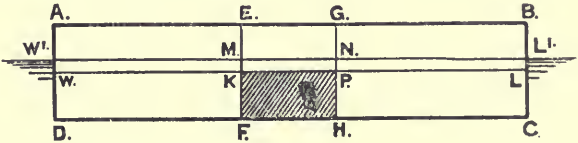

Fig. 22.

This vessel has two water-tight athwartship bulkheads in the middle portion, EF and GH. A hole is made in the bottom or side below water somewhere between these bulkheads. We will take a definite case, and work it out in detail to illustrate the principles involved in such a problem.

Length of box-shaped vessel	... ...	100 feet.
Breadth ,, ,,	... ...	20 ,,
Depth ,, ,,	... ...	20 ,,
Draught ,, ,,	... ...	10 ,,
Distance of bulkheads apart	... ...	20 ,,

If the vessel is assumed to be floating in salt water, its weight must be—

$$\frac{100 \times 20 \times 10}{35} = \frac{20000}{35} \text{ tons}$$

Now, this weight remains the same after the bilging as before, but the buoyancy has been diminished by the opening of the compartment KPHF to the sea. This lost buoyancy must be made up by the vessel sinking in the water until the volume of displacement is the same as it originally was. Suppose W′L′ to be the new water-line, then the new volume of displacement is given by the addition of the volumes of W′MFD and NL′CH, or, calling *d* the new draught of water in feet—

$$(40 \times 20 \times d) + (40 \times 20 \times d) = 1600d \text{ cubic feet}$$

The original volume of displacement was—

$$100 \times 20 \times 10 = 20{,}000 \text{ cubic feet}$$
$$\therefore 1600\, d = 20{,}000$$
$$\therefore d = \tfrac{200}{16} = 12'\ 6''$$

that is, the new draught of water is 12′ 6″, or the vessel will sink a distance of 2′ 6″.

The problem may be looked at from another point of view. The lost buoyancy is 20 × 20 × 10 cubic feet = 4000 cubic feet; this has to be made up by the volumes W′MKW and NL′LP, or the area of the intact water-plane multiplied by the increase in draught. Calling x the increase in draught, we shall have—

$$80 \times 20 \times x = 4000$$
$$x = \tfrac{4000}{1600} = 2\tfrac{1}{2} \text{ feet}$$
$$= 2'\ 6''$$

which is the same result as was obtained above.

If the bilged compartment contains stores, etc., the amount of water which enters from the sea will be less than if the compartment were quite empty. The volume of the lost displacement will then be given by the volume of the compartment up to the original water-line less the volume occupied by the stores.

Thus, suppose the compartment bilged in the above example to contain coal, stowed so that 44 cubic feet of it will weigh one ton, the weight of the solid coal being taken at 80 lbs. to the cubic foot.

1 cubic foot of coal, if solid, weighs 80 lbs.
1 „ „ as stowed „ $\frac{2240}{44} = 51$ lbs.

Therefore in every cubic foot of the compartment there is—

$\frac{51}{80}$ cubic feet solid coal
$\frac{29}{80}$ „ space into which water will find its way

The lost buoyancy is therefore—

$$\frac{29}{80} \times 4000 = 1450 \text{ cubic feet}$$

The area of the intact water-plane will also be affected in the same way; the portion of the water-plane between the bulkheads will contribute—

$$\frac{51}{80} \times 20 \times 20 = 255 \text{ square feet to the area}$$

The area of the intact waterplane is therefore—

$$1600 + 255 = 1855 \text{ square feet}$$

The sinkage in feet is therefore—

$$\frac{1450}{1855} = 0{\cdot}78, \text{ or } 9{\cdot}36 \text{ inches}$$

In the case of a ship the same principles apply, supposing the compartment to be a central one, and we have—

$$\left.\begin{array}{c}\text{Sinkage of vessel}\\ \text{in feet}\end{array}\right\} = \frac{\text{volume of lost buoyancy in cubic feet}}{\text{area of intact water-plane in square feet}}$$

In the case of a compartment bilged which is not in the middle of the length, change of the trim occurs. The method of calculating this for any given case will be dealt with in Chapter IV.

In the above example, if the transverse bulkheads EF and GH had stopped just below the new water-line W′L′, it is evident that the water would flow over their tops, and the vessel would sink. But if the tops were connected by a watertight flat, the water would then be confined to the space, and the vessel would remain afloat.

Velocity of Inflow of Water into a Vessel on Bilging.—

Let A = area of the hole in square feet;

d = the distance the centre of the hole below the surface in feet;

v = initial rate of inflow of the water in feet per second.

Then $v = 8\sqrt{d}$ nearly

and consequently the volume of water passing through the hole per second $= 8\sqrt{d} \times A$ cub. ft.

Thus, if a hole 2 square feet in area, 4 feet below the water-line, were made in the side of a vessel, the amount of water, approximately, that would flow into the vessel would be as follows:—

$$\text{Cubic feet per second} = 8 \times \sqrt{4} \times 2$$
$$= 32$$
$$\text{Cubic feet per minute} = 32 \times 60$$
$$\text{Tons of water per minute} = \frac{32 \times 60}{35}$$
$$= 54{\cdot}85$$

Weights of Materials.—The following table gives average weights which may be used in calculating the weights of materials employed in shipbuilding:—

Steel	490	lbs. per	cubic foot.
Wrought iron	480	,,	,,
Cast iron	445	,,	,,
Copper	550	,,	,,
Brass	530	,,	,,
Zinc	445	,,	,,
Gunmetal	528	,,	,,
Lead	712	,,	,,
Elm (English)	35	,,	,,
,, (Canadian)	45	,,	,,
Fir (Dantzic)	36	,,	,,
Greenheart	72	,,	,,
Mahogany	40–48	,,	,,
,, (for boats) ...	35	,,	,,
Oak (English)	52	,,	,,
,, (African)	62	,,	,,
Pine (Pitch)	40	,,	,,
,, (red)	36	,,	,,
,, (yellow)	30	,,	,,
Teak	50	,,	,,

It follows, from the weights per cubic foot of iron and steel given above, that an iron plate 1 inch thick weighs 40 lbs. per square foot, and a steel plate 1 inch thick weighs 40·8 lbs. per square foot.

The weight per square foot may be obtained for other thicknesses from these values, and we have the following :—

Thickness in inches.	Weight per square foot in pounds.	
	Iron.	Steel.
$\frac{1}{4}$	10	10·2
$\frac{3}{8}$	15	15·3
$\frac{1}{2}$	20	20·4
$\frac{5}{8}$	25	25·5
$\frac{3}{4}$	30	30·6
$\frac{7}{8}$	35	35·7
1	40	40·8

It is convenient to have the weight of steel per square foot when specified in one-twentieths of an inch, as is the case in Lloyd's rules—

Thickness in inches.	Weight per square foot in pounds.	Thickness in inches.	Weight per square foot in pounds.
$\frac{1}{20}$	2·04	$\frac{11}{20}$	22·44
$\frac{2}{20}$	4·08	$\frac{12}{20}$	24·48
$\frac{3}{20}$	6·12	$\frac{13}{20}$	26·52
$\frac{4}{20}$	8·16	$\frac{14}{20}$	28·56
$\frac{5}{20} = \frac{1}{4}$	10·20	$\frac{15}{20} = \frac{3}{4}$	30·60
$\frac{6}{20}$	12·24	$\frac{16}{20}$	32·64
$\frac{7}{20}$	14·28	$\frac{17}{20}$	34·68
$\frac{8}{20}$	16·32	$\frac{18}{20}$	36·72
$\frac{9}{20}$	18·36	$\frac{19}{20}$	38·76
$\frac{10}{20} = \frac{1}{2}$	20·40	$\frac{20}{20} = 1$	40·80

Examples to Chapter I.

1. A plate has the form shown in Fig. 23. What is its weight if its weight per square foot is 10 lbs.? *Ans.* 95 lbs.

2. The material of an armour plate weighs 490 lbs. a cubic foot. A certain plate is ordered 400 lbs. per square foot: what is its thickness?

Ans. 9·8 inches.

3. Steel armour plates, as in the previous question, are ordered 400 lbs. per square foot instead of 10 inches thick. What is the saving of weight per 100 square feet of surface of this armour?

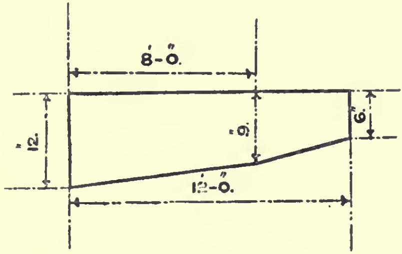

Fig. 23.

Ans. 833 lbs., or 0·37 ton.

4. An iron plate is of the dimensions shown in Fig. 24. What is its area? If two lightening holes 2′ 3″ in diameter are cut in it, what will its area then be?

Ans. $33\frac{3}{4}$ square feet; 25·8 square feet.

5. A hollow pillar is 4 inches external diameter and $\frac{3}{8}$ inch thick. What is its sectional area, and what would be the weight in pounds of 10 feet of this pillar if made of wrought iron?

Ans. 4·27 square inches; 142 lbs.

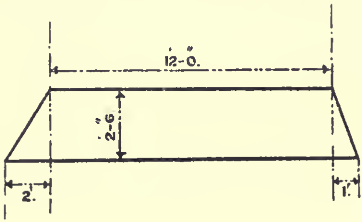

Fig. 24.

6. A steel plate is of the form and dimensions shown in Fig. 25. What is its weight? (A steel plate $\frac{5}{8}$ inch thick weighs 25·5 lbs. per square foot.)

Ans. 1267 lbs.

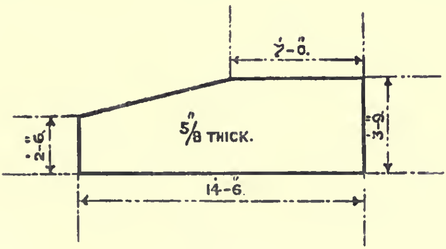

Fig. 25.

7. A wrought-iron armour plate is 15′ 3″ long, 3′ 6″ wide, and $4\frac{1}{2}$ inches thick. Calculate its weight in tons.

Ans. 4·29 tons.

8. A solid pillar of iron of circular section is 6′ 10″ long and $2\frac{1}{4}$ inches in diameter. What is its weight?

Ans. $90\frac{1}{2}$ lbs.

9. A Dantzic fir deck plank is 22 feet long and 4 inches thick, and tapers in width from 9 inches at one end to 6 inches at the other. What is its weight?

Ans. 165 lbs.

10. A solid pillar of iron is 7′ 3″ long and $2\frac{3}{4}$ inches diameter. What is its weight?

Ans. 143 lbs.

11. The total area of the deck plan of a vessel is 4500 square feet. What would be the surface of deck plank to be worked, if there are—

4 hatchways, each $4' \times 2\frac{1}{2}'$
2 ,, ,, $10' \times 6'$

and two circular skylights, each 4 feet in diameter, over which no plank is to be laid?

Ans. 4314·86 square feet.

12. A pipe is 6 inches diameter inside. How many cubic feet of water will a length of 100 feet of this pipe contain?

Ans. 19·6 cubic feet.

13. A mast 90 feet in length and 3 feet external diameter, is composed of 20 lb. plating worked flush-jointed on three T-bars, each $5'' \times 3'' \times 15\frac{1}{2}$ lbs. per foot. Estimate the weight, omitting straps, and rivet heads.

Ans. $9\frac{1}{2}$ tons nearly.

14. A curve has the following ordinates, 1′ 4″ apart: 10·86, 13·53, 14·58, 15·05, 15·24, 15·28, 15·22 feet respectively. Draw this curve, and find its area—

(1) By Simpson's first rule;
(2) By Simpson's second rule.

Ans. (1) 116·07 square feet; (2) 116·03 square feet.

15. The semi-ordinates in feet of a vessel's midship section, starting from the load water-line, are 26·6, 26·8, 26·8, 26·4, 25·4, 23·4, and 18·5 feet respectively, the ordinates being 3 feet apart. Below the lowest ordinate there is an area for one side of the section of 24·6 square feet. Find the area of the midship section, using—

(1) Simpson's first rule;
(2) Simpson's second rule.

Ans. (1) 961 square feet; (2) 960·7 square feet.

16. The internal dimensions of a tank for holding fresh water are $8' \, 0'' \times 3' \, 6'' \times 2' \, 6''$. How many tons of water will it contain?

Ans. 1·94.

17. The *half*-ordinates of a deck plan in feet are respectively $1\frac{1}{4}$, $5\frac{1}{2}$, $10\frac{1}{4}$, $13\frac{1}{2}$, $14\frac{3}{4}$, $14\frac{1}{2}$, $12\frac{1}{2}$, 9, and $3\frac{1}{2}$, and the length of the plan is 128 feet. Find the area of the deck plan in square yards.

Ans. 296.

18. Referring to the previous question, find the area in square feet of the portion of the plan between the ordinates $1\frac{1}{4}$ and $5\frac{1}{2}$.

Ans. 106·7.

19. The half-ordinates of the midship section of a vessel are 22·3, 22·2, 21·7, 20·6, 17·2, 13·2, and 8 feet in length respectively. The common interval between consecutive ordinates is 3 feet between the first and fifth ordinates, and 1′ 6″ between the fifth and seventh. Calculate the total area of the section in square feet.

Ans. 586·2 square feet.

20. Obtain the total area included between the first and fourth ordinates of the section given in the preceding question.

Ans. 392·8 square feet.

21. The semi-ordinates of the load water-plane of a vessel are 0·2, 3·6, 7·4, 10, 11, 10·7, 9·3, 6·5, and 2 feet respectively, and they are 15 feet apart. What is the area of the load water-plane?

Ans. 1808 square feet.

22. Referring to the previous question, what weight must be taken out of the vessel to lighten her $3\frac{1}{2}$ inches?

What additional immersion would result by placing 5 tons on board?

Ans. 15 tons; 1·16 inch.

23. The "tons per inch immersion" of a vessel when floating in salt water at a certain water-plane is 44·5. What is the area of this plane?

Ans. 18,690 square feet.

24. A curvilinear area has ordinates 3 feet apart of length 9·7, 10·0, and 13·3 feet respectively. Find—

(1) The area between the first and second ordinates.

(2) The area between the second and third ordinates.

(3) Check the addition of these results by finding the area of the whole figure by Simpson's first rule.

25. Assuming the truth of the five-eight rule for finding the area between two consecutive ordinates of a curve, prove the truth of the rule known as Simpson's first rule.

26. A curvilinear area has the following ordinates at equidistant intervals of 18 feet: 6·20, 13·80, 21·90, 26·40, 22·35, 14·70, and 7·35 feet. Assuming that Simpson's first rule is correct, find the percentage of error that would be involved by using—

(1) The trapezoidal rule;

(2) Simpson's second rule.

Ans. (1) 1·2 per cent.; (2) 0·4 per cent.

27. A compartment for containing fresh water has a mean section of the form shown in Fig. 26. The length of the compartment is 12 feet. How many tons of water will it contain?

Ans. 17 tons.

28. A compartment 20 feet long, 20 feet broad, and $8\frac{1}{4}$ feet deep, has to be lined with teak 3 inches in thickness. *Estimate* the amount of teak required in cubic feet, and in tons.

Ans. 365 cubic feet; 8·1 tons.

29. The areas of the water-line sections of a vessel in square feet are respectively 2000, 2000, 1600, 1250, and 300. The common interval between them is $1\frac{1}{2}$ foot. Find the displacement of the vessel in tons in salt water, neglecting the small portion below the lowest water-line section.

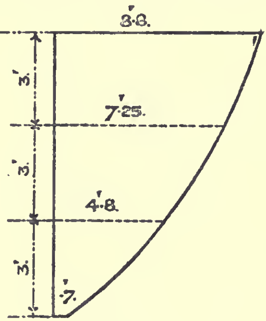

Fig. 26.

Ans. $264\frac{2}{7}$ tons.

30. A series of areas, 17′ 6″ apart, contain 0·94, 2·08, 3·74, 5·33, 8·27, 12·14, 16·96, 21·82, 24·68, 24·66, 22·56, 17·90, 12·66, 8·40, 5·69, 3·73, 2·61, 2·06, 0 square feet respectively. Find the volume of which the above are the sectional areas.

Ans. 3429 cubic feet.

31. Show how to estimate the change in the mean draught of a vessel in going from salt to river water, and *vice versâ*.

A vessel floats at a certain draught in river water, and when floating in sea water without any change in lading, it is found that an addition of 175 tons is required to bring the vessel to the same draught as in river water. What is the displacement after the addition of the weight named?

Ans. 11,200 tons.

32. The vertical sections of a vessel 10 feet apart have the following areas: 10, 50, 60, 70, 50, 40, 20 square feet. Find the volume of displacement, and the displacement in tons in salt and fresh water.

Ans. 2966 cubic feet; 84·7 tons, 82·4 tons.

33. A cylinder is 500 feet long, 20 feet diameter, and floats with the axis in the water-line. Find its weight when floating thus in salt water. What weight should be taken out in order that the cylinder should float with its axis in the surface if placed into fresh water?

Ans. 2244 tons; 62 tons.

34. A vessel is 500 feet long, 60 feet broad, and floats at a mean draught of 25 feet when in salt water. Make an approximation to her draught when she passes into river water. (Coefficient of displacement, 0·5; coefficient of L.W.P., 0·6.)

Ans. 25′ 4″.

35. A piece of teak is 20 feet long, 4½ inches thick, and its breadth tapers from 12 inches at one end to 9 inches at the other end. What is its weight, and how many cubic feet of water would it displace if placed into fresh water (36 cubic feet to the ton)?

Ans. 328 lbs.; 5¼ cubic feet nearly.

36. The area of a water-plane is 5443 square feet. Find the tons per inch immersion. Supposing 40 tons placed on board, how much would the vessel sink?

State any slight error that may be involved in any assumption made. If 40 tons were taken out, would the vessel rise the same amount? What further information would you require to give a more accurate answer?

Ans. 12·96 tons; 3·1 inches nearly.

37. Bilge keels are to be fitted to a ship whose tons per inch are 48. The estimated weight of the bilge keels is 36 tons, and the volume they occupy is 840 cubic feet. What will be the increase of draught due to fitting these bilge keels?

Ans. ¼ inch.

38. The tons per inch of a vessel at water-lines 2 feet apart are 19·45, 18·51, 17·25, 15·6, 13·55, 10·87, and 6·52, the lowest water-line being 18 inches above the underside of flat keel. Draw the curve of tons per inch immersion to scale, and estimate the number of tons necessary to sink the vessel from a draught of 12 feet to a draught of 13′ 6″.

Ans. 344 tons.

39. The steamship *Umbria* is 500 feet long, 57 feet broad, 22′ 6″ draught, 9860 tons displacement, 1150 square feet area of immersed midship section. Find—

(1) Block coefficient of displacement.
(2) Prismatic ,, ,,
(3) Midship-section coefficient.

Ans. (1) 0·538; (2) 0·6; (3) 0·896.

40. The steamship *Orient* is 445 feet long, 46 feet broad, 21′ 4½″ draught mean; the midship section coefficient is 0·919, the block coefficient of displacement is 0·621. Find—

(1) Displacement in tons.
(2) Area of immersed midship section.
(3) Prismatic coefficient of displacement.

Ans. (1) 7763 tons; (2) 904 square feet; (3) 0·675.

41. A vessel is 144 feet long, 22′ 6″ broad, 9 feet draught; displacement, 334 tons salt water; area of midship section, 124 square feet. Find—

(1) Block coefficient of displacement.
(2) Prismatic ,, ,,
(3) Midship-section coefficient.

Ans. (1) 0·4; (2) 0·655; (2) 0·612.

42. Find the displacement in tons in salt water, area of the immersed midship section, prismatic coefficient of displacement, having given the following particulars: Length, 168 feet; breadth, 25 feet; draught, 10′ 6″; midship-section coefficient, 0·87; block coefficient of displacement, 0·595.

Ans. 750 tons; 228·5 square feet; 0·685.

43. A vessel in the form of a box, 100 feet long, 10 feet broad, and 20 feet depth, floats at a draught of 5 feet. Find the draught if a central compartment 10 feet long is bilged below water.

Ans. 5′ $6\frac{2}{3}$″.

44. In a given ship, pillars in the hold can be either solid iron $4\frac{3}{4}$ inches diameter, or hollow iron 6 inches diameter and half inch thick. Find the saving in weight for every 100 feet length of these pillars, if hollow pillars are adopted instead of solid, neglecting the effect of the solid heads and heels of the hollow pillars.

Ans. 1·35 ton.

45. What is the solid contents of a tree whose girth (circumference) is 60 inches, and length is 18 feet?

Ans. 35·8 cubic feet nearly.

46. A portion of a cylindrical steel stern shaft casing is $12\frac{3}{4}$ feet long, $1\frac{1}{4}$ inch thick, and its external diameter is 14 inches. Find its weight in pounds.

Ans. 2170 lbs.

47. A floating body has a water-plane whose semi-ordinates 25 feet apart are 0·3, 8, 12, 10, 2 feet respectively, and every square station is in the form of a circle with its centre in the water-plane. Find the volume of displacement ($\pi = \frac{22}{7}$).

Ans. 12,414 cubic feet.

48. A quadrant of 16 feet radius is divided by means of ordinates parallel to one radius, and the following distances away: 4, 8, 10, 12, 13, 14, 15 feet respectively. The lengths of these ordinates are found to be 15·49, 13·86, 12·49, 10·58, 9·33, 7·75, and 5·57 feet respectively. Find—

(1) The exact area to two places of decimals.
(2) The area by using only ordinates 4 feet apart.
(3) The area by using also the half-ordinates.
(4) The area by using all the ordinates given above.
(5) The area as accurately as it is possible, supposing the ordinate 12·49 had not been given.

Ans. (1) 201·06; (2) 197·33; (3) 199·75; (4) 200·59; (5) 200·50.

49. A cylindrical vessel 50 feet long and 16 feet diameter floats at a constant draught of 12 feet in salt water. Using the information given in the previous question, find the displacement in tons.

Ans. 231 tons nearly.

50. A bunker 24 feet long has a mean section of the form of a trapezoid, with length of parallel sides 3 feet and 4·8 feet, and distance between them 10·5 feet. Find the number of tons of coal contained in the bunker, assuming

1 ton to occupy 43 cubic feet. If the parallel sides are perpendicular to one of the other sides, and the side 4·8 feet long is at the top of the section, where will the top of 17 tons of coal be, supposing it to be evenly distributed?

(This latter part should be done by a process of trial and error.)

Ans. 22·8 tons; 2′ 3″ below the top.

51. The sections of a ship are 20 feet apart. A coal-bunker extends from 9 feet abaft No. 8 section to 1 foot abaft No. 15 section, the total length of the bunker thus being 132 feet. The areas of sections of the bunker at Nos. 8, 11, and 15 are found to be 126, 177, and 145 square feet respectively. With this information given, estimate the capacity of the bunker, assuming 44 cubic feet of coal to go to the ton. Stations numbered from forward.

Ans. 495 tons.

52. The tons per inch immersion at water-lines 2 feet apart are 18·09, 16·80, 15·15, 13·15, 10·49, and 6·48. The draught of water to the top water-line is 11′ 6″, and below the lowest water-line there is a displacement of 75·3 tons. Find the displacement in tons, and construct a curve of displacement.

Ans. 1712 tons.

53. A tube 35 feet long, 16 feet diameter, closed at the ends, floats in salt water with its axis in the surface. Find approximately the thickness of the tube, supposed to be of iron, neglecting the weight of the ends.

Ans. 0·27 foot.

54. Find the floating power of a topmast, length 64 feet, mean diameter 21 inches, the wood of the topmast weighing 36 lbs. per cubic foot.

(The floating power of a spar is the weight it will sustain, and this is the difference between its own weight and that of the water it displaces. In constructing a raft, it has to be borne in mind that all the weight of human beings is to be placed *on* it, and that a great quantity of provisions and water may be safely carried *under* it. For instance, a cask of beef slung beneath would be 116 lbs., above 300 lbs. See "Sailor's Pocket-book," by Admiral Bedford.)

Ans. 4310 lbs.

CHAPTER II.

MOMENTS, CENTRE OF GRAVITY, CENTRE OF BUOYANCY, DISPLACEMENT TABLE, PLANIMETER, ETC.

Principle of Moments.—The *moment* of a force about any given line is the product of the force into the perpendicular distance of its line of action from that line. It may also be regarded as the *tendency to turn* about the line. A man pushes at the end of a capstan bar (as Fig. 27) with a

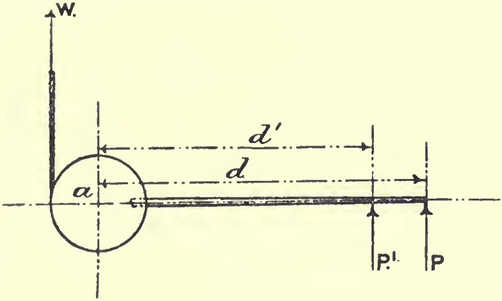

FIG. 27.

certain force. The tendency of the capstan to turn about its axis is given by the force exerted by the man multiplied by his distance from the centre of the capstan, and this is the *moment* of the force about the axis. If P is the force exerted by the man in pounds (see Fig. 27), and d is his distance from the axis in feet, then—

The moment about the axis $= P \times d$ foot-lbs.

The same moment can be obtained by a smaller force with a larger leverage, or a larger force with a smaller leverage, and the moment can be increased :—

(1) By increasing the force;

(2) By increasing the distance of the force from the axis.

If, in addition, there is another man helping the first man, exerting a force of P′ lbs. at a distance from the axis of d' feet, the total moment about the axis is—

$$(P \times d) + (P' \times d') \text{ foot-lbs.}$$

We must now distinguish between moments tending to turn one way and those tending to turn in the opposite direction.

Thus, in the above case, we may take a rope being wound on to the drum of the capstan, hauling a weight W lbs. If the radius of the drum be a feet, then the rope tends to turn the capstan in the opposite direction to the men, and the moment about the axis is given by—

$$W \times a \text{ foot-lbs.}$$

If the weight is just balanced, then there is no tendency to turn, and hence no moment about the axis of the capstan, and leaving out of account all consideration of friction, we have—

$$(P \times d) + (P' \times d') = W \times a$$

The most common forces we have to deal with are those caused by gravity, or the attraction of bodies to the earth. This is known as their weight, and the direction of these forces must all be parallel at any given place. If we have a number of weights, W_1, W_2, and W_3, on a beam at A, B, and C (Fig. 28),

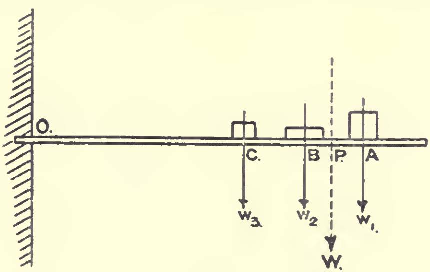

Fig. 28.

whose end is fixed at O, the moment of these weights about O is given by—

$$(W_1 \times AO) + (W_2 \times BO) + (W_3 \times CO)$$

This gives the tendency of the beam to turn about O, due to the weights W_1, W_2, and W_3 placed upon it, and the beam must be strong enough at O in order to resist this tendency, or, as it is termed, the *bending moment*. Now, we can evidently place a single weight W, equal to the sum of the weights W_1, W_2, and W_3, at some point on the beam so that its moment about O shall be the same as that due to the three weights. If P be this point, then we must have—

$$W \times OP = (W_1 \times OA) + (W_2 \times OB) + (W_3 \times OC)$$

or, since $W = W_1 + W_2 + W_3$

$$OP = \frac{(W_1 \times OA) + (W_2 \times OB) + (W_3 \times OC)}{W_1 + W_2 + W_3}$$

Example.—Four weights, 30, 40, 50, 60 lbs. respectively, are placed on a beam fixed at one end, O, at distances from O of 3, 4, 5, 6 feet respectively. Find the bending moment at O, and also the position of a single weight equal to the four weights which will give the same bending moment.

$$\begin{aligned} \text{Bending moment at O} &= (30 \times 3) + (40 \times 4) + (50 \times 5) + (60 \times 6) \\ &= 90 + 160 + 250 + 360 \\ &= 860 \text{ foot-lbs.} \\ \text{Total weight} &= 180 \text{ lbs.} \\ \therefore \text{position of single weight} &= \tfrac{860}{180} = 4\tfrac{7}{9} \text{ feet from O} \end{aligned}$$

Centre of Gravity.—The single weight W above, when placed at P, has the same effect on the beam at O and at any other point of the beam, as the three weights W_1, W_2, and W_3. The point P is termed the *centre of gravity* of the weights W_1, W_2, and W_3. Thus we may define the centre of gravity of a number of weights as follows:—

The centre of gravity of a system of weights is that point at which we may regard the whole system as being concentrated, and at which the same effect is produced as by the original system of weights.

This definition will apply to the case of a solid body, since we may regard it as composed of a very large number of small particles, each of which has a definite weight and occupies a definite position. A homogeneous solid has the same density throughout its volume; and all the solids with which we have to deal are taken as homogeneous unless otherwise specified.

It follows, from the above definition of the centre of gravity, that if a body is suspended at its centre of gravity, it would be perfectly balanced and have no tendency to move away from any position in which it might be placed.

To Find the Position of the Centre of Gravity of a number of Weights lying in a Plane.—Two lines are drawn in the plane at right angles, and the moment of the system of weights is found successively about each of these lines. The total weight being known, the distance of the centre of gravity from each of these lines is found, and consequently the position of the centre of gravity definitely fixed.

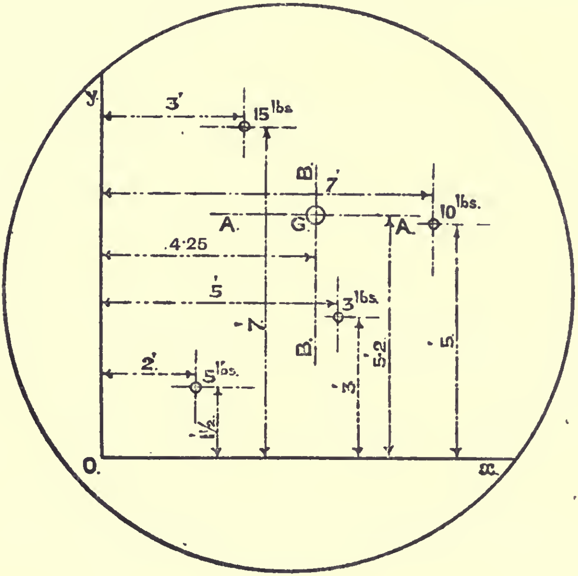

FIG. 29.

The following example will illustrate the principles involved: Four weights, of 15, 3, 10, and 5 lbs. respectively, are lying on a table in definite positions as shown in Fig. 29. Find the position of the centre of gravity of these weights. (If the legs of the table were removed, this would be the place where we should attach a rope to the table in order that it should remain horizontal, the weight of the table being neglected.)

Draw two lines, Ox, Oy, at right angles on the table in any convenient position, and measure the distances of each of the weights from Ox, Oy respectively: these distances are indicated in the figure. The total weight is 33 lbs. The moment of the weights about Ox is—

$$(15 \times 7) + (3 \times 3) + (10 \times 5) + (5 \times 1{\cdot}5) = 171{\cdot}5 \text{ foot-lbs.}$$

The distance of the centre of gravity from $Ox = \frac{171{\cdot}5}{33} = 5{\cdot}2$ feet

If we draw a line AA a distance of 5·2 feet from Ox, the centre of gravity of the weights must be somewhere in the line AA.

Similarly, we take moments about Oy, finding that the moment is 150 foot-lbs., and the distance of the centre of gravity from Oy is—

$$\tfrac{140}{33} = 4{\cdot}25 \text{ feet}$$

If we draw a line BB a distance of 4·55 feet from Oy, the centre of gravity of the weights must be somewhere in the line BB. The point G, where AA and BB meet, will be the centre of gravity of the weights.

Centres of Gravity of Plane Areas.—A plane area has length and breadth, but no thickness, and in order to give a definite meaning to what is termed its centre of gravity, the area is supposed to be the surface of a thin lamina or plate of homogeneous material of uniform thickness. With this supposition, the centre of gravity of a plane area is that point at which it can be suspended and remain in equilibrium.

Centres of Gravity of Plane Figures.

Circle.—The centre of gravity of a circle is obviously at its centre.

Square and Rectangle.—The centre of gravity of either of these figures is at the point where the diagonals intersect.

Rhombus and Rhomboid.—The centre of gravity of either of these figures is at the point where the diagonals intersect.

Triangle.—Take the triangle ABC, Fig. 30. Bisect any two sides BC, AC in the points D and E. Join AD, BE. The point G where these two lines intersect is the centre of gravity of the triangle. It can be proved that the point G is situated so that DG is one-third DA, and EG is one-third EB. We therefore have the following rules:—

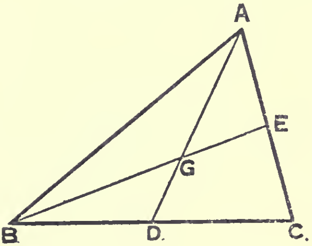

FIG. 30.

1. *Bisect any two sides of the triangle, and join the points thus obtained to the opposite angular points. Then the point in which these two lines intersect is the centre of gravity of the triangle.*

2. *Bisect any side of the triangle, and join the point thus obtained with the opposite angular point. The centre of gravity of the triangle will be on this line, and at a point at one-third its length measured from the bisected side.*

Trapezium.—Let ABCD, Fig. 31, be a trapezium. By joining the corners A and C we can divide the figure into two triangles, ADC, ABC. The centres of gravity, E and F, of these triangles can be found as indicated above. Join EF. The centre of gravity of the whole figure must be somewhere in the line EF. Again, join the corners D and B, thus dividing the figure into two triangles ADB, CDB. The centres of gravity, H and K, of these triangles can be found. The centre of gravity of the whole figure must be somewhere in the line HK; therefore the point G, where the lines HK and EF intersect, must be the centre of gravity of the trapezium.

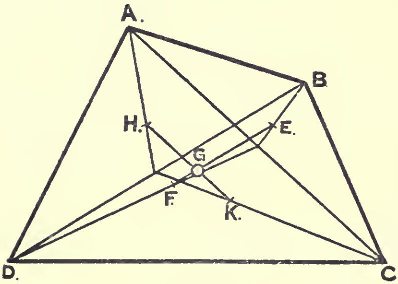

FIG. 31.

The following is a more convenient method of finding the centre of gravity of a trapezium.

Let ABCD, Fig. 32, be a trapezium. Draw the diagonals AC, BD, intersecting at E. In the figure CE is greater than

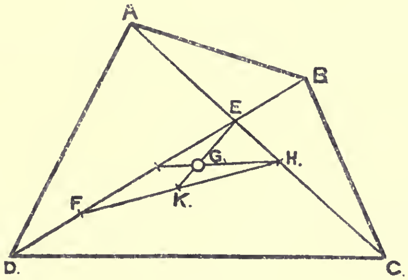

FIG. 32.

EA, and DE is greater than EB. Make CH = EA and DF = EB. Join FH. Then the centre of gravity of the triangle EFH will also be the centre of gravity of the trapezium ABCD.

(A useful exercise in drawing would be to take a trapezium on a large scale and find its centre of gravity by each of the above methods. If the drawing is accurately done, the point should be in precisely the same position as found by each method.)

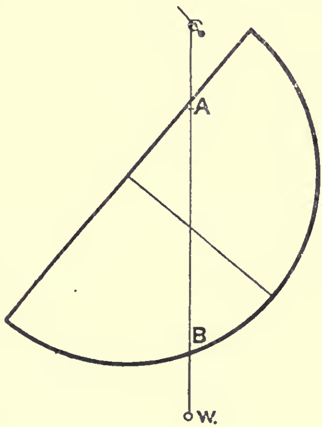

FIG. 33.

To find the Centre of Gravity of a Plane Area by Experiment.—Draw out the area on a piece of cardboard or stiff paper, and cut out the shape. Then suspend the cardboard as indicated in Fig. 33, a small weight, W, being allowed to hang plumb. A line drawn behind the string AW must pass through the centre of gravity. Mark on the cardboard two points on the string, as A and B, and join. Then the centre of gravity must lie on AB. Now suspend the cardboard by another point, C, as in Fig. 34, and draw the line CD immediately behind the string of the plumb-bob W. Then also the centre of gravity must lie on the line CD. Consequently it follows that the point of intersection G of the lines AB and CD must be the centre of gravity of the given area.

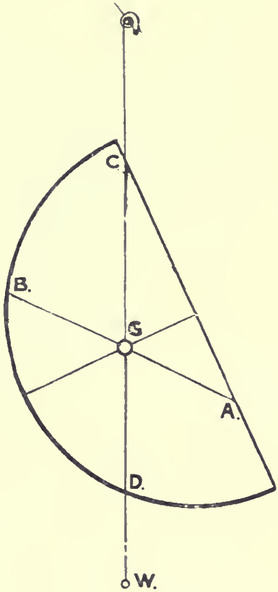

FIG. 34.

Example.—Set out the section of a beam on a piece of stiff paper, and find by experiment the position of its centre of gravity, the beam being formed of a bulb plate 9 inches deep and $\frac{1}{2}$ inch thick, having two angles on the upper edge, each $3'' \times 3'' \times \frac{1}{2}''$.

Ans. 3 inches from the top.

Centres of Gravity of Solids formed of Homogeneous Material.

Sphere.—The centre of gravity of a sphere is at its centre.

Cylinder.—The centre of gravity of a cylinder is at one-half its height from the base, on the line joining the centres of gravity of the ends.

Pyramid or Cone.—The centre of gravity of a pyramid or cone is at one-fourth the height of the apex from the base, on the line joining the centre of gravity of the base to the apex.

Moment of an Area.

The geometrical moment of a plane area relatively to a given axis, is the product of its area into the perpendicular distance of its centre of gravity from the given axis. It follows that the position of the centre of gravity is known relatively to the given axis if we know the geometrical moment about the axis and also the area, for the distance will be the moment divided by the area. It is usual to speak of the moment of an area about a given axis when the geometrical moment is really meant.

To find the Position of the Centre of Gravity of a Curvilinear Area with respect to one of its Ordinates.—Let AEDO, Fig. 35, be a plane curvilinear area, and

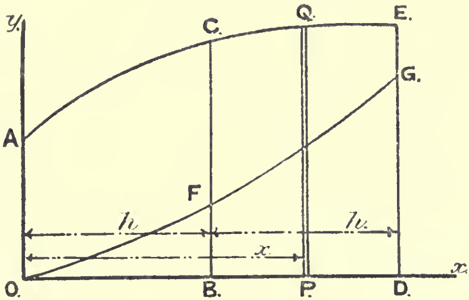

Fig. 35.

we wish to find its centre of gravity with respect to the end ordinate, OA. To do this, we must first find the moment of the total area about OA, and this divided by the area of the figure itself will give the distance of the centre of gravity from OA. Take any ordinate, PQ, a distance of x from OA, and at PQ draw a strip Δx wide. Then the area of the strip is $y \times \Delta x$ very nearly, and the moment of the strip about OA is $(y \times \Delta x)x$ very nearly.

If now Δx be made indefinitely small, the moment of the strip about OA will be—

$$y \,.\, x \,.\, dx$$

Now, we can imagine the whole area divided up into such strips, and if we added up the moments about OA of all such strips, we should obtain the total moment about OA. Therefore, using the notation we employed for finding the area of a plane curvilinear figure on p. 14, we shall have—

Moment of the total area about OA $= \int y \,.\, x \,.\, dx$

The expression for the *area* is—

$$\int y \,.\, dx$$

and this is of the same form as the expression for the moment. Therefore, instead of y we put yx through Simpson's rule in the ordinary way, and the result will be the moment about OA. Set off on BC a length BF = BC × h, and on DE a length DG = DE × $2h$. Then draw through all such points a curve, as OFG.[1] Any ordinate of this curve will give the ordinate of the original curve at that point multiplied by its distance from OA. The area of an elementary strip of this new curve will be $y \cdot x \cdot dx$, and the total area of the new curve will be $\int y \cdot x \cdot dx$, or the moment of the original figure about OA. Therefore, to find the moment of a curvilinear figure about an end ordinate, we take each ordinate and multiply it by its distance from the end ordinate. These products, put through Simpson's rule in the ordinary way, will give the moment of the figure about the end ordinate. This moment divided by the area will give the distance of the centre of gravity of the area from the end ordinate.

Example.—A midship section has semi-ordinates, 1′ 6″ apart, commencing at the L.W.L., of length 8·60, 8·10, 6·95, 4·90, 2·75, 1·50, 0·70 feet respectively. Find the area of the section and the distance of its C.G. from the L.W.L.

Number of ordinates.	Length of ordinates.	Simpson's multipliers.	Function of ordinates.	Number of intervals from No. 1 ord.	Products for moment.
1	8·60	1	8·60	0	0·0
2	8·10	4	32·40	1	32·40
3	6·95	2	13·90	2	27·80
4	4·90	4	19·60	3	58·80
5	2·75	2	5·50	4	22·00
6	1·50	4	6·00	5	30·00
7	0·70	1	0·70	6	4·20
			86·70	Function of moment	175·20

The half-area will be given by 86·70 × ($\frac{1}{3}$ × 1·5) = 43·35 square feet
and the whole area is 86·70 „

The arrangement above is adopted in order to save labour in

[1] If the original curve is a parabola of the second order, this curve will be one of the third order, and it can be proved that Simpson's first rule will integrate a parabola of the third order.

finding the moment of the area. In the fourth column we have the functions of the ordinates, or the ordinates multiplied successively by their proper multipliers. In the fifth column is placed, not the actual distance of each ordinate from the No. 1 ordinate, but the *number of intervals* away, and the distance apart is brought in at the end. In the sixth column the products of the functions in column 4 and the multipliers in column 5 are placed. It will be noticed that we have put the ordinates through Simpson's multipliers first, and then multiplied by the numbers in the fifth column after. This is the reverse to the rule given in words above, which was put into that form in order to bring out the principle involved more plainly. The final result will, of course, be the same in either case, the method adopted giving the result with the least amount of labour, because column 4 is wanted for finding the area. The sum of the products in column 6 will not be the moment required, because it has to be multiplied as follows: First, by one-third the common interval, and second, by the distance apart of the ordinates.

$$\left.\begin{array}{l}\text{The moment of the half-area}\\ \quad\text{about the L.W.L.}\end{array}\right\} = 175\cdot20 \times (\tfrac{1}{3} \times 1\tfrac{1}{2}) \times 1\tfrac{1}{2}$$
$$= 131\cdot4$$

and the distance of the C.G. of the half-area from the L.W.L. is—

$$\text{Moment} \div \text{area} = \frac{131\cdot4}{43\cdot35} = 3\cdot03 \text{ feet}$$

It will be noticed that we have multiplied both columns 4 and 6 by one-third the common interval, the distance of the C.G. from No. 1 ordinate being obtained by—

$$\frac{175\cdot20 \times (\frac{1}{3} \times 1\cdot5) \times 1\cdot5}{86\cdot70 \times (\frac{1}{3} \times 1\cdot5)}$$

The expression $\frac{1}{3} \times 1\cdot5$ is common to both top and bottom, and so can be cancelled out, and we have—

$$\frac{175\cdot20 \times 1\cdot5}{86\cdot70} = 3\cdot03 \text{ feet}$$

The position of the centre of gravity of the half-area with regard to the L.W.L. is evidently the same as that of the whole area.

When finding the centre of gravity of a large area, such as a water-plane of a vessel, it is usual to take moments about the middle ordinate. This considerably simplifies the work, because the multipliers in column 5 are not so large.

Example.—The semi-ordinates of the load water-plane of a vessel 395 feet long are, commencing from forward, 0, 10·2, 20·0, 27·4, 32·1, 34·0, 33·8, 31·7, 27·6, 20·6, 9·4. Find the area and the distance of its C.G. from the middle ordinate.

In addition to the above, there is an appendage abaft the last ordinate, having an area of 153 square feet, and whose C.G. is 5·6 feet abaft the last ordinate. Taking this appendage into account, find the area and the position of the C.G. of the water-plane.

Number of ordinates.	Length of ordinates.	Simpson's multipliers.	Function of ordinates.	Number of interval from mid. ord.	Product for moment.
1	0·0	1	0·0	5	0·0
2	10·2	4	40·8	4	163·2
3	20·0	2	40·0	3	120·0
4	27·4	4	109·6	2	219·2
5	32·1	2	64·2	1	64·2
6	34·0	4	136·0	0	566·6
7	33·8	2	67·6	1	67·6
8	31·7	4	126·8	2	253·6
9	27·6	2	55·2	3	165·6
10	20·6	4	82·4	4	329·6
11	9·4	1	9·4	5	47·0
			732·0		863·4

The half-area will be given by—

$$732·0 \times (\tfrac{1}{3} \times 39·5) = 9638 \text{ square feet}$$

The fifth column gives the number of intervals away from the middle ordinate, and the products are obtained for the forward portion adding up to 566·6, and they are obtained for the after portion adding up to 863·4. This gives an excess aft of 863·4 − 566·6 = 296·8. The distance of the C.G. abaft the middle ordinate is then given by—

$$\frac{296·8 \times 39·5}{732·0} = 16·01 \text{ feet}$$

The area of both sides is 19,276 square feet.

The second part of the question takes into account an appendage abaft No. 11 ordinate, having an area of 153 square feet.

The total area will then be—

$$19{,}276 + 153 = 19{,}429 \text{ square feet}$$

To find the position of the C.G. of the whole water-plane, we take moments about No. 6 ordinate, the distance of the C.G. of the appendage from it being—

$$197·5 + 5·6 = 203·1 \text{ feet}$$

Moment of main area abaft No. 6 ordinate $= 19{,}276 \times 16·01 = 308{,}609$

,, appendage ,, ,, $= 153 \times 203·1 = 31·074$

∴ total moment abaft No. 6 ordinate $= 308{,}609 + 31{,}074$

$= 339{,}683$

and the distance of the centre of gravity of the whole area abaft No. 6 ordinate $= \dfrac{339683}{19429} = 17·48$ feet

To find the Position of the Centre of Gravity of a Curvilinear Area contained between Two Consecutive Ordinates with respect to the Near End Ordinate.—The rule investigated in the previous paragraph for finding the centre of gravity of an area about its end ordinate fails when applied to such a case as the above. For instance, try the following example :—

A curve has ordinates 10, 9, 7 feet long, 4 feet apart. To find the position of the centre of gravity of the portion between the two first ordinates with respect to the end ordinate.

Ordinates.	Simpson's multipliers.	Functions.	Multipliers for moment.	Products for moment.
10	5	50	0	0
9	8	72	1	72
7	−1	−7	2	−14
		115		58

Centre of gravity from the end ordinate would be—

$$\frac{58 \times 4}{115} = 2\tfrac{2}{115} \text{ feet}$$

Now this is evidently wrong, since the shape of the curve is such that the centre of gravity ought to be slightly less than 2 feet from the end ordinate.

We must use the following rule :—

To ten times the middle ordinate add three times the near end ordinate and subtract the far end ordinate. Multiply the *remainder by one-twenty-fourth the square of the common interval. The product will be the moment about the end ordinate.*

Using y_1, y_2, y_3, for the lengths of the ordinates, and h the common interval, the moment of the portion between the ordinates y_1 and y_2 about the ordinate y_1 is given by—

$$\frac{h^2}{24}(3y_1 + 10y_2 - y_3)$$

We will now apply this rule to the case considered above.

Ordinates.	Area.		Moment.	
	Simpson's multipliers.	Functions.	Simpson's multipliers.	Functions.
10	5	50	3	30
9	8	72	10	90
7	− 1	7	− 1	− 7
		115		113

$$\text{Moment} = 113 \times \tfrac{16}{24}$$
$$\text{Area} = 115 \times \tfrac{4}{12}$$

Therefore distance of the centre of gravity from the end ordinate is—

$$\frac{113 \times \frac{16}{24}}{115 \times \frac{4}{12}} = \frac{113 \times 2 \times 3}{115 \times 3}$$
$$= \tfrac{226}{115} = 1{\cdot}965 \text{ feet}$$

This result is what one might expect by considering the shape of the curve.

To find the Position of the Centre of Gravity of a Curvilinear Area with respect to its Base.—Let DABC, Fig. 36, be a plain curvilinear area. We wish to find the distance of its centre of gravity from the base DC. To do this, we must first find the moment of the figure about DC and divide it by the area. Take any ordinate PQ, and at PQ draw a consecutive ordinate giving a strip Δx wide. Then the area of the strip is—

$$y \times \Delta x \text{ very nearly}$$

and regarding it as a rectangle, its centre of gravity is at a distance of $\frac{1}{2}y$ from the base. Therefore the moment of the strip about the base is—

$$\tfrac{1}{2}y^2 \times \Delta x$$

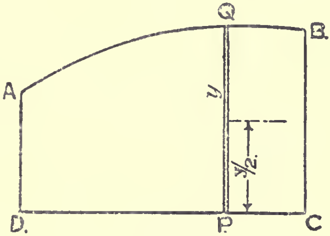

FIG. 36.

If now we consider the strip to be indefinitely thin, its moment about the base will be—

$$\tfrac{1}{2}y^2 \,.\, dx$$

and the moment of the total area about the base must be the sum of the moments of all such strips, or—

$$\int \tfrac{1}{2}y^2 \,.\, dx$$

This expression for the moment is of the same form as that for the area, viz. $\int y \,.\, dx$. Therefore, instead of y we put $\frac{1}{2}y^2$[1] through Simpson's rule in the ordinary way, and the result will be the moment of the curve about DC.

Example.—An athwartship coal-bunker is 6 feet long in a fore-and-aft direction. It is bounded at the sides by two longitudinal bulkheads 34 feet apart, and by a horizontal line at the top. The bottom is formed by the inner bottom of the ship, and is in the form of a curve having vertical ordinates measured from the top of 12·5, 15·0, 16·0, 16·3, 16·4, 16·3, 16·0, 15·0, 12·5 feet respectively, the first and last ordinates being on the bulkheads. Find—

(1) The number of tons of coal the bunker will hold.

(2) The distance of the centre of gravity of the coal from the top.

The inner bottom is symmetrical either side of the middle line, so we need only deal with one side. The work is arranged as follows:—

Ordinates.	Simpson's multipliers.	Functions of ordinates.	Squares of ordinates.	Simpson's multipliers.	Functions of squares.
16·4	1	16·4	269	1	269
16·3	4	65·2	266	4	1064
16·0	2	32·0	256	2	512
15·0	4	60·0	225	4	900
12·5	1	12·5	156	1	156
		Function of area 186·1			Function of moment 2901

[1] This assumes that Simpson's first rule, which will most probably be used, will correctly integrate a parabola of the fourth order, which can be shown to be the case for all practical purposes.

$$\text{Common interval} = 4{\cdot}25 \text{ feet}$$
$$\text{Half-area of section} = 186{\cdot}1 \times \tfrac{1}{3} \times 4{\cdot}25 \text{ square feet}$$
$$\text{Volume of bunker} = 186{\cdot}1 \times \frac{4{\cdot}25 \times 2 \times 6}{3} \text{ cubic feet}$$
$$\text{Number of tons of coal} = 186{\cdot}1 \times \tfrac{17}{44}$$
$$= 72 \text{ tons}$$
$$\text{Moment of half-area below top} = 2901 \times \frac{1}{2} \times \frac{4{\cdot}25}{3}$$
$$\text{And distance of C.G. from the top} = \frac{\text{moment}}{\text{area}}$$
$$= \frac{2901 \times \frac{1}{2} \times \frac{4{\cdot}25}{3}}{186{\cdot}1 \times \frac{4{\cdot}25}{3}}$$
$$= 7{\cdot}8 \text{ feet}$$

In the first three columns we proceed in the ordinary way for finding the area. In the fourth column is placed, not the half-squares, but the squares of the ordinates in column 1, the multiplication by $\frac{1}{2}$ being brought in at the end. These squares are then put through Simpson's multipliers, and the addition of column 6 will give a function of the moment of the area about the base. This multiplied by $\frac{1}{2}$ and by $\frac{1}{3}$ the common interval gives the actual moment. This moment divided by the area gives the distance of the centre of gravity we want. It will be noticed that $\frac{1}{3}$ the common interval comes in top and bottom, so that we divide the function of the moment 2901 by the function of the area 186·1, and then multiply by $\frac{1}{2}$ to get the distance of centre of gravity required.

It is not often required in practice to find the centre of gravity of an area with respect to its base, because most of the areas we have to deal with are symmetrical either side of a centre line (as water-planes), but the problem sometimes occurs, the question above being an example.

To find the Position of the Centre of Gravity of an Area bounded by a Curve and Two Radii.—We have already seen (p. 15) how to find the area of a figure such as this. It is simply a step further to find the position of the centre of gravity with reference to either of the bounding radii. Let OAB, Fig. 13, be a figure bounded by a curve, AB, and two bounding radii, OA, OB. Take any radius OP, the angle BOP being called θ, and the length of OP being called r.

Draw a consecutive radius, OP′; the angle POP′ being indefinitely small, we may call it $d\theta$. Using the assumptions we have already employed in finding areas, the area POP′ = $\frac{1}{2}r^2 . d\theta$, POP′ being regarded as a triangle. The centre of gravity of POP′ is at g, and $Og = \frac{2}{3}r$, and gm is drawn perpendicular to OB, and $gm = \frac{2}{3}r . \sin\theta$ (see p. 87).

$$\left.\begin{array}{l}\text{The moment of the area}\\ \text{POP}' \text{ about OB}\end{array}\right\} = (\tfrac{1}{2}r^2 . d\theta) \times (\tfrac{2}{3}r . \sin\theta)$$
$$= \tfrac{1}{3}r^3 . \sin\theta . d\theta$$

The moment of the whole figure about OB is the sum of the moments of all such small areas as POP′, or, using the ordinary rotation—

$$\tfrac{1}{3}\int r^3 . \sin\theta . d\theta$$

This is precisely similar in form to the expression we found for the area of such a figure as the above (see p. 15), viz.—

$$\tfrac{1}{2}\int r^2 . d\theta$$

so that, instead of putting $\frac{1}{2}r^2$ through Simpson's rule, measuring r at equidistant angular intervals, we put $\frac{1}{3}r^3 . \sin\theta$ through the rule in a similar way. This will be best illustrated by the following example:—

Example.—Find the area and position of centre of gravity of a quadrant of a circle with reference to one of its bounding radii, the radius being 10 feet.

We will divide the quadrant by radii 15° apart, and thus be able to use Simpson's first rule.

Number of radius.	Length of radius.	(Radius)^2.	Simpson's multipliers.	Product for area.	(Radius)^3.	Angle from first radius.	Sin of angle.	Product $r^3 \times \sin\theta$.	Simpson's multipliers.	Product for moment.
1	10	100	1	100	1000	0	0·0	0	1	0
2	10	100	4	400	1000	15	0·258	258	4	1,032
3	10	100	2	200	1000	30	0·500	500	2	1,000
4	10	100	4	400	1000	45	0·707	707	4	2,828
5	10	100	2	200	1000	60	0·866	866	2	1,732
6	10	100	4	400	1000	75	0·965	965	4	3,860
7	10	100	1	100	1000	90	1·000	1000	1	1,000

Function of area 1800 Function of moment 11,452

The circular measure of $180° = \pi = 3{\cdot}1416$

$$,, \quad ,, \quad 15° = \frac{3{\cdot}1416}{12}$$

$$\therefore \text{ area} = 1800 \times \tfrac{1}{2} \times \left(\frac{1}{3} \times \frac{3{\cdot}1416}{12}\right)$$

$$= 78{\cdot}54 \text{ square feet}$$

$$\text{Moment of area about the first radius} = 11{,}452 \times \frac{1}{3} \times \left(\frac{1}{3} \times \frac{3{\cdot}1416}{12}\right)$$

therefore distance of centre of gravity from the first radius is—

$$\text{Moment} \div \text{area} = \frac{11{,}452 \times \frac{1}{3} \times \left(\frac{1}{3} \times \frac{3{\cdot}1416}{12}\right)}{1800 \times \frac{1}{2} \times \left(\frac{1}{3} \times \frac{3{\cdot}1416}{12}\right)}$$

$$= \frac{11452 \times 2}{1800 \times 3} = 4{\cdot}24 \text{ feet}$$

The exact distance of the centre of gravity of a quadrant from either of its bounding radii is $\frac{4}{3\pi}$ times the radius, and if this is applied to the above example, it will be found that the result is correct to two places of decimals, and would have been more correct if we had put in the values of the sines of the angles to a larger number of decimal places.

Centre of Gravity of a Solid Body which is bounded by a Curved Surface and a Plane.—In the first chapter we saw that the finding the volume of such a solid as this was similar in principle to the finding the area of a plane curve, the only difference being that we substitute areas for simple ordinates, and as a result get the volume required. The operation of finding the centre of gravity of a volume in relation to one of the dividing planes is precisely similar to the operation of finding the centre of gravity of a curvilinear area in relation to one of its ordinates. This will be illustrated by the following example:—

Example.—A coal-bunker has sections 17′ 6″ apart, and the areas of these sections, commencing from forward, are 98, 123, 137, 135, 122 square feet respectively. Find the volume of the bunker, and the position of its centre of gravity in a fore-and-aft direction.

Areas.	Simpson's multipliers.	Functions of areas.	Number of intervals from forward.	Products for moments.
98	1	98	0	0
123	4	492	1	492
137	2	274	2	548
135	4	540	3	1620
122	1	122	4	488
		1526		3148

$$\text{Volume} = 1526 \times \tfrac{1}{3} \times 17\tfrac{1}{2} = 8902 \text{ cubic feet}$$

$$\text{moment} = 3148 \times \tfrac{1}{3} \times 17\tfrac{1}{2} \times 17\tfrac{1}{2}$$

$$\therefore \left.\begin{array}{l}\text{distance of centre of gravity}\\ \text{from forward end}\end{array}\right\} = \frac{3148 \times 17\cdot5}{1526} = 36\cdot2 \text{ feet}$$

It is always advisable to roughly check any result such as this; and if this habit is formed, it will often prevent mistakes being made. The total length of this bunker is $4 \times 17' \ 6'' = 70$ feet, and the areas of the sections show that the bunker is fuller aft than forward, and so, on the face of it, we should expect the position of the centre of gravity to be somewhat abaft the middle of the length; and this is shown to be so by the result of the calculation. Also as regards the volume. This must be less than the volume of a solid 70 feet long, and having a constant section equal to the area of the middle section of the bunker. The volume of such a body would be $70 \times 137 = 9590$ cubic feet. The volume, as found by the calculation, is 8902 cubic feet, thus giving a coefficient of $\frac{8902}{9590} = 0{\cdot}93$ nearly, which is a reasonable result to expect.

Centre of Buoyancy.—The centre of buoyancy of a vessel is the centre of gravity of the underwater volume, or, more simply, the centre of gravity of the displaced water. This has nothing whatever to do with the centre of gravity of the ship herself. The centre of buoyancy is determined solely by the shape of the underwater portion of the ship. The centre of gravity of the ship is determined by the distribution of the weights forming the structure, and of all the weights she has on board. Take the case of two sister ships built from the same lines, and each carrying the same weight of cargo and floating at the same water-line. The centre of buoyancy of each of these ships must necessarily be in the same position. But suppose they are engaged in different trades—the first, say, carrying a cargo of steel rails and other heavy weights, which are stowed low down. The second, we may suppose, carries a cargo of homogeneous materials, and this has to be stowed much higher than the cargo in the first vessel. It is evident that the centre of gravity in the first vessel must be much lower down than in the second, although as regards form they are precisely similar. This distinction between the centre of buoyancy and the centre of gravity is a very important one, and should always be borne in mind.

To find the Position of the Centre of Buoyancy of a Vessel in a Fore-and-aft Direction, having given the Areas of Equidistant Transverse Sections.—The following example will illustrate the principles involved :—

Example.—The underwater portion of a vessel is divided by transverse sections 10 feet apart of the following areas, commencing from forward : 0·2, 22·7, 48·8, 73·2, 88·4, 82·8, 58·7, 26·2, 3·9 square feet respectively. Find the position of the centre of buoyancy relative to the middle section.

Number of station.	Area of section.	Simpson's multipliers.	Functions of area.	Number of intervals from middle.	Product for moment.
1	0·2	1	0·2	4	0·8
2	22·7	4	90·8	3	272·4
3	48·8	2	97·6	2	195·2
4	73·2	4	292·8	1	292·8
					761·2
5	88·4	2	176·8	0	
6	82·8	4	331·2	1	331·2
7	58·7	2	117·4	2	234·8
8	26·2	4	104·8	3	314·4
9	3·9	1	3·9	4	15·6

Function of displacement 1215·5   Function of moment } 896·0

$$\text{Volume of displacement} = 1215 \cdot 5 \times \tfrac{10}{3}$$
$$\text{excess of products aft} = 896 \cdot 0 - 761 \cdot 2 = 134 \cdot 8$$
$$\text{moment aft} = 134 \cdot 8 \times \tfrac{10}{3} \times 10$$

$$\text{C.B. abaft middle} = \frac{134 \cdot 8 \times \frac{10}{3} \times 10}{1215 \cdot 5 \times \frac{10}{3}}$$
$$= \frac{134 \cdot 8 \times 10}{1215 \cdot 5} = 1 \cdot 11 \text{ feet}$$

The centre of gravity of a plane area is fully determined when we know its position relative to two lines in the plane, which are generally taken at right angles to one another. The centre of gravity of a volume is fully determined when we know its position relative to three planes, which are generally taken at right angles to one another. In the case of the underwater volume of a ship, we need only calculate the position of its centre of gravity relative to (1) the load water-plane, and (2) an athwartship section (usually the section amidships), because, the two sides of the ship being identical, the centre of gravity of the displacement must lie in the middle-line longitudinal plane of the ship.

Approximate Position of the Centre of Buoyancy.— In vessels of ordinary form, it is found that the distance of the centre of buoyancy below the L.W.L. varies from about $\frac{8}{20}$ to $\frac{9}{20}$ of the mean moulded draught, the latter being the case in vessels of full form. For yachts and vessels of unusual form, such a rule as this cannot be employed.

Example.—A vessel 13′ 3″ mean draught has her C.B. 5·34 feet below L.W.L.

Here the proportion of the draught is—

$$\frac{5\cdot34}{13\cdot25} = 0\cdot403 = \frac{8\cdot06}{20}$$

This is an example of a fine vessel.

Example.—A vessel 27′ 6″ mean draught has her C.B. 12·02 feet below L.W.L.

Here the proportion of the draught is—

$$\frac{12\cdot02}{27\cdot5} = \frac{8\cdot75}{20}$$

This is an example of a fuller vessel than the first case.

Normand's Approximate Formula for the Distance of the Centre of Buoyancy below the Load Waterline.[1]

Let V = volume of displacement up to the load-line in cubic feet;

A = the area of the load water-plane in square feet;

d = the mean draught (to top of keel) in feet.

[1] See a paper in *Transactions of the Institution of Naval Architects*, by Mr. S. W. F. Morrish, M.I.N.A., in 1892.

Then centre of buoyancy below L.W.L. $= \frac{1}{3}\left(\frac{d}{2} + \frac{V}{A}\right)$

This rule gives exceedingly good results for vessels of ordinary form. In the early stages of a design the above particulars would be known as some of the elements of the design, and so the vertical position of the centre of buoyancy can be located very nearly indeed. In cases in which the stability of the vessel has to be approximated to, it is important to know where the C.B. is, as will be seen later when we are dealing with the question of stability.

The rule is based upon a very ingenious assumption, and the proof is given in Appendix A, p. 249.

The Area of a Curve of Displacement divided by the Load Displacement gives the Distance of the Centre of Buoyancy below the Load Water-line.—This is an interesting property of the curve of displacement. A demonstration of it will be found in Appendix A, p. 247.

Displacement Sheet.—We now proceed to investigate the method that is very generally employed in practice to find the displacement of a vessel, and also the position of its centre of buoyancy both in a longitudinal and a vertical direction. The calculation is performed on what is termed a "*Displacement Sheet*" or "*Displacement Table*," and a specimen calculation is given at the end of the book for a single-screw tug of the following dimensions:—

Length between perpendiculars	75′ 0″
Breadth moulded	14′ 6″
Depth moulded	8′ 3″
Draught moulded forward	5′ 5″
,, ,, aft	6′ 2″
,, ,, mean	5′ 9½″

The sheer drawing of the vessel is given on Plate I. This drawing consists of three portions—the body plan, the half-breadth plan, and the sheer. The sheer plan shows the ship in side elevation, the load water-line being horizontal, and the keel, in this case, sloping down from forward to aft. The ship is supposed cut by a number of transverse vertical planes, which are shown in the sheer plan as straight lines, numbered

1, 2, 3, etc. Now, each of these transverse sections of the ship has a definite shape, and the form of each half-section to the outside of frames is shown in the body-plan, the sections being numbered as in the sheer. The sections of the forward end form what is termed the "*fore-body*," and those of the after end the "*after-body*." Again, the ship may be supposed to be cut by a series of equidistant horizontal planes, of which the load water-plane is one. The shape of the curve traced on each of these planes by the moulded surface of the ship is given in the half-breadth plan, and the curves are numbered A, 1, 2, 3, etc., to agree with the corresponding lines in the sheer and body plan. Each of these plans must agree with the other two. Take a special station, for example, No. 4. The breadth of the ship at No. 4 station at the level of No. 3 water-plane is Oa' in the body-plan, but it is also given in the half-breadth plan by Oa, and therefore Oa must exactly equal Oa'. The process of making all such points correspond exactly is known as "*fairing*." For full information as to the methods adopted in fairing, the student is referred to the works on "Laying-off" given below.[1] For purposes of reference, the dimensions of the vessel and other particulars are placed at the top of the displacement sheet. The water-lines are arranged on the sheer drawing with a view to this calculation, and in this case are spaced at an equidistant spacing apart of 1 foot, with an intermediate water-line between Nos. 5 and 6. The number of water-lines is such that Simpson's first rule can be used, and the multipliers are, commencing with the load water-plane—

$$1 \quad 4 \quad 2 \quad 4 \quad 1\tfrac{1}{2} \quad 2 \quad \tfrac{1}{2}$$

The close spacing near the bottom is very necessary to ensure accuracy, as the curvature of the midship sections of the vessel is very sharp as the bottom is approached, and, as we saw on p. 13, Simpson's rules cannot accurately deal with areas such as these unless intermediate ordinates are introduced. Below No. 6 water-plane there is a volume the depth of which increases as we go aft, and the sections of this volume are very

[1] "Laying Off," by Mr. S. J. P. Thearle; "Laying Off," by Mr. T. H. Watson.

nearly triangles. This volume is dealt with separately on the left-hand side of the table, and is termed an "*appendage.*"

In order to find the volume of displacement between water-planes 1 and 6, we can first determine the areas of the water-planes, and then put these areas through Simpson's rule. To find the area of any of the water-planes, we must proceed in the ordinary manner and divide its length by ordinates so that Simpson's rule (preferably the first rule) can be used. In the case before us, the length is from the after-edge of the stem to the forward edge of the body post, viz. 71 feet, and this length is divided into ten equal parts, giving ordinates to each of the water-planes at a distance apart of 7·1 feet. The displacement-sheet is arranged so that we can put the lengths of the semi-ordinates of the water-planes in the columns headed respectively L.W.L., 2W.L., 3W.L., etc., the semi-ordinates at the several stations being placed in the same line as the number of ordinates given at the extreme left of the table. The lengths of the semi-ordinates are shown in italics. Thus, for instance, the lengths of the semi-ordinates of No. 3 W.L., as measured off, are 0·05, 1·82, 4·05, 5·90, 6·90, 7·25, 7·04, 6·51, 5·35, 2·85, and 0·05 feet, commencing with the forward ordinate No. 1, and these are put down in italics[1] as shown beneath the heading 3 W.L. in the table. The columns under the heading of each W.L. are divided into two, the semi-ordinates being placed in the first column. In the second column of each water-line is placed the product obtained by multiplying the semi-ordinate by the corresponding multiplier to find the area. These multipliers are placed at column 2 at the left, opposite the numbers of the ordinates. We have, therefore, under the heading of each water-line what we have termed the "*functions of ordinates,*" and if these functions are added up, we shall obtain what we have termed the "*function of area.*"

Taking No. 3 W.L. as an instance, the "*function*" of its area is 144·10, and to convert this "*function*" into the actual area, we must multiply by one-third the common interval to complete Simpson's first rule, *i.e.* by $\frac{1}{3} \times 7{\cdot}1$; and also by 2

[1] In practice, it is advisable to put down the lengths of the semi-ordinates in some distinctive colour, such as red.

to obtain the area of the water-plane on both sides of the ship. We should thus obtain the area of No. 3 W.L.—

$$144 \cdot 10 \times \tfrac{1}{3} \times 7 \cdot 1 \times 2 = 682 \cdot 07 \text{ square feet}$$

The functions of the area of each water-plane are placed at the bottom of the columns, the figures being, starting with the L.W.L., 163·70, 155·36, 144·10, 128·74, 105·67, 87·27, and 60·97. To get the actual areas of each of the water-planes, we should, as above, multiply each of these functions by $\frac{1}{3} \times 7 \cdot 1 \times 2$. Having the areas, we could proceed as on p. 20 to find the volume of displacement between No. 1 and No. 6 water-lines, but we do not proceed quite in this way; we put the "*functions of areas*" through Simpson's rule, and multiply afterwards by $\frac{1}{3} \times 7 \cdot 1 \times 2$, the same result being obtained with much less work. Below the "functions of areas" are placed the Simpson's multipliers, and the products 163·70, 621·44, etc., are obtained. These products added up give 1951·83. This number is a function of the volume of displacement, this volume being given by first multiplying it by one-third the vertical interval, *i.e.* $\frac{1}{3} \times 1$; and then by $\frac{1}{3} \times 7 \cdot 1 \times 2$, as seen above. The volume of displacement between No. 1 W.L. and No. 6 W.L. is therefore—

$$1951 \cdot 83 \times (\tfrac{1}{3} \times 1) \times (\tfrac{1}{3} \times 7 \cdot 1) \times 2 = 3079 \cdot 5 \text{ cubic feet}$$

$$\left.\begin{array}{c}\text{and the displacement in} \\ \text{tons (salt water)}^{1}\end{array}\right\} = \frac{3079 \cdot 5}{35} = 87 \cdot 98 \text{ tons}$$

We have thus found the displacement by dividing the volume under water by a series of equidistant horizontal planes; but we could also find the displacement by dividing the under-water volume by a series of equidistant vertical planes, as we saw in Chapter I. This is done on the displacement sheet, an excellent check being thus obtained on the accuracy of the work. Take No. 4 section, for instance: its semi-ordinates, commencing with the L.W.L., are 6·40, 6·24, 5·90, 5·32, 4·30, 3·40, and 2·25 feet. These ordinates are already put down opposite No. 4 ordinate. If these are multiplied successively by the multipliers, 1, 4, 2, 4, $1\frac{1}{2}$, 2, $\frac{1}{2}$, and the sum of the

[1] Thirty-five cubic feet of salt water taken to weigh one ton.

functions of ordinates taken, we shall obtain the "*function of area*" of No. 4 section between the L.W.L. and 6 W.L. This is done in the table by placing the functions of ordinates immediately below the corresponding ordinate, the multiplier being given at the head of each column. We thus obtain a series of horizontal rows, and these rows are added up, the results being placed in the column headed "*Function of areas.*" Each of these functions multiplied by one-third the common interval, *i.e.* $\frac{1}{3} \times 1$, and then by 2 for both sides, will give the areas of the transverse sections between the L.W.L. and 6 W. L.; but, as before, this multiplication is left till the end of the calculation. These functions of areas are put through Simpson's multipliers, the products being placed in the column headed "*Multiples of areas.*" This column is added up, giving the result 1951·83. To obtain the volume of displacement, we multiply this by $(\frac{1}{3} \times 1) \times 2 \times (\frac{1}{3} \times 7{\cdot}1)$. It will be noticed that we obtain the number 1951·83 by using the horizontal water-lines and the vertical sections; and this must evidently be the case, because the displacement by either method must be the same. The correspondence of these additions forms the check, spoken of above, of the accuracy of the work. We thus have the result that the volume of displacement from L.W.L. to 6 W.L. is 3079·5 cubic feet, and the displacement in tons of this portion 87·98 tons in salt water. This is termed the "Main solid," and forms by far the greater portion of the displacement.

We now have to consider the portion we have left out below No. 6 water-plane. Such a volume as this is termed an "*appendage.*" The sections of this appendage are given in the body-plan at the several stations. The form of these sections are traced off, and by the ordinary rules their areas are found in square feet. We have, therefore, this volume divided by a series of equidistant planes the same as the main solid, and we can put the areas of the sections through Simpson's rule and obtain the volume. This calculation is done on the left-hand side of the sheet, the areas being placed in column 3, and the functions of the areas in column 4. The addition of these functions is 49·99, and this multiplied by $\frac{1}{3} \times 7{\cdot}1$ gives the volume of the appendage in cubic feet, viz. 118·3; and this volume divided by 35 gives the number of tons the appendage displaces in salt water, viz. 3·38 tons. The total displacement is thus obtained by adding together the main solid and the appendage, giving 91·36 tons in salt water. The displacement in fresh water (36 cubic feet to the ton) would be 88·8 tons.

The sheer drawing for this vessel as given on Plate I. was drawn to the frame line, *i.e.* to the moulded dimensions of the ship; but the actual ship is fuller than this, because of the outer bottom plating, and this plating will contribute a small amount to the displacement, but this is often neglected. Some sheer drawings, on the other hand, are drawn so that the lines include a mean thickness of plating outside the frame line, and when this is the case, the displacement sheet gives the actual displacement, including the effect of the plating. For a sheathed ship this is also true; in this latter case, the displacement given by the sheathing would be too great to be neglected. When the sheer drawing is drawn to the outside of sheathing, or to a mean thickness of plating, it is evident that the ship must be laid off on the mould loft floor, so that, when built, she shall have the form given by the sheer drawing.

We now have to find the position of the centre of buoyancy both in a fore-and-aft and in a vertical direction. (It must be in the middle-line plane of the ship, since both sides are symmetrical.) Take first the fore-and-aft position. This is found with reference to No. 6 station. The functions of the areas of the sections are 0·55, 23·055, etc., and in the column headed "Multiples of areas" we have these functions put through Simpson's multipliers. We now multiply these multiples by the number of intervals they respectively are from No. 6 station, viz. 5, 4, etc., and thus obtain a column headed "Moments." This column is added up for the fore body, giving 1505·43, and for the after body, giving 1913·02, the difference being 407·59 in favour of the after body. To get the actual moment of the volume abaft No. 6 station, we should multiply this difference by $(\frac{1}{3} \times 1)$ for the vertical direction, $(\frac{1}{3} \times 7·1)$ for the fore-and-aft direction, and by 2 for both sides, and then by 7·1, since we have only multiplied by the number of intervals away, and not by the actual distances, or $407 \cdot 59 \times (\frac{1}{3} \times 1) \times (\frac{1}{3} \times 7 \cdot 1) \times 2 \times 7 \cdot 1$. The volume, as we have seen above, is given by—

$$1951 \cdot 83 \times (\tfrac{1}{3} \times 1) \times (\tfrac{1}{3} \times 7 \cdot 1) \times 2$$

The distance of the centre of gravity of the main solid from No. 6 station will be—

$$\text{Moment} \div \text{volume}$$

But on putting this down we shall see that we can cancel out, leaving us with—

$$\frac{407 \cdot 59 \times 7 \cdot 1}{1951 \cdot 83} = 1 \cdot 48 \text{ feet}$$

which is the distance of the centre of gravity of the main solid abaft No. 6 station. The distance of the centre of gravity of the appendage abaft No. 6 station is 4·0 feet; the working is shown on the left-hand side of the table, and requires no further explanation. These results for the main solid and for the appendage are combined together at the bottom; the displacement of each in tons is multiplied by the distance of its centre of gravity abaft No. 6 station, giving the moments. The total moment is 143·73, and the total displacement is 91·36 tons, and this gives the centre of gravity of the total displacement, or what we term the *centre of buoyancy*, C.B., 1·57 feet abaft No. 6 station.

Now we have to consider the vertical position of the C.B., and this is determined with reference to the load waterline. For the main solid the process is precisely similar to that adopted for finding the horizontal position, with the exception that we take our moments all below the load waterplane, the number of intervals being small compared with the horizontal intervals. We obtain, as indicated on the sheet, the centre of gravity of the main solid at a distance of 2·21 feet below the L.W.L. For the appendage, we proceed as shown on the left-hand side of the sheet. When finding the areas of the sections of the appendage, we spot off as nearly as possible the centre of gravity of each section, and measure its distance below No. 6 W.L. If the sections happen to be triangles, this will, of course, be one-third the depth. These distances are placed in a column as shown, and the "functions of areas" are respectively multiplied by them, *e.g.* for No. 4 station the function of the area is 5·92, and this is multiplied by 0·22, the distance of the centre of gravity of the section of the appendage below No. 6 W.L. We thus obtain a column which, added up, gives a total of 13·78. To get the actual moment, we only have to multiply this by $\frac{1}{3} \times 7·1$. The volume of the appendage is $49·99 \times (\frac{1}{3} \times 7·1)$. So that the distance of the centre of gravity of the whole appendage below No. 6 W.L. is given by moment ÷ volume, or $\frac{13·78}{49·99} = 0·27$ feet, and therefore the centre of gravity of the appendage is 5·27 feet below the L.W.L. The results for the main solid and for the appendage are combined together in the table at the bottom, giving the final position of the C.B. of the whole displacement as 2·32 feet below the L.W.L.

It will be of interest at this stage to test the two approximations that were given on p. 63 for the distance of the C.B. below the L.W.L. The first was that this distance would be from $\frac{8}{20}$ to $\frac{9}{20}$ of the mean draught to top of keel (*i.e.* the mean moulded draught). For this vessel the distance is 2·32 feet, and the mean moulded draught is 5′ 9½″, or 5·8 feet, and so we have the ratio $\frac{2·32}{5·8}$, or exactly $\frac{8}{20}$. The second approximation (Normand's), p. 63, was—

$$\frac{1}{3}\left(\frac{d}{2}+\frac{V}{A}\right)$$

All these are readily obtainable from the displacement sheet, and if worked out its value is found to be 2·29 feet. This agrees well with the actual result, 2·32 feet, the error being 3 in 232, or less than 1½ per cent.

For large vessels a precisely similar displacement-sheet is prepared, but it is usual to add in the effect of other appendages besides that below the lowest W.L. A specimen calculation is shown below. In this case the sheer drawing was made to include a mean thickness of plating. The appendages are—

Before fore perpendicular (ram bow).
Abaft after perpendicular.
Rudder.
Shaft-tubes, etc. (including propellers, shafts, swells, and struts).
Bilge keels (if fitted).

The effect of these appendages, outside the naked hull, is to increase the displacement by 61·5 tons, and to throw the C.B. aft from 8·44 feet to 8·88 feet abaft the middle ordinate. The effect on the vertical position of the C.B. is of very small amount.

SUMMARY.

Item.	Displacement in tons.	Below L.W.L.		From Middle Ordinate.			
				Forward.		Aft.	
		Lever.	Moment.	Lever.	Moment.	Lever.	Moment.
Main portion	13,319·6	11·11	147,981	—	—	8·58	114,282
Below 8 W.L.	921·0	25·47	23,458	—	—	6·45	5,940
Abaft A.P.	34·1	3·60	123	—	—	195·05	6,651
Before F.P.	12·2	13·77	168	192·80	2,352	—	—
Rudder	4·0	18·60	74	—	—	196·30	785
Shaft-tubes, etc.	11·2	16·80	188	—	—	149·00	1,679
	14,302·1		)171,992		2,352		129,337

12·02 feet below L.W.L.

129,337
2,352
)126,985
8·88 feet aft.

The total displacement up to the L.W.L. is 14,302 tons. The centre of buoyancy is 12·02 feet below the L.W.L. and 8·88 feet abaft the middle ordinate.

Graphic or Geometrical Method of calculating Displacement and Position of Centre of Buoyancy.—There is one property of the curve, known as the "*parabola of the second order*" (see p. 6), that can be used in calculating by a graphic method the area of a figure bounded by such a curve. Let BFC, Fig. 37, be a curve bounding the figure ABCD, and suppose the curve is a "*parabola of the second order.*" Draw the ordinate EF midway between AB and DC; then the following is a property of the curve BFC:—the area of the segment BCF is given by two-thirds the product of the deflection GF and the base AD, or—

$$\text{Area BCF} = \tfrac{2}{3} \times \text{GF} \times \text{AD}$$

Make GH $= \tfrac{2}{3}$ GF. Then—

$$\text{Area BCF} = \text{GH} \times \text{AD}$$

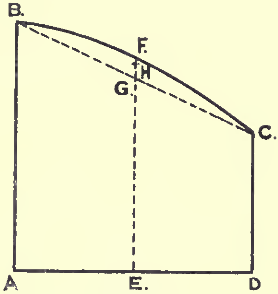

FIG. 37.

Now, the area of the trapezoid ABCD is given by AD × EG, and consequently—

$$\text{The area ADCFB} = \text{AD} \times \text{EH}$$ [1]

Thus, if we have a long curvilinear area, we can divide it up as for Simpson's first rule, and set off on each of the intermediate ordinates two-thirds the deflection of the curve above or below the straight line joining the extremities of the dividing ordinates. Then add together on a strip of paper all such distances as EH right along, and the sum multiplied by the

[1] This property may be used to prove the rule known as Simpson's first rule. Call AB, EF, DC respectively y_1, y_2, y_3. Then we have—

$$\text{EG} = \frac{y_1 + y_3}{2} \text{ and FG} = y_2 - \text{EG}$$

$$\therefore \text{FG} = y_2 - \frac{y_1 + y_3}{2}$$

$$\text{HG} = \tfrac{2}{3}y_2 - \frac{y_1 + y_3}{3}$$

$$\text{EH} = \text{EG} + \text{GH}$$

$$= \left(\frac{y_1 + y_3}{2}\right) + \left(\frac{2y_2 - y_1 - y_3}{3}\right)$$

$$= \tfrac{1}{6}(y_1 + 4y_2 + y_3)$$

and calling AE $= h$, we have—

$$\text{Area ADCFB} = \frac{h}{3}(y_1 + 4y_2 + y_3)$$

which is the same expression as given by Simpson's first rule.

distance apart of the dividing ordinates, as AD, will give the area required. Thus in Fig. 38, AB is divided into equal parts

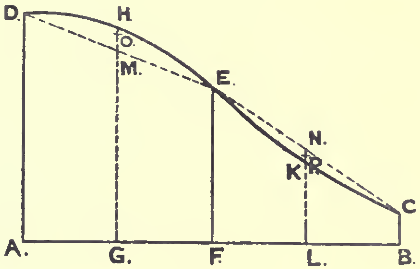

FIG. 38.

as shown. D and E are joined, also E and C: MO is set off = $\frac{2}{3}$HM, and NP is set off = $\frac{2}{3}$NK. Then—

$$\text{Area ADEF} = \text{AF} \times \text{GO}$$
$$\text{and area FECB} = \text{FB} \times \text{LP}$$
$$\text{and the whole area ABCD} = \text{AF} \times (\text{GO} + \text{LP})$$

We can represent the area ABCD by a length equal to GO + LP on a convenient scale, if we remember that this length has to be multiplied by AF to get the area. This principle can be extended to finding the areas of longer figures, such as water-planes, and we now proceed to show how the displacement and centre of buoyancy of a ship can be determined by its use. The assumption we made at starting is supposed to hold good with all the curves we have to deal, *i.e.* that the portions between the ordinates are supposed to be "*parabolas of the second order*." This is also the assumption we make when using Simpson's first rule for finding displacement in the ordinary way.

Plate I. represents the ordinary sheer drawing of a vessel, and the underwater portion is divided by the level water-planes shown by the half-breadth plan. The areas of each of these planes can be determined graphically as above described, the area being represented by a certain length obtained by the addition of all such lengths as GO, etc., Fig. 38, the interval being constant for all the water-planes. Let AB, Fig. 39, be set vertically to represent the extreme moulded draught of the vessel. Draw BC at right angles to AB, to represent on a convenient scale the area of the L.W.L. obtained as above. Similarly, DE, FG are set out to represent on the same scale the areas of water-planes 2 and 3, and so on for each water-plane. A curve drawn through all such points as C, E, and G

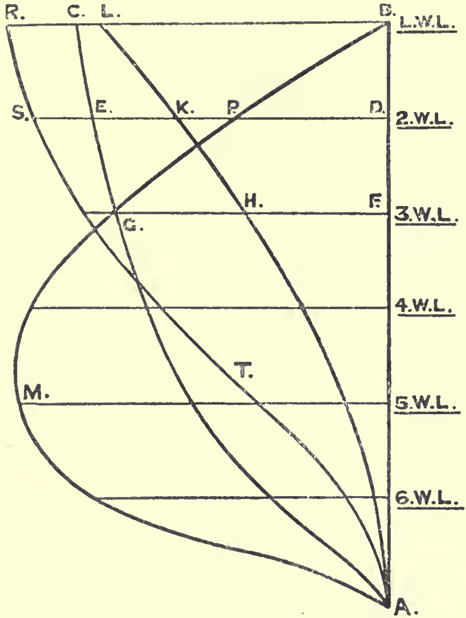

Fig. 39.

will give a "*curve of areas of water-planes.*" Now, the area of this curve up to the L.W.L. gives us the volume of displacement up to the L.W.L., as we have seen in Chapter I., and we can readily find the area of the figure ABCEG by the graphic method, and this area will give us the displacement up to the L.W.L. Similarly, the area of ADEG will give the displacement up to 2 W.L., and so on. Therefore set off BL to represent on a convenient scale the area of the figure ABCE, DK on the same scale to represent the area ADEG, and so on. Then a curve drawn through all such points as L, K will give us a "*curve of displacement*," and the ordinate of this curve at any draught will give the displacement at that draught, BL being the load displacement.

We now have to determine the distance of the centre of buoyancy below the L.W.L., and to find this we must get the *moment* of the displacement about the L.W.L. and divide this by the volume of displacement below the L.W.L. We now construct a curve, BPMA, such that the ordinate at any draught represents the area of the water-plane at that draught multiplied by the depth of the water-plane below the L.W.L. Thus DP represents on a convenient scale the area of No. 2 water-plane multiplied by DB, the distance below the L.W.L. The ordinate of this curve at the L.W.L. must evidently be zero. This curve is a curve of "*moments of areas of water-planes*" about the L.W.L. The area of this curve up to the L.W.L. will evidently be the moment of the load displacement about the L.W.L., and thus the length BR is set out to equal on a convenient scale the area of BPMA. Similarly, DS is set out to represent, on the same scale, the area of DPMA, and thus the moment of the displacement up to 2 W.L. about the L.W.L. These areas are found graphically as in the preceding cases. Thus a curve RSTA can be drawn in, and BR ÷ BL, or moment of load displacement about L.W.L. ÷ load displacement, gives us the depth of the centre of buoyancy for the load displacement below the L.W.L.

Exactly the same course is pursued for finding the displacement and the longitudinal position of the centre of buoyancy, only in this case we use a curve of areas of transverse sections instead of a curve of areas of water-planes, and we get the moments of the transverse areas about the middle ordinate. Fig. 40 gives the forms the various curves take for the fore body. AA is the "curve of areas of transverse sections;" BB is the "curve of displacement" for the fore body, OB being the displacement of the fore body. CC is the curve of "moments of areas of transverse sections" about No. 6 ordinate; DD is the curve of "moment of displacement" about No. 6 ordinate, OD being the moment of the fore-body displacement about No. 6 ordinate. Similar curves can be drawn for the after body, and the difference of the moments of the fore and after bodies divided by the load displacement will give the distance

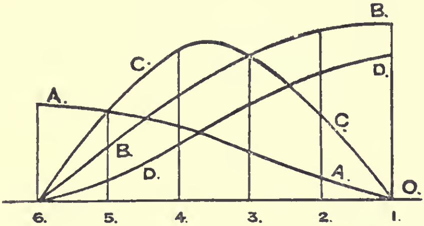

FIG. 40.

of the centre of buoyancy forward or aft of No. 6 ordinate, as the case may be. The total displacement must be the same as found by the preceding method.

Method of finding Areas by Means of the Planimeter.—This instrument is frequently employed to find the area of plane curvilinear figures, and thus the volume of displacement of a vessel can be determined. One form of the instrument is shown in diagram by Fig. 41. It is supported at three places: first, by a weighted pin, which is fixed in position by being pressed into the paper; second, by a wheel, which actuates a circular horizontal disc, the wheel and disc both being graduated; and third, by a blunt pointer. The instrument is placed on the drawing, the pin is fixed in a convenient position, and the pointer is placed on a marked spot A on the boundary of the curve of which the area is required. The reading given by the wheel and disc is noted. On passing round the boundary of the area with the pointer (the same way as the hands of a clock) back to the starting-point, another reading is obtained. The difference of the two readings is *proportional* to the area of the figure, the multiplier required to convert the difference into the area depending on the instrument and on the scale to which the figure is drawn. Particulars concerning the necessary multipliers are given with the instrument; but it is a good practice to pass round figures of known area to get accustomed to its use.

By the use of the planimeter the volume of displacement of a vessel can very readily be determined. The body plan is taken, and the L.W.L. is marked on. The pointer of the instrument is then passed round each section in turn, up to the L.W.L., the readings being tabulated. If the differences of the readings were each multiplied by the proper multiplier, we should obtain the area of each of the transverse sections, and so, by direct application of Simpson's rules, we should find the

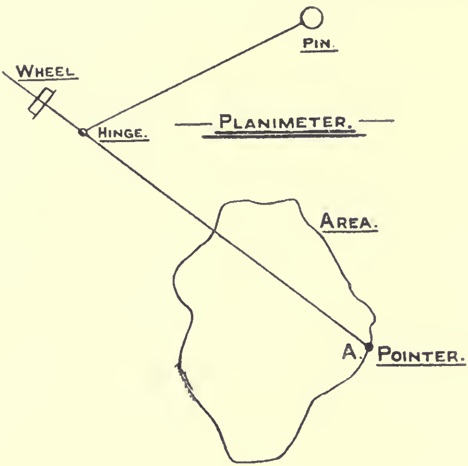

FIG. 41.

required volume of displacement. Or we could put the actual difference of readings through Simpson's multipliers, and multiply at the end by the constant multiplier.

It is frequently the practice to shorten the process as follows: The body-plan is arranged so that Simpson's first rule will be used, *i.e.* an odd number of sections is employed. The pointer is passed round the first and last sections, and the reading is recorded. It is then passed round *all* the even sections, 2, 4, 6, etc., and the reading is recorded. Finally, it is passed round *all* the odd sections except the first and last, viz. 3, 5, 7, etc., and the reading is put down. The differences of the readings are found and put down in a column. The first difference is multiplied by 1, the next difference is multiplied by 4, and the last by 2. The sum of these products is then multiplied for Simpson's first rule, and then by the proper multiplier for the instrument and scale used. The work can conveniently be arranged thus:

Numbers of sections.	Readings.	Differences of readings.	Simpson's multipliers.	Products.
Initial reading	5,124	—	—	—
1, 21	5,360	236	1	236
2, 4, 6, 8, 10, 12, 14, 16, 18, 20 ...	18,681	13,321	4	53,284
3, 5, 7, 9, 11, 13, 15, 17, 19	31,758	13,077	2	26,154
				79,674

The multiplier for the instrument and scale of the drawing used and to complete the use of Simpson's first rule is $\frac{6}{5}$; so that the volume of displacement is $79{,}674 \times \frac{6}{5}$ cubic feet, and the displacement in tons is $79{,}674 \times \frac{6}{5} \times \frac{1}{35} = 2732$ tons.

There are two things to be noticed in the use of the planimeter: first, it is not necessary to set the instrument to the exact zero, which is somewhat troublesome to do; and second, the horizontal disc must be watched to see how many times it makes the complete revolution, the complete revolution meaning a reading of 10,000.

It is also possible to find the vertical position of the centre of buoyancy by means of the planimeter. By the method above described we can determine the displacement up to each water-line in succession, and so draw in on a convenient scale the ordinary curve of displacement. Now we can run round this curve with the planimeter and find its area. This area divided by the top ordinate (*i.e.* the load displacement) will give the distance of the centre of buoyancy below the load-line (see p. 64).

To find the centre of buoyancy in a fore-and-aft direction, it is necessary to tabulate the differences for each section, and treat these differences in precisely the same way as the "functions of areas of vertical sections" are treated in the ordinary displacement sheet.

Method of approximating to the Area of the Wetted Surface[1] **by "Kirk's" Analysis.**—The ship is assumed to be represented by a block model, shaped as shown in Fig. 42, formed of a parallel middle body and a

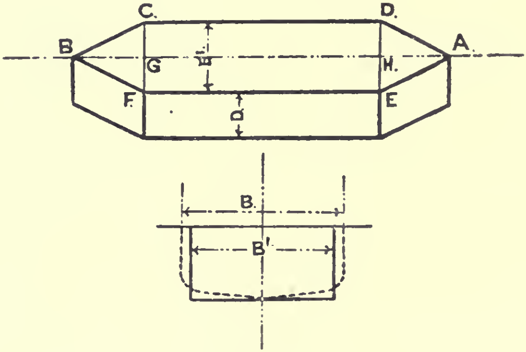

FIG. 42.

tapered entrance and run which are taken as of equal length. The depth of the model is equal to the mean draught, and the length of the model is equal to the length of the vessel. The breadth is not equal to the breadth of the vessel, but is equal to area of immersed midship section ÷ mean draught. The displacement of the model is made equal to that of the vessel. We then have—

$$\left.\begin{array}{l}\text{Volume of displace-}\\ \quad\text{ment}\end{array}\right\} = V, \text{ say}$$

$$= AG \times \text{area of midship section}$$

$$\text{or } AG = \frac{V}{\text{area of midship section}}$$

$$\therefore \left.\begin{array}{l}\text{length of entrance}\\ \quad\text{or run}\end{array}\right\} = \text{length of ship} - \frac{V}{\text{area of midship section}}$$

$$= L - \frac{V}{B' \times D}$$

[1] The area of wetted surface can be closely approximated to by putting a curve of girths (modified for the slope of the level lines, see p. 192) through Simpson's rule.

where L = length of ship;
B′ = breadth of model;
D = mean draught.

Having found these particulars, the surface of the model can be readily calculated.

$$\text{Area of bottom} = \text{AG} \times \text{B}'$$
$$\text{Area of both sides} = 2(\text{GH} + 2\text{AE}) \times \text{mean draught}$$

The surface of a model formed in this way approximates very closely to the actual wetted surface of the vessel. It is stated that in very fine ships the surface of the model exceeds the actual wetted surface by about 8 per cent., for ordinary steamers by about 3 per cent., and for full ships by 2 per cent.

By considering the above method, we may obtain an approximate formula for the wetted surface—

$$\text{Area of bottom} = \frac{\text{V}}{\text{D}}$$
$$\text{Area of sides} = 2\text{L}'\text{D}$$

where L′ is the length along ADCB. Then—

$$\text{Surface} = 2\text{L}'\text{D} + \frac{\text{V}}{\text{D}}$$

This gives rather too great a result, as seen above; and if we take—

$$\text{Surface} = 2\text{LD} + \frac{\text{V}}{\text{D}}$$

we shall get the area of the wetted surface slightly in excess, but this will allow for appendages, such as keels, etc.

Since $\text{V} = k\,.\,\text{LBD}$, where k is the block coefficient of displacement, we may write—

$$\text{Surface} = 2\text{LD} + k\,.\,\text{LB}$$

Approximate Formulæ for finding Wetted Surface.—Mr. Denny gives the following formula for the area of wetted surface :—

$$1 \cdot 7LD + \frac{V}{D}$$

which is seen to be very nearly that obtained above.

Mr. Taylor, in his work on "Resistance and Propulsion of Ships," gives the following formula :—

$$15 \cdot 6\sqrt{WL}$$

where W is the displacement in tons.

Approximate Method of determining the Mean Wetted Girth of Ships, given by Mr. A. Blechynden, M.I.N.A. (*Transactions of Institution of Naval Architects*, 1888)—

Let M = midship wetted girth measured on midship section in feet;

L = length between perpendiculars in feet;

V = volume of displacement in cubic feet;

S = area of midship section in square feet;

D = moulded draught in feet;

c = prismatic coefficient of fineness $= \frac{V}{L \times S}$ (see p. 30)

m = mean wetted girth in feet.

$$\text{Then } m = 0 \cdot 95cM + 2(1 - c)D$$

Examples to Chapter II.

1. A ship has the following weights placed on board :—

20 tons	...	...	100 feet before amidships	
45 ,,	...	...	80 ,,	,,
15 ,,	...	...	40 ,,	,,
60 ,,	...	...	50 feet abaft	,,
40 ,,	...	...	80 ,,	,,
30 ,,	...	...	110 ,,	,,

Show that these weights will have the same effect on the trim of the ship as a single weight of 210 tons placed $15\frac{5}{7}$ feet abaft amidships.

2. Six weights are placed on a drawing-board. The weights are 3, 4, 5, 6, 7, 8 lbs. respectively. Their respective distances from one edge are 5, $4\frac{1}{2}$, 4, $3\frac{1}{4}$, 3, 2 feet respectively, and from the edge at right angles, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$ feet respectively. The drawing-board weighs 6 lbs., and is 6 feet long and 3 feet broad. Find the position where a single support would need to be placed in order that the board should remain horizontal.

Ans. 3·27 feet from short edge, 1·58 feet from long edge.

3. An area bounded by a curve and a straight line is divided by ordinates 4 feet apart of the following lengths: 0, 12·5, 14·3, 15·1, 15·5, 15·4, 14·8, 14·0, 0 feet respectively. Find—
   (1) Area in square feet.
   (2) Position of centre of gravity relative to the first ordinate.
   (3) Position of the centre of gravity relative to the base.

Ans. (1) 423 square feet; (2) 16·27 feet; (3) 7·24 feet.

4. A triangle ABC has its base BC 15 feet long, and its height 25 feet. A line is drawn 10 feet from A parallel to the base, meeting AB and AC in D and E. Find the distance of the centre of gravity of DBCE from the apex.

Ans. 18·57 feet.

5. The semi-ordinates of a water-plane in feet, commencing from the after end, are 5·2, 10·2, 14·4, 17·9, 20·6, 22·7, 24·3, 25·5, 26·2, 26·5, 26·6, 26·3, 25·4, 23·9, 21·8, 1·88, 15·4, 11·5, 7·2, 3·3, 2·2. The distance apart is 15 feet. Find the area of the water-plane, and the position of the centre of gravity in relation to the middle ordinate.

Ans. 11,176 square feet; 10·15 feet abaft middle.

6. Find the area and transverse position of the centre of gravity of "half" a water-line plane, the ordinates in feet being 0·5, 6, 12, 16, 12, 10, and 0·5 respectively, the common interval being 15 feet.

Ans. 885 square feet; 6·05 feet.

7. The areas of sections 17′ 6″ apart through a bunker, commencing from forward, are 65, 98, 123, 137, 135, 122, 96 square feet respectively. The length of bunker is 100 feet, and its fore end is 1′ 6″ forward of the section whose area is 65 square feet. Draw in a curve of sectional areas, and obtain, by using convenient ordinates, the number of cubic feet in the bunker, and the number of tons of coal it will contain, assuming that 43 cubic feet of coal weigh 1 ton. Find also the position of the C.G. of the coal relative to the after end of the bunker.

Ans. 272 tons; 46½ feet from the after end.

8. The tons per inch in salt water of a vessel at water-lines 3 feet apart, commencing with the L.W.L., are 31·2, 30·0, 28·35, 26·21, 23·38, 19·5, 12·9. Find the displacement in salt and fresh water and the position of the C.B. below the L.W.L., neglecting the portion below the lowest W.L. Draw in the tons per inch curve for salt water to a convenient scale, and *estimate* from it the weight necessary to be taken out in order to lighten the vessel 2′ 3½″ from the L.W.L. The mean draught is 20′ 6″.

Ans. 5405 tons; 5255 tons; 8·01 feet; 847 tons.

9. In the preceding question, calling the L.W.L. 1, find the displacement up to 2 W.L., 3 W.L., and 4 W.L., and draw in a curve of displacement from the results you obtain, and check your answer to the latter part of the question.

10. The tons per inch of a ship's displacement at water-lines 4 feet apart, commencing at the L.W.L., are 44·3, 42·7, 40·5, 37·5, 33·3. Find number of tons displacement, and the depth of C.B. below the top W.L.

Ans. 7670 tons; 7·6 feet.

11. The ship in the previous question has two water-tight transverse bulkheads 38 feet apart amidship, and water-tight flats at 4 feet below and 3 feet above the normal L.W.L. If a hole is made in the side 2 feet below the L.W.L., how much would the vessel sink, taking the breadth of the L.W.L. amidships as 70 feet? Indicate the steps where, owing to insufficient information, you are unable to obtain a perfectly accurate result.

Ans. 8 inches.

12. The areas of transverse sections of a coal-bunker 19 feet apart are

respectively 63·2, 93·6, 121·6, 108·8, 94·8 square feet, and the centres of gravity of these sections are 10·8, 11·6, 12·2, 11·7, 11·2 feet respectively below the L.W.L. Find the number of tons of coal the bunker will hold, and the vertical position of its centre of gravity (44 cubic feet of coal to the ton).

Ans. 174·3 tons; 11·68 feet below L.W.L.

13. A vessel is 180 feet long, and the transverse sections from the load water-line to the keel are semicircles. Find the longitudinal position of the centre of buoyancy, the ordinates of the load water-plane being 1, 5, 13, 15, 14, 12, and 10 feet respectively.

Ans. 106·2 feet from the finer end.

14. Estimate the distance of the centre of buoyancy of a vessel below the L.W.L., the vessel having 22′ 6″ mean moulded draught, block coefficient of displacement 0·55, coefficient of fineness of L.W.L. 0·7 (use Normand's formula, p. 63).

Ans. 9·65 feet.

15. A vessel of 2210 tons displacement, 13′ 6″ draught, and area of load water-plane 8160 square feet, has the C.B., calculated on the displacement sheet, at a distance of 5·43 feet below the L.W.L. Check this result.

16. The main portion of the displacement of a vessel has been calculated and found to be 10,466 tons, and its centre of gravity is 10·48 feet below the L.W.L., and 5·85 feet abaft the middle ordinate. In addition to this, there are the following appendages :—

	tons.		
Below lowest W.L.	263,	24·8 ft. below L.W.L.,	4·4 ft. abaft mid. ord.
Forward	5,	12·0 ,, ,,	202 ft. forward of mid. ord.
Stern	16,	2·8 ,, ,,	201 ft. abaft mid. ord.
Rudder	16,	17·5 ,, ,,	200 ,, ,,
Bilge keels ...	20,	20 ,, ,,	0 ,, ,,
Shafting, etc. ...	18,	15 ,, ,,	140 ,, ,,

Find the total displacement and position of the centre of buoyancy.

Ans. 10,804 tons; C.B. 6·5 abaft mid. ord., 10·86 ft. below L.W.L.

17. The displacements of a vessel up to water-planes 4 feet apart are 10,804, 8612, 6511, 4550, 2810, 1331, and 263 tons respectively. The draught is 26 feet. Find the distance of the centre of buoyancy below the load water-line. Would you call the above a fine or a full ship?

Ans. 10·9 feet nearly.

18. The load displacement of a ship is 5000 tons, and the centre of buoyancy is 10 feet below the load water-line. In the light condition the displacement of the ship is 2000 tons, and the centre of gravity of the layer between the load and light lines is 6 feet below the load-line. Find the vertical position of the centre of buoyancy below the light line in the light condition.

Ans. 4 feet, assuming that the C.G. of the layer is at half its depth.

19. Ascertain the displacement and position of the centre of buoyancy of a floating body of length 140 feet, depth 10 feet, the forward section being a triangle 10 feet wide at the deck and with its apex at the keel, and the after section a trapezoid 20 feet wide at the deck and 10 feet wide at the keel, the sides of the vessel being plane surfaces; draught of water may be taken as 7 feet.

Ans. 238 tons; 56·3 feet before after end, 3 feet below water-line.

20. Show by experiment or otherwise that the centre of gravity of a

quadrant of a circle 3 inches radius is 1·8 inches from the right angle of the quadrant.

21. A floating body has a constant triangular section, vertex downwards, and has a constant draught of 12 feet in fresh water, the breadth at the water-line being 24 feet. The keel just touches a quantity of mud of specific gravity 2. The water-level now falls 6 feet. How far will the body sink into the mud?

Ans. 4 feet $11\frac{1}{4}$ inches.

CHAPTER III.

CONDITIONS OF EQUILIBRIUM, TRANSVERSE METACENTRE, MOMENT OF INERTIA, TRANSVERSE BM, INCLINING EXPERIMENT, METACENTRIC HEIGHT, ETC.

Trigonometry.—The student of this subject will find it a distinct advantage, especially when dealing with the question of stability, if he has a knowledge of some of the elementary portions of trigonometry. The following are some properties which should be thoroughly grasped:—

Circular Measure of Angles.—The degree is the unit generally employed for the measurement of angles. A right angle is divided into 90 equal parts, and each of these parts is termed a "*degree.*" If two lines, as OA, OB, Fig. 43, are inclined to each other, forming the angle AOB, and we draw at any radius OA an arc AB from the centre O, cutting OA, OB in A and B, then length of arc AB ÷ radius OA is termed the *circular measure* of the angle AOB. Or, putting it more shortly—

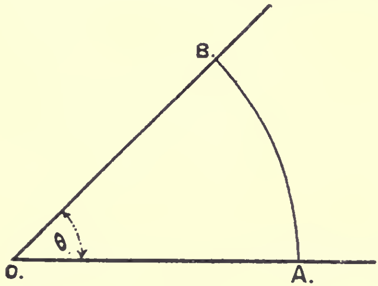

Fig. 43.

$$\text{Circular measure} = \frac{arc}{radius}$$

$$\left.\begin{array}{l}\text{The circular measure of four right} \\ \quad\text{angles, or 360 degrees}\end{array}\right\} = \frac{\text{circumference of a circle}}{\text{radius}}$$

$$= 2\pi$$

$$\text{The circular measure of a right angle} = \frac{\pi}{2}$$

Since 360 degrees = 2π in circular measure, then the angle whose circular measure is unity is—

$$\frac{360}{2\pi} = 57{\cdot}3 \text{ degrees}$$

The circular measure of 1 degree is $\frac{2\pi}{360} = 0{\cdot}01745$, and thus the circular measure of any angle is found by multiplying the number of degrees in it by 0·01745.

Trigonometrical Ratios,[1] *etc.*—Let BOC, Fig. 44, be any angle; take any point P in one of the sides OC, and draw PM perpendicular to OB. Call the angle BOC, θ.[2]

PM is termed the perpendicular.

OM is termed the base.

OP is termed the hypotenuse.

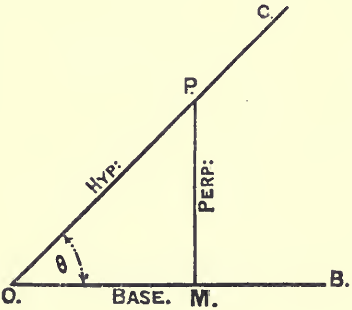

Fig. 44.

Then—

$$\frac{PM}{OP} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \textit{sine}\ \theta, \text{ usually written } \textit{sin}\ \theta$$

$$\frac{OM}{OP} = \frac{\text{base}}{\text{hypotenuse}} = \textit{cosine}\ \theta, \text{ usually written } \textit{cos}\ \theta$$

$$\frac{PM}{OM} = \frac{\text{perpendicular}}{\text{base}} = \textit{tangent}\ \theta, \text{ usually written } \textit{tan}\ \theta$$

These ratios will have the same value wherever P is taken on the line OC.

[1] An aid to memory which is found of assistance by many in learning these ratios is—

*Sin per*plexes *hyp*ocrites
Cos of *base hyp*ocrisy.

[2] θ is a Greek letter (*theta*) often used to denote an angle.

$$\text{We can write } \sin\theta = \frac{\text{per.}}{\text{hyp.}}$$
$$\cos\theta = \frac{\text{base}}{\text{hyp.}}$$
$$\text{and also } \tan\theta = \frac{\sin\theta}{\cos\theta}$$

There are names for the inversions of the above ratios, which it is not proposed to use in this work.

For small angles, the value of the angle θ in circular measure is very nearly the same as the values of sin θ and tan θ. This will be seen by comparing the values of θ, sin θ, and tan θ for the following angles:—

Angle in degrees.	Angle in circular measure.	Sin θ.	Tan θ.
2	0·0349	0·0349	0·0349
4	0·0698	0·0697	0·0699
6	0·1047	0·1045	0·1051
8	0·1396	0·1392	0·1405
10	0·1745	0·1736	0·1763

Up to 10° they have the same values to two places of decimals, and for smaller angles the agreement in value is closer still.

Further information can be obtained by reference to a pocket-book, as "Mackrow's" or "Molesworth's," or an elementary text-book on trigonometry.

Conditions that must hold in the Case of a Vessel floating freely, and at Rest in Still Water. — We saw in Chapter I. that, for a vessel floating in still water, the weight of the ship with everything she has on board must equal the weight of the displaced water. To demonstrate this, we imagined the cavity left by the ship when lifted out of the water to be filled with water (see Fig. 17). Now, the upward support of the surrounding water must exactly balance the weight of the water poured in. This weight may be regarded as acting downwards through its centre of gravity, or, as we now term it, the centre of buoyancy. Consequently, the upward support of the water, or the buoyancy, must act through the centre of buoyancy. All the horizontal pressures of the water on the surface of the ship must evidently balance among themselves. We therefore have the following forces acting upon the ship :—

(1) The weight acting downwards through the C.G.;

(2) The upward support of the water, or, as it is termed, the buoyancy, acting upwards through the C.B.;

and for the ship to be at rest, these two forces must act in the same line and counteract each other. Consequently, we also have the following condition :—

The centre of gravity of the ship, with everything she has on board, must be in the same vertical line as the centre of buoyancy.

If a rope is pulled at both ends by two men exerting the same strength, the rope will evidently remain stationary; and this is the case with a ship floating freely and at rest in still water. She will have no tendency to move of herself so long as the C.G. and the C.B. are in the same vertical line.

Definition of Statical Stability. — The statical stability of a vessel is the tendency she has to return to the upright when inclined away from that position. It is evident that under ordinary conditions of service a vessel cannot always remain upright; she is continually being forced away from the upright by external forces, such as the action of the wind and the waves. It is very important that the ship shall have such qualities that these inclinations that are forced upon her shall not affect her safety; and it is the object of the present chapter to discuss how these qualities can be secured and made the subject of calculation so far as small angles of inclination are concerned.

A ship is said to be in *stable equilibrium* for a given direction of inclination if, on being slightly inclined in that direction from her position of rest, she tends to return to that position.

A ship is said to be in *unstable equilibrium* for a given direction of inclination if, on being slightly inclined in that direction from her position of rest, she tends to move away farther from that position.

A ship is said to be in *neutral or indifferent equilibrium* for a given direction of inclination if, on being slightly inclined in that direction from her position of rest, she neither tends to return to nor move farther from that position.

These three cases are represented by the case of a heavy sphere placed upon a horizontal table.

1. If the sphere is weighted so that its C.G. is out of the centre, and the C.G. is vertically below the centre, it will be in *stable equilibrium.*

2. If the same sphere is placed so that its C.G. is vertically above the centre, it will be in *unstable equilibrium.*

3. If the sphere is formed of homogeneous material so that its C.G. is at the centre, it will be in *neutral or indifferent equilibrium.*

Transverse Metacentre.—We shall deal first with transverse inclinations, because they are the more important, and deal with inclinations in a longitudinal or fore-and-aft direction in the next chapter.

Let Fig. 45 represent the section of a ship steadily inclined

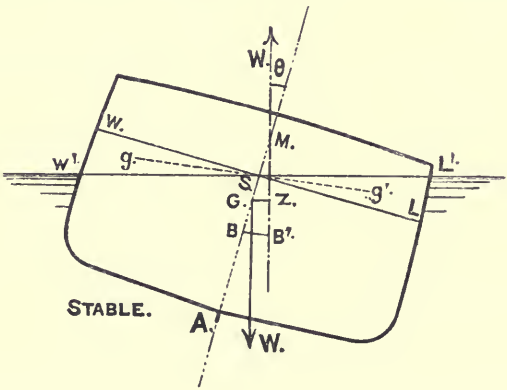

FIG. 45.

at a small angle from the upright by some external force, such as the wind. The vessel has the same weight before and after the inclination, and consequently has the same volume of displacement. We must assume that no weights on board shift, and consequently the centre of gravity remains in the same position in the ship. But although the total volume of displacement remains the same, the shape of this volume changes, and consequently the centre of buoyancy will shift from its original position. In the figure the ship is represented by the section, WAL being the immersed section when upright, WL being the position of the water-line on the ship. On being inclined, W′L′ becomes the water-line, and W′AL′ represents tbe immersed volume of the ship, which, although different in shape, must have the same volume as the original immersed volume WAL.

The wedge-shaped volume represented by WSW′, which has come out of the water, is termed the "*emerged*" or "*out*" wedge. The wedge-shaped volume represented by LSL′, which has gone into the water, is termed the "*immersed*" or "*in*" wedge. Since the ship retains the same volume of displacement, it follows that the volume of the emerged wedge WSW′ is equal to the volume of the immersed wedge LSL′. It is only for small angles of inclination that the point S, where the water-lines intersect, falls on the middle line of the vessel. For larger angles it moves further out, as shown in Fig. 77.

Now consider the vessel inclined at a small angle from the upright, as in Fig. 45. The new volume of displacement W′AL′ has its centre of buoyancy in a certain position, say B′. This position might be calculated from the drawings in the same manner as we found the point B, the original centre of buoyancy; but we shall see shortly how to fix the position of the point B′ much more easily.

B′ being the new centre of buoyancy, the upward force of the buoyancy must act through B′, while the weight of the ship acts vertically down through G, the centre of gravity of the ship. Suppose the vertical through B′ cuts the middle line of the ship in M; then we shall have two equal forces acting on the ship, viz.—

(1) Weight acting vertically down through the centre of gravity.

(2) Buoyancy acting vertically up through the new centre of buoyancy.

But they do not act in the same vertical line. Such a system of forces is termed a *couple*. Draw GZ perpendicular to the vertical through B′. Then the equal forces act at a distance from each other of GZ. This distance is termed the *arm* of the couple, and the *moment* of the couple is W × GZ. On looking at the figure, it is seen that the couple is tending to take the ship back to the upright. If the relative positions of G and M were such that the couple acted as in Fig. 46, the couple would tend to take the ship farther away from the upright; and again, if G and M coincided, we should have the forces acting in the same vertical line, and consequently no

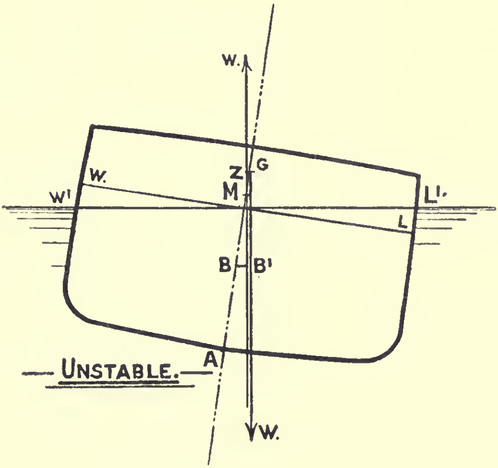

FIG. 46.

couple at all, and the ship would have no tendency to move either to the upright or away from it.

We see, therefore, that for a ship to be in stable equilibrium for any direction of inclination, it is necessary that the metacentre be above the centre of gravity of the ship. We now group together the three conditions which must be fulfilled in order that a ship may float freely and at rest in stable equilibrium—

(1) The weight of water displaced must equal the total weight of the ship (see p. 21).

(2) The centre of gravity of the ship must be in the same vertical line as the centre of gravity of the displaced water (centre of buoyancy) (see p. 89).

(3) The centre of gravity of the ship must be below the metacentre.

For small transverse inclinations, M is termed the *transverse metacentre*, which we may accordingly define as follows :—

For a given plane of flotation of a vessel in the upright condition, let B be the centre of buoyancy, and BM the vertical through it. Suppose the vessel inclined transversely through a very small angle, retaining the same volume of displacement, B′ being the new centre of buoyancy, and B′M the vertical through it, meeting BM in M. Then this point of intersection, M, is termed the *transverse metacentre.*

There are two things in this definition that should be noted : (1) the angle of inclination is supposed very small, and (2) the volume of displacement remains the same.

It is found that, for all practical purposes, in ordinary ships the point M does not change in position for inclinations up to as large as 10° to 15°; but beyond this it takes up different positions.

We may now say, with reference to a ship's initial stability or stability in the upright condition—

(1) If G is below M, the ship is in stable equilibrium.

(2) If G is above M, the ship is in unstable equilibrium.

(3) If G coincides with M, the ship is in neutral or indifferent equilibrium.

We thus see how important the relative positions of the centre of gravity and the transverse metacentre are as affecting a ship's initial stability. The distance GM is termed the *transverse metacentric height*, or, more generally, simply the *metacentric height.*

We have seen that for small angles M remains practically in a constant position, and consequently we may say GZ $= \text{GM} . \sin \theta$ for angles up to 10° to 15°, say. GZ is the arm of the couple, and so we can say that the moment of the couple is—

$$W \times \text{GM} . \sin \theta$$

If M is above G, this moment tends to right the ship, and we may therefore say that the *moment of statical stability* at the angle θ is—

$$W \times GM . \sin \theta$$

This is termed the *metacentric method* of determining a vessel's stability. It can only be used at small angles of inclination to the upright, viz. up to from 10 to 15 degrees.

Example.—A vessel of 14,000 tons displacement has a metacentric height of $3\frac{1}{2}$ feet. Then, if she is steadily inclined at an angle of 10°, the tendency she has to return to the upright, or, as we have termed it, the moment of statical stability, is—

$$14{,}000 \times 3\cdot5 \times \sin 10° = 8506 \text{ foot-tons}$$

We shall discuss later how the distance between G and M, or the metacentric height, influences the behaviour of a ship, and what its value should be in various cases; we must now investigate the methods which are employed by naval architects to determine the distance for any given ship.

There are two things to be found, viz. (1) the position of G, the centre of gravity of the vessel; (2) the position of M, the transverse metacentre.

Now, G depends solely upon the vertical distribution of the weights forming the structure and lading of the ship, and the methods employed to find its position we shall deal with separately; but M depends solely upon the form of the ship, and its position can be determined when the geometrical form of the underwater portion of the ship is known. Before we proceed with the investigation of the rules necessary to do this, we must consider certain geometrical principles which have to be employed.

Centre of Flotation.—*If a floating body is slightly inclined so as to maintain the same volume of displacement, the new water-plane must pass through the centre of gravity of the original water-plane.* In order that the same volume of displacement may be retained, the volume of the immersed wedge SLL_1, Fig 47, must equal the volume of the emerged wedge SWW_1. Call y an ordinate on the immersed side, and y' an ordinate on the emerged side of the water-plane. Then the areas of the sections of the immersed and emerged wedges are respectively (since $LL_1 = y \,.\, d\theta$, $WW_1 = y' \,.\, d\theta$, $d\theta$ being the small angle of inclination)—

$$\tfrac{1}{2}y^2 \,.\, d\theta, \qquad \tfrac{1}{2}(y')^2 \,.\, d\theta$$

and using the notation we have already employed—

$$\text{Volume of immersed wedge} = \tfrac{1}{2}\int y^2 \,.\, d\theta \,.\, dx$$
$$\text{,, emerged ,,} = \tfrac{1}{2}\int (y')^2 \,.\, d\theta \,.\, dx$$

and accordingly—

$$\tfrac{1}{2}\int y^2 \,.\, d\theta \,.\, dx = \tfrac{1}{2}\int (y')^2 \,.\, d\theta \,.\, dx$$
$$\text{or } \tfrac{1}{2}\int y^2 \,.\, dx = \tfrac{1}{2}\int (y')^2 \,.\, dx$$

But $\tfrac{1}{2}\int y^2 \,.\, dx$ is the moment of the immersed portion of the water-plane about the intersection, and $\tfrac{1}{2}\int (y')^2 \,.\, dx$ is the moment of the emerged portion of the water-plane about the intersection (see p. 57); therefore the moment of one side of the water-plane about the intersection is the same as the moment of the other side, and consequently the line of intersection passes through the centre of gravity of the water-plane. The centre of gravity of the water-plane is termed the *centre of flotation.* In whatever direction a ship is inclined, transversely, longitudinally, or in any intermediate direction, through a small angle, the line of intersection of the new water-plane with the original water-plane must always pass through the centre of flotation. For transverse inclinations of a ship the line of intersection is the centre line of the water-plane; for longitudinal inclinations the fore-and-aft position of the centre of flotation has to be calculated, as we shall see when we deal with longitudinal inclinations.

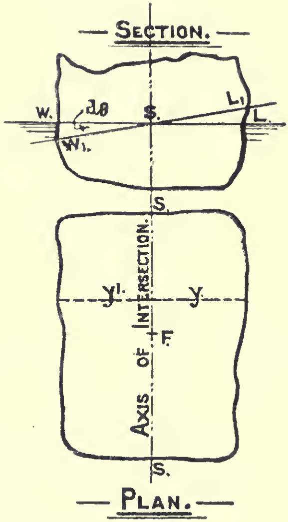

FIG. 47.

Shift of the Centre of Gravity of a Figure due to the Shift of a Portion of the Figure.—Let ABCD, Fig. 48, be a square with its centre at G; this point will also be its centre of gravity. Suppose one corner of the square EF is taken away and placed in the position FK, forming a new figure, ADKHGE. We wish to find the centre of gravity of this new figure. The centre of gravity of the original figure was at G, and a portion of it, EF, with its centre of gravity at g, has been shifted so that its centre of gravity now is at g'. Then this important principle holds good—

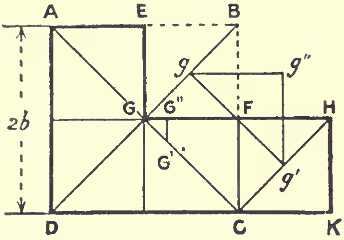

FIG. 48.

The centre of gravity of the figure will shift to G′, such that GG′ is parallel to gg', and if A be the original area of the square, and a be the area shifted—

$$GG' = \frac{a \times gg'}{A}$$

In this case, if $2b$ be a side of the square—

$$A = 4b^2$$
$$a = b^2$$
$$gg' = b\sqrt{2}$$
$$\text{and therefore } GG' = \frac{b^2 \times b\sqrt{2}}{4b^2} = 0{\cdot}353b$$

In the same way, gg'' being the horizontal shift of the centre of gravity of the corner EF, the horizontal shift of the centre of gravity of the whole area will be given by—

$$GG'' = \frac{a \times gg''}{A}$$

$$\text{In this case } gg'' = b$$
$$\text{and therefore } GG'' = \tfrac{1}{4}b$$

The same principle applies to the shift of the centre of gravity of a volume or a weight due to the shift of a portion of it. The small portion multiplied by its shift is equal to the whole body multiplied by its shift, and the shifts are in parallel directions.

The uses that are made of this will become more apparent as we proceed, but the following examples will serve as illustrations :—

Example.—A vessel weighing W tons has a weight w tons on the deck. This is shifted transversely across the deck a distance of d feet, as in Fig. 49. Find the shift of the C.G. of the vessel both in direction and amount.

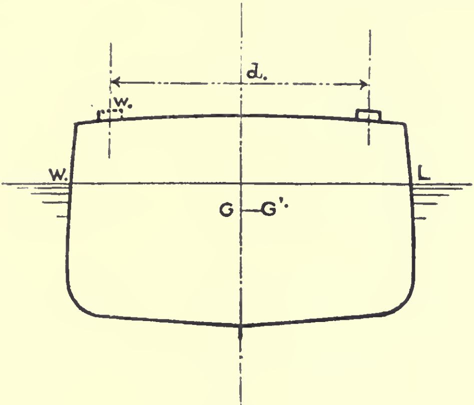

FIG. 49.

G will move to G′ such that GG′ will be parallel to the line joining the original and final positions of the weight w;

$$\text{and } GG' = \frac{w \times d}{W}$$

If $w = 70$ tons, $d = 30$ feet, W = 5000 tons, then—

$$GG' = \frac{70 \times 30}{5000} = \frac{21}{50} \text{ feet} = 0{\cdot}42 \text{ foot}$$

Example.—In a vessel of 4000 tons displacement, suppose 100 tons of coal to be shifted so that its C.G. moves 18 feet transversely and $4\frac{1}{2}$ feet vertically. Find the shift of the C.G. of the vessel.

The C.G. will move horizontally an amount equal to $\frac{100 \times 18}{4000} = 0{\cdot}45$ ft.

and vertically an amount equal to $\frac{100 \times 4{\cdot}5}{4000} = 0{\cdot}1125$ ft.

Moment of Inertia.—We have dealt in Chapter II. with the *moment* of a force about a given point, and we defined it as the product of the force and the perpendicular distance of its line of action from the point; also the moment of an area about a given axis as being the area multiplied by the distance of its centre of gravity from the axis. We could find the moment of a large area about a given axis by dividing it into a number of small areas and summing up the moments of all these small areas about the axis. In this we notice that the area or force is multiplied simply by the distance. Now we have to go a step further, and imagine that each small area is multiplied by the *square of its distance* from a given axis. If all such products are added together for an area, we should obtain not the simple moment, but what may be termed the

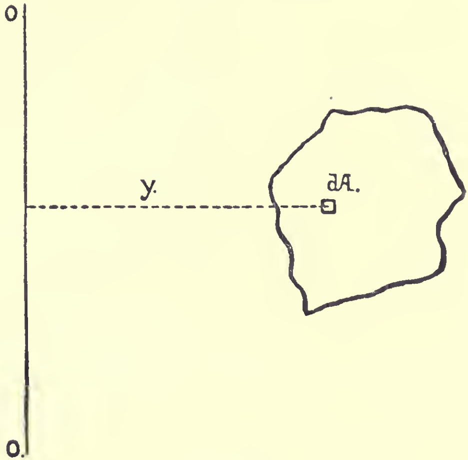

FIG. 50.

moment of the second degree, or more often the *moment of inertia* of the area about the given axis.[1] We therefore define the moment of inertia of an area about a given axis as follows:—

[1] This is the geometrical moment of inertia. Strictly speaking, moment of inertia involves the mass of the body. We make here the same assumption that we did in simple moments (p. 47), viz. that the area is the surface of a very thin lamina or plate of homogeneous material of uniform thickness.

Imagine the area divided into very small areas, and each such small area multiplied by the square of its distance from the given axis; then, if all these products be added together, we shall obtain the moment of inertia of the total area about the given axis.

Thus in Fig. 50, let OO be the axis. Take a very small area, calling it dA, distance y from the axis. Then the sum of all such products as $dA \times y^2$, or (using the notation we have employed) $\int y^2 . dA$, will be the moment of inertia of the area about the axis OO.

To determine this for any figure requires the application of advanced mathematics, but the result for certain regular figures are given below.

It is found that we can always express the moment of inertia, often written I, of a plane area about a given axis by the expression—

$$nAh^2$$

where A is the area of the figure;

h is the depth of the figure perpendicular to the axis;
n is a coefficient depending on the shape of the figure and the position of the axis.

First, when the axis is through the centre of gravity of the figure parallel to the base, as in Figs. 51 and 52—

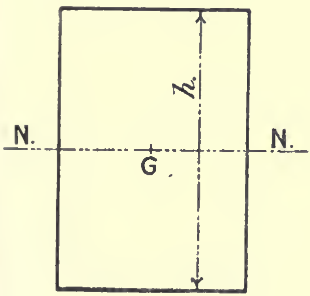

FIG. 51.

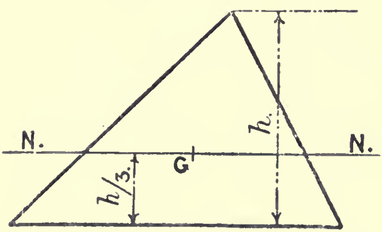

FIG. 52.

for a circle $n = \frac{1}{16}$, so that $I = \frac{1}{16}Ah^2$
for a rectangle $n = \frac{1}{12}$, „ $I = \frac{1}{12}Ah^2$
for a triangle $n = \frac{1}{18}$, „ $I = \frac{1}{18}Ah^2$

Second, when the axis is one of the sides—

for a rectangle $n = \frac{1}{3}$, so that $I = \frac{1}{3}Ah^2$

for a triangle $n = \frac{1}{6}$, " $I = \frac{1}{6}Ah^2$

Example.—Two squares of side a are joined to form a rectangle. The I of each square about the common side is—

$$\tfrac{1}{3}(a^2)a^2 \qquad (a^2 = \text{area})$$

the I of both about the common side will be the sum of each taken separately, or—

$$\tfrac{2}{3}a^4$$

If, however, we took the whole figure and treated it as a rectangle, its I about the common side would be—

$$\tfrac{1}{12}(2a^2)(2a)^2 = \tfrac{2}{3}a^4 \qquad (\text{area} = 2a^2)$$

which is the same result as was obtained before.

To find the moment of inertia of a plane figure about an axis parallel to and a given distance from an axis through its centre of gravity.

Suppose the moment of inertia about the axis NN passing through the centre of gravity of the figure (Fig. 53) is I_0, the area of the figure is A, and OO, the given axis, is parallel to NN and a distance y from it. Then the moment of inertia (I) of the figure about OO is given by—

$$I = I_0 + Ay^2$$

The moment of inertia of an area about any axis is therefore determined by adding to the moment of inertia of the area about a parallel axis through the centre of gravity, the product of the area into the square of the distance between the two axes. We see from this that the moment of inertia of a figure about an axis through its own centre of gravity is always less than about any other axis parallel to it.

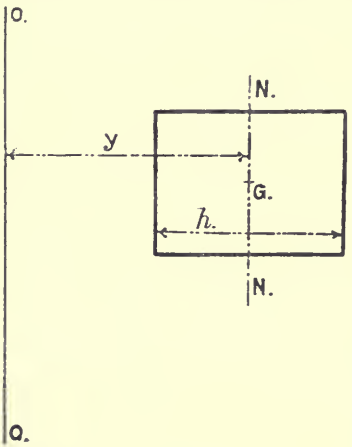

FIG. 53.

Example.—Having given the moment of inertia of the triangle in Fig. 52 about the axis NN through the centre of gravity as $\frac{1}{18}Ah^2$, find the moment of inertia about the base parallel to NN.

Applying the above rule, we have—

$$I = \frac{1}{18}Ah^2 + A\left(\frac{h}{3}\right)^2$$
$$= \frac{1}{6}Ah^2$$

which agrees with the value given above for the moment of inertia of a triangle about its base.

Example.—Find the moment of inertia of a triangle of area A and height h about an axis through the vertex parallel to the base.

Ans. $\frac{1}{2}Ah^2$.

Example.—A rectangle is 4 inches long and 3 inches broad. Compare the ratio of its moment of inertia about an axis through the centre parallel to the long and short sides respectively.

Ans. 9 : 16.

Example.—A square of 12 inches side has another symmetrical square of half its area cut out of the centre. Compare the moments of inertia about an axis through the centre parallel to one side of, the original square, the square cut out, the remaining area.

Ans. As 4 : 1 : 3, the ratio of the areas being 4 : 2 : 2.

This last example illustrates the important fact that if an area is distributed away from the centre of gravity, the moment of inertia is very much greater than if the same area were massed near the centre of gravity.

To find the Moment of Inertia of a Plane Curvilinear Figure (as Fig. 36, p. 57) **about its Base.**—Take a strip PQ of length y and breadth (indefinitely small) dx. Then, if we regard PQ as a rectangle, its moment of inertia about the base DC is—

$$\tfrac{1}{3}(y \,.\, dx)y^2 = \tfrac{1}{3}y^3 \,.\, dx \qquad (y \,.\, dx = \text{area})$$

and the moment of inertia of the whole figure about DC will be the sum of all such expressions as this ; or—

$$\int \tfrac{1}{3}y^3 \,.\, dx$$

that is, we put the third part of the cubes of the ordinates of the curve through either of Simpson's rules. For the water-plane of a ship (for which we usually require to find the moment of inertia about the centre line), we must add the moment of inertia of both sides together: and, since these are symmetrical, we have—

$$I = \tfrac{2}{3}\int y^3 \,.\, dx \qquad (y = \text{semi-ordinate of water-plane})$$

In finding the moment of inertia of a water-plane about the centre line, the work is arranged as follows :—

Number of ordinate.	Semi-ordinates of water-plane.	Cubes of semi-ordinates.	Simpson's multipliers.	Functions of cubes.
1	0·05	—	1	—
2	4·65	101	4	404
3	10·05	1015	2	2,030
4	14·30	2924	4	11,696
5	16·75	4699	2	9,398
6	17·65	5498	4	21,992
7	17·40	5268	2	10,536
8	16·20	4252	4	17,008
9	13·55	2488	2	4,976
10	9·65	899	4	3,596
11	3·65	49	1	49
				81,685

Common interval = 28 feet

Moment of inertia $= 81{,}685 \times \frac{2}{3} \times \frac{28}{3} = 508{,}262$ [1]

The semi-ordinates are placed in column 2, and the cubes of these are placed in column 3. It is not necessary, in ordinary cases, to put any decimal places in the cube; the nearest whole number is sufficient. It is best to take the cubes out of Barlow's tables or out of a pocket-book, as "Mackrow," since the labour of cubing the numbers is very great. These cubes are put through Simpson's multipliers in the ordinary way, giving column 5. The sum of the functions of cubes has to be treated as follows: First there is the multiplier for Simpson's rule, viz. $\frac{1}{3} \times 28$, and then the $\frac{2}{3}$ of the expression $\frac{2}{3}\int y^3 . dx$, which takes into account both sides. The multiplier, therefore, is $\frac{2}{3} \times \frac{28}{3}$, and the sum of the numbers in column 5 multiplied by this will give the moment of inertia required.

Approximation to the Moment of Inertia of a Ship's Water-plane about the Centre Line.—We have seen that for certain regular figures we can express the moment of inertia about an axis through the centre of gravity in the form $n\text{A}h^2$, where n is a coefficient varying for each figure. We can, in the same way, express the I of a water-plane area

[1] This calculation for the L.W.P. is usually done on the displacement sheet.

about the centre line, but it is not convenient to use the area as we have done above. We know that the area can be expressed in the form—

$$k \times L \times B$$

where L is the extreme length;
B „ „ breadth;
k is a coefficient of fineness;

so that we can write—

$$I = nLB^3$$

where n is a new coefficient that will vary for different shapes of water-planes. If we can find what the values of the coefficient n are for ordinary water-planes, it would be very useful in checking our calculation work. Taking the case of a L.W.P. in the form of a rectangle, we should find that $n =$ 0·08, and for a L.W.P. in the form of two triangles, $n = 0·02$.

These are two extreme cases, and we should expect for ordinary ships the value of the coefficient n would lie between these values. This is found to be the case, and we may take the following approximate values for the value of n in the formula $I = nLB^3$:—

For ships whose load water-planes are extremely fine ...	0·04
„ „ „ „ moderately fine ...	0·05
„ „ „ „ very full	0·06

For the water-plane whose moment of inertia we calculated above, we have, length 280 feet, breadth 35·3 feet, and I = 508,262 in foot-units. Therefore the value of the coefficient n is—

$$\frac{508262}{280 \times (35·3)^3} = 0·041$$

Formula for finding the Distance of the Transverse Metacentre above the Centre of Buoyancy (BM). — We have already discussed in Chapter II. how the position of the centre of buoyancy can be determined if the underwater form of the ship is known, and now we proceed to discuss how the distance BM is found. Knowing this, we are able to fix the position of the transverse metacentre in the ship.

Let Fig. 45, p. 90, represent a ship heeled over to a very small angle θ (much exaggerated in the figure).

B is the centre of buoyancy in the upright position when floating at the water-line WL.

B′ is the centre of buoyancy in the inclined position when floating at the water-line W′L′.

v is the volume of either the immersed edge LSL′ or the emerged wedge WSW′.

V is the total volume of displacement.

g is the centre of gravity of the emerged wedge.

g' is the centre of gravity of the immersed wedge.

Then, using the principle given on p. 96, BB′ will be parallel to gg', and—

$$BB' = \frac{v \times gg'}{V}$$

since the new displacement is formed by taking away the wedge WSW′ from the original displacement, and putting it in the position LSL′.

Now for the very small angle of inclination, we may say that—

$$\frac{BB'}{BM} = \sin \theta$$

$$\text{or } BB' = BM \sin \theta$$

so that we can find BM if we can determine the value of $v \times gg'$, since V, the volume of displacement, is known.

Let Fig. 54 be a section of the vessel; wl, $w'l'$, the original and new water-lines respectively, the angle of inclination being very small. Then we may term wSw' the emerged triangle, and lSl' the immersed triangle, being transverse sections of the emerged and immersed wedges, and ww', ll' being for all practical purposes straight lines. If y be the half-breadth of the water-line at this section, we can say $ww' = ll' = y \sin \theta$, and the area of either of the triangles is—

$$\tfrac{1}{2}y \times y \sin \theta = \tfrac{1}{2}y^2 \sin \theta$$

Let a, a' be the centres of gravity of the triangles wSw', lSl' respectively; then we can say, seeing that θ is very small, that

$aa' = \frac{4}{3}y$, since the centre of gravity of a triangle is two-thirds the height from the apex. The new immersed section being regarded as formed by the transference of the triangle wSw'

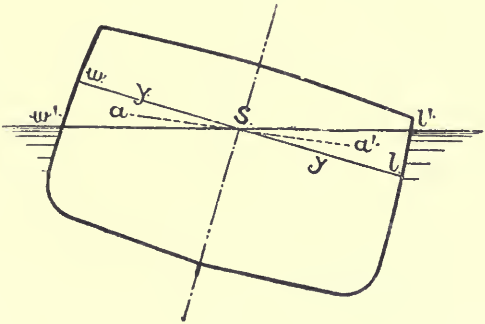

FIG. 54.

to the position occupied by the triangle lSl', the moment of transference is—

$$(\tfrac{1}{2}y^2 \sin\theta) \times \tfrac{4}{3}y = \tfrac{2}{3}y^3 \sin\theta$$

and for a very small length dx of the water-line the moment will be—

$$\tfrac{2}{3}y^3 \sin\theta \,.\, dx$$

since the small volume is $\frac{1}{2}y^2 \sin\theta \,.\, dx$, and the shift of its centre of gravity is $\frac{4}{3}y$. If now we summed all such expressions as this for the whole length of the ship, we should get the moment of the transference of the wedge, or $v \times gg'$. Therefore we may say, using the ordinary notation—

$$\begin{aligned} v \times gg' &= \int \tfrac{2}{3}y^3 \sin\theta \,.\, dx \\ &= \tfrac{2}{3} \sin\theta \int y^3 \,.\, dx \end{aligned}$$

therefore we have—

$$\text{BB}' = \text{BM} \sin\theta = \frac{v \times gg'}{\text{V}} = \frac{\frac{2}{3} \sin\theta \int y^3 \,.\, dx}{\text{V}}$$

$$\text{or BM} = \frac{\frac{2}{3}\int y^3 \,.\, dx}{\text{V}}$$

But the numerator of this expression is what we have found to be the moment of inertia of a water-plane about its centre line, y being a semi-ordinate; therefore we can write—

$$BM = \frac{I}{V}$$

We have seen, on p. 101, how the moment of inertia of a water-plane is found for any given case, and knowing the volume of displacement, we can then determine the distance BM, and so, knowing the position of the C.B., fix the position of the transverse metacentre in the ship.

Example.—A lighter is in the form of a box, 120 feet long, 30 feet broad, and floats at a draught of 10 feet. Find its transverse BM.

In this case the water-plane is a rectangle 120′ × 30′, and we want its I about the middle line. Using the formula for the I of a rectangle about an axis through its centre parallel to a side, $\frac{1}{12}Ah^2$, we have—

$$I = \frac{1}{12} \times 3600 \times 900 \qquad (h = 30)$$
$$= 270,000$$

$$V, \text{ the volume of displacement}, = 120 \times 30 \times 10 = 36,000$$

$$\therefore BM = \frac{270,000}{36,000} = 7 \cdot 5 \text{ feet}$$

Example.—A pontoon of 10 feet draught has a constant section in the form of a trapezoid, breadth at the water-line 30 feet, breadth at base 20 feet, length 120 feet. Find the transverse BM.

Ans. 9 feet.

It will be noticed that the water-plane in this question is the same as in the previous question, but the displacement being less, the BM is greater. M is therefore higher in the ship for two reasons. BM is greater and B is higher in the second case.

Example.—A raft is formed of two cylinders 5 feet in diameter, parallel throughout their lengths, and 10 feet apart, centre to centre. The raft floats with the axes of the cylinders in the surface. Find the transverse BM.

We shall find that the length does not affect the result, but we will suppose the length is l feet. We may find the I of the water-plane in two ways. It consists of two rectangles each $l' \times 5'$, and their centre lines are 10 feet apart.

1. The water-plane may be regarded as formed by cutting a rectangle $l' \times 5'$ out of a rectangle $l' \times 15'$;

$$\therefore I = \frac{1}{12}(l \times 15) \times 15^2 - \frac{1}{12}(l \times 5) \times 5^2$$
$$= \frac{1}{12}l(15^3 - 5^3)$$
$$= \frac{3250}{12}l$$

this being about a fore-and-aft axis at the centre of the raft.

2. We may take the two rectangles separately, and find the I of each about the centre line of the raft, which is 5 feet from the line through the centre of each rectangle. Using the formula—

$$I = I_0 + Ay^2$$
$$= \frac{1}{12}(l \times 5)5^2 + (l \times 5)5^2$$
$$= \frac{1625}{12}l$$

and for both rectangles the moment of inertia will be twice this, or $\frac{3250}{12}l$, as obtained above.

We have to find the volume of displacement, which works out to $\frac{275}{14}l$ cubic feet. The distance BM is therefore—

$$\frac{3250}{12}l \div \frac{275}{14}l = 13{\cdot}8 \text{ feet}$$

Example.—A raft is formed of three cylinders, 5 feet in diameter, parallel and symmetrical throughout their lengths, the breadth extreme being 25 feet. The raft floats with the axes of the cylinders in the surface. Find the transverse BM.

The moment of inertia of the water-plane of this raft is best found by using the formula $I = I_0 + Ay^2$ for the two outside rectangles, and adding it to I_0, the moment of inertia of the centre rectangle about the middle line. We therefore have for the whole water-plane $I = \frac{4125}{4}l$, where $l =$ the length; and the volume of displacement being $\frac{825}{28}l$, the value of BM will be 35 feet.

Approximate Formula for the Height of the Transverse Metacentre above the Centre of Buoyancy.—The formula for BM is—

$$BM = \frac{I}{V}$$

We have seen that we may express I as $n\text{LB}^3$, where n is a coefficient which varies for different shapes of water-planes, but which will be the same for two ships whose water-planes are similar.

We have also seen that we may express V as $k\text{LBD}$, where D is the mean moulded draft (to top of keel amidships), and k is a coefficient which varies for different forms, but which will be the same for two ships whose under-water forms are similar. Therefore we may say—

$$\begin{aligned} BM &= \frac{n \times L \times B^3}{k \times L \times B \times D} \\ &= a \cdot \frac{B^2}{D} \end{aligned}$$

where a is a coefficient obtained from the coefficients n and k. Sir William White, in the "Manual of Naval Architecture," gives the value of a as being between 0·08 and 0·1, a usual value for merchant ships being 0·09. The above formula shows very clearly that the breadth is more effective than the draught in determining what the value of BM is in any given case. It will also be noticed that the length is not brought in.

The ship for which the moment of inertia of a water-plane was calculated on p. 102, had a displacement of 1837 tons up to that water-plane. The value of BM is therefore—

$$\frac{508262}{1837 \times 35} = 7 \cdot 91 \text{ feet}$$

The breadth and mean draught were 35·3 and $13\frac{1}{4}$ feet respectively. Consequently the value of the coefficient a is 0·084.

To prove that a Homogeneous Log of Timber of Square Section and Specific Gravity 0·5 cannot float in Fresh Water with One of its Faces Horizontal.— The log having a specific gravity of 0·5 will float, and will float with half its substance immersed. The condition that it shall float in stable equilibrium, as regards transverse inclination, in any position is that the transverse metacentre shall be above the centre of gravity.

Let the section be as indicated in Fig. 55, with side length $2a$. And suppose the log is placed in the water with one side of this section horizontal. Then the draught-line will be at a distance a from the bottom, and the log, being homogeneous, *i.e.* of the same quality all through, will have its C.G. in the middle at G, at a distance also of a from the bottom. The centre of buoyancy will be at a distance of $\frac{a}{2}$ from the bottom. The height of the transverse metacentre above the centre of buoyancy is given by—

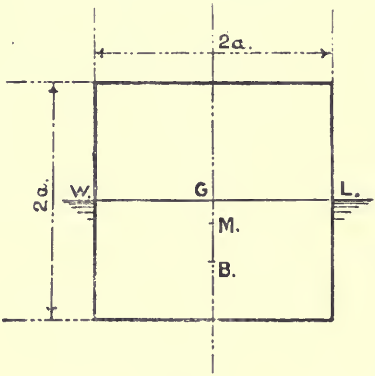

FIG. 55.

$$\text{BM} = \frac{\text{I}}{\text{V}}$$

where I = moment of inertia of water-plane about a longitudinal axis through its centre

V = volume of displacement in cubic feet.

Now, the water-plane of the log is a rectangle of length l and breadth $2a$, and therefore—

$$\text{its I} = \frac{1}{12} l . 2a(2a)^2 = \frac{8}{12} la^3$$
$$\text{and V} = l . 2a . a = 2la^2$$
$$\therefore \text{BM} = \frac{8}{12} la^3 \div 2la^2 = \frac{1}{3} a$$
$$\text{But BG} = \frac{1}{2} a$$

therefore the transverse metacentre is below the centre of gravity, and consequently the log cannot float in the position given.

If, now, the log be assumed floating with one corner downward, it will be found by a precisely similar method that—

$$\text{BG} = 0{\cdot}471a$$
$$\text{and BM} = 0{\cdot}943a$$

Thus in this case the transverse metacentre is above the centre of gravity, and consequently the log will float in stable equilibrium.

It can also be shown by similar methods that the position of stable equilibrium for all directions of inclination of a cube composed of homogeneous material of specific gravity 0·5 is with one corner downwards.

Metacentric Diagram.—We have seen how the position of the transverse metacentre can be determined for any given ship floating at a definite water-line. It is often necessary, however, to know the position of the metacentre when the ship is floating at some different water-line; as, for instance, when coal or stores have been consumed, or when the ship is in a light condition. It is usual to construct a diagram which will show at once, for any given mean draught which the vessel may have, the position of the transverse metacentre. Such a diagram is shown in Fig. 56, and it is constructed in the following manner: A line W_1L_1 is drawn to represent the load water-line, and parallel to it are drawn W_2L_2, W_3L_2, W_4L_4 to represent the water-lines Nos. 2, 3, and 4, which are used for calculating the displacement, the proper distance apart, a convenient scale being ½ inch to 1 foot. A line L_1L_4 is drawn cutting these level lines, and inclined to them at an angle of 45°. Through the points of intersection L_1, L_2, L_3, L_4, are drawn vertical lines as shown. The ship is then supposed to float successively at these water-lines, and the position of the centre of buoyancy and the distance of the transverse metacentre above the C.B.

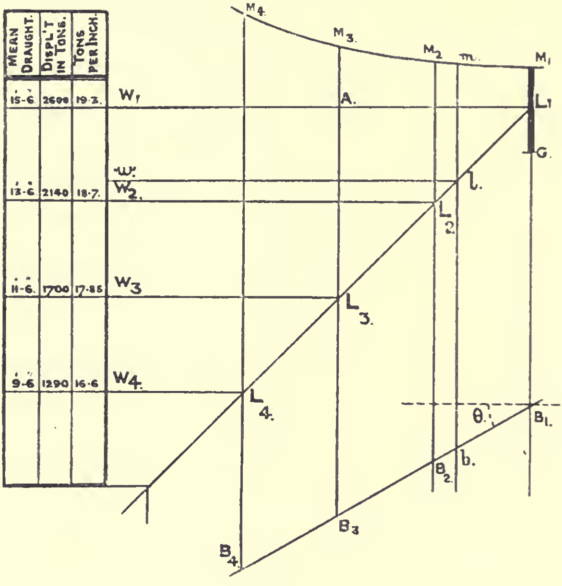

Fig. 56.

calculated for each case. The methods employed for finding the position of the C.B. at the different water-lines have already been dealt with in Chapter II. On the vertical lines are then set down from the L.W.L. the respective distances of the centres of buoyancy below the L.W.L. Thus L_1B_1 is the distance when floating at the L.W.L., and AB_3 the distance when floating at No. 3 W.L. In this way the points B_1, B_2, B_3, B_4 are obtained; and if the calculations are correct, a fair line can be drawn passing through all these spots as shown. Such a curve is termed the *curve of centres of buoyancy*. It is usually found to be rather a flat curve, being straight near the load-line condition. The distance BM for each water-line is then set up from B_1, B_2, B_3, B_4 respectively, giving the points M_1, M_2, M_3, M_4. A curve can then be drawn through these points, which is termed *the curve of transverse metacentres*. Now, suppose the ship is floating at some intermediate water-line—say *wl*: through *l*, where *wl* cuts the 45° line, draw a vertical cutting the curves of centres of buoyancy and metacentres in *b* and *m* respectively. Then *m* will be the position of the transverse metacentre of the ship when floating at the water-line *wl*. It will be noticed that we have supposed the ship to float always with the water-plane parallel to the L.W.P.; that is to say, she does not alter trim. For water-planes not parallel to the L.W.P. we take the mean draught (*i.e.* the draughts at the fore-and-aft perpendiculars are added together and divided by 2), and find the position of M on the metacentric diagram for the water-plane, parallel to the L.W.P., corresponding to this mean draught. Unless the change of trim is very considerable, this is found to be correct enough for all practical purposes. Suppose, however, the ship trims very much by the stern,[1] owing to coal or stores forward being consumed, the shape of her water-plane will be very different from the shape it would have if she were floating at her normal trim or parallel to the L.W.P.; generally the water-plane will be fuller under these circumstances, and the moment of inertia will be greater, and consequently M higher in the ship, than would be given on the metacentric diagram. When a ship is inclined, an operation that will be described later, she is frequently in an unfinished condition, and trims considerably by the stern. It is necessary to know the position of the transverse metacentre accurately for this condition, and

[1] This would be the case in the following: A ship is designed to float at a draught of 17 feet forward and 19 feet aft, or, as we say, 2 feet by the stern. If her draught is, say, 16 feet forward and 20 feet aft, she will have the same mean draught as designed, viz. 18 feet, but she will trim 2 feet more by the stern.

consequently the metacentric diagram cannot be used, but a separate calculation made for the water-plane at which the vessel is floating.

On the metacentric diagram is placed also the position of the centre of gravity of the ship under certain conditions. For a merchant ship these conditions may vary considerably owing to the nature of the cargo carried. There are two conditions for which the C.G. may be readily determined, viz. the light condition, and the condition when loaded to the load-line with a homogeneous cargo. The *light condition* may be defined as follows: No cargo, coal, stores, or any weights on board not actually forming a part of the hull and machinery, but including the water in boilers and condensers. The draught-lines for the various conditions are put on the metacentric diagram, and the position of the centre of gravity for each condition placed in its proper vertical position. The various values for GM, the metacentric height, are thus obtained:

On the left of the diagram are placed, at the various water-lines, the mean draught, displacement, and tons per inch.

There are two forms of section for which it is instructive to construct the metacentric diagram.

1. A floating body of constant rectangular section.

2. A floating body of constant triangular section, the apex of the triangle being at the bottom.

1. For a body having a constant rectangular section, the moment of inertia of the water-plane is the same for all draughts, but the volume of displacement varies. Suppose the rectangular box is 80 feet long, 8 feet broad, 9 feet deep. Then the moment of inertia of the water-plane for all draughts is—

$$\frac{1}{12}(80 \times 8) \times 8^2 = \frac{10240}{3}$$

The volumes of displacement are as follows:—

Draught 6 inches	... ...	$V = 80 \times 8 \times \frac{1}{2}$ cubic feet
,, 1 foot ...	... ...	$V = 80 \times 8$,,
,, 2 feet ...	... ...	$V = 80 \times 8 \times 2$,,
,, 4 ,, ...	... ...	$V = 80 \times 8 \times 4$,,
,, 7 ,, ...	... ...	$V = 80 \times 8 \times 7$,,
,, 9 ,, ...	... ...	$V = 80 \times 8 \times 9$,,

and the values of BM are therefore as follows :—

Draught 6 inches	... ...	BM = 10·66 feet
,, 1 foot	... ...	BM = 5·33 ,,
,, 2 feet	... ...	BM = 2·66 ,,
,, 4 ,,	... ...	BM = 1·33 ,,
,, 7 ,,	... ...	BM = 0·76 ,,
,, 9 ,,	... ...	BM = 0·59 ,,

The centre of buoyancy is always at half-draught, so that its locus or path will be a straight line,[1] and if the values obtained above are set off from the centres of buoyancy at the various water-lines, we shall obtain the curve of transverse metacentres as shown in Fig. 57 by the curve AA, the line BB being the corresponding locus of the centres of buoyancy.

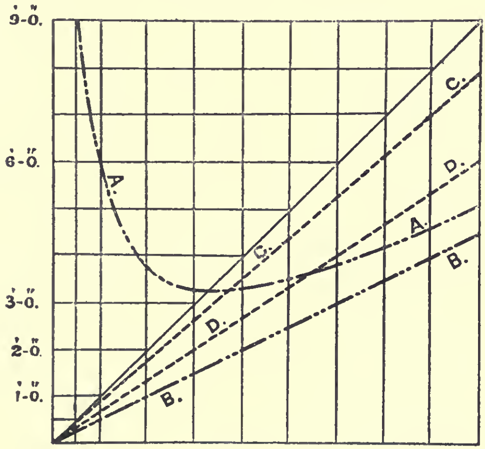

Fig. 57.

2. For a floating body with a constant triangular section, the locus of centres of buoyancy is also a straight line, because it is always two-thirds the draught above the base.[1] Suppose the triangular section to be 10 feet broad at the top and 9 feet deep, the length of the body being 120 feet. In this case we must calculate the moment of inertia of each water-plane and the volume of displacement up to each. The results are found to be as follows :—

[1] This may be seen by finding a few spots on this locus.

Draught 1 foot	... ...	BM = 0·20 feet
,, 2 feet	... ...	BM = 0·41 ,,
,, 4 ,,	... ...	BM = 0·82 ,,
,, 6 ,,	... ...	BM = 1·23 ,,
,, 9 ,,	... ...	BM = 1·85 ,,

These values are set up from the respective centres of buoyancy, and give the locus of transverse metacentres, which is found to be a straight line, as shown by CC in Fig. 57, DD being the locus of centres of buoyancy.

Approximation to Locus of Centres of Buoyancy on the Metacentric Diagram.—We have seen (p. 63) how the distance of the centre of buoyancy below the L.W.L. can be approximately determined. The locus of centres of buoyancy in the metacentric diagram is, in most cases, very nearly straight for the portion near the load-line, and if we could obtain easily the direction the curve takes on leaving the position for the load water-line, we should obtain a very close approximation to the actual curve itself. It might be desirable to obtain such an approximation in the early stages of a design, when it would not be convenient to calculate the actual positions of the centre of buoyancy, in order to accurately construct the curve.

Let θ be the angle the tangent to the curve of buoyancy at the load condition makes with the horizontal, as in Fig. 56;

A, the area of the load water-plane in square feet;

V, the volume of displacement up to the load water-line in cubic feet;

h, the distance of the centre of buoyancy of the load displacement below the load water-line in feet.

Then the direction of the tangent to the curve of buoyancy is given by—

$$\tan \theta = \frac{Ah}{V}$$

Each of the terms in the latter expression are known or can be readily approximated to,[1] and we can thus determine the inclination at which the curve of centres of buoyancy will start, and this will closely follow the actual curve.

[1] See Example 39, p. 131, for a further approximation.

In a given case—

$$A = 7854 \text{ square feet}$$
$$h = 5{\cdot}45 \text{ feet}$$
$$V = 2140 \times 35 \text{ cubic feet}$$

so that—

$$\tan\theta = \frac{7854 \times 5{\cdot}45}{2140 \times 35}$$
$$= 0{\cdot}572\,[1]$$

Finding the Metacentric Height by Experiment. Inclining Experiment.—We have been dealing up to the present with the purely geometrical aspect of initial stability, viz. the methods employed and the principles involved in finding the position of the transverse metacentre. All that is needed in order to determine this point is the form of the underwater portion of the vessel. But in order to know anything about the vessel's initial stability, we must also know the vertical position of the centre of gravity of the ship, and it is to determine this point that the *inclining experiment* is performed. This is done as the vessel approaches completion, when weights that have yet to go on board can be determined together with their final positions. Weights are shifted transversely across the deck, and by using the principle explained on p. 97, we can tell at once the horizontal shift of the centre of gravity of the ship herself due to this shift of the weights on board. The weight of the ship can be determined by calculating the displacement up to the water-line she floats at, during the experiment. (An approximate method of determining this displacement when the vessel floats out of her designed trim

[1] The best way to set off this tangent is, not to find the angle θ in degrees and then set it off by means of a protractor, but to set off a horizontal line of 10 feet long (on a convenient scale), and from the end set down a vertical line 5·72 feet long on the same scale. This will give the inclination required, for $\tan\theta = \frac{\text{per.}}{\text{base}} = \frac{5{\cdot}72}{10} = 0{\cdot}572$.

This remark applies to any case in which an angle has to be set off very accurately. A table of tangents is consulted and the tangent of the required angle is found, and a similar process to the above is gone through.

will be found on p. 140.) Using the notation employed on p. 97, and illustrated by Fig. 49, we have—

$$GG' = \frac{w \times d}{W}$$

Now, unless prevented by external forces, it is evident that the vessel must incline over to such an angle that the centre of gravity G′ and the centre of buoyancy B′ are in the same vertical line (see Fig. 58), and, the angle of inclination being small,

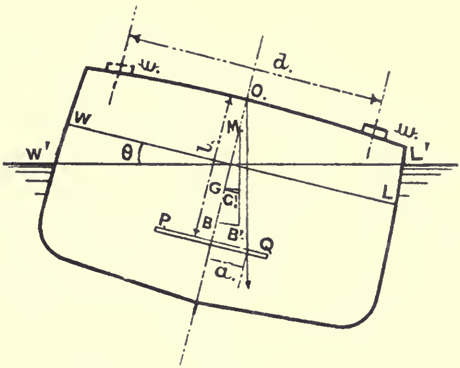

Fig. 58.

M will be the transverse metacentre. If now we call θ the angle of inclination to the upright, GM being the "metacentric height"—

$$\tan\theta = \frac{GG'}{GM}$$

$$GM = \frac{GG'}{\tan\theta}$$

$$= \frac{w \times d}{W \times \tan\theta}$$

using the value found above for GG′. The only term that we do not yet know in this expression is tan θ, and this is found in the following manner: At two or three convenient positions in the ship[1] (such as at bulkheads or down hatchways) plumb-bobs are suspended from a point in the middle line of the ship, and at a convenient distance from the point of suspension a horizontal batten is fixed, with the centre line of the ship marked on it, as shown by PQ in Fig. 58. Before the ship is inclined, the plumb-line should coincide, as nearly as possible, with the centre-line of the ship—that is to say, the ship should be practically upright. When the ship is heeled over to the angle θ, the plumb-line will also be inclined at the same angle, θ, to the original vertical or centre line of the ship, and if l be the distance of the horizontal batten below the point of suspension in inches, and a the deviation of the plumb-line along the batten, also in inches, the angle θ is at once determined, for—

$$\tan\theta = \frac{a}{l}$$

so that we can write—

$$GM = \frac{w \times d}{W \times \frac{a}{l}}$$

In practice it is convenient to check the results obtained by dividing the weight w into four equal parts, placing two sets on one side and two sets on the other side, arranged as in Fig. 59.

The experiment is then performed in the following order:—

(*a*) See if the ship is floating upright, in which case the plumb-lines will coincide with the centre of the ship.

(*b*) The weight (1), Fig. 59, is shifted from port to starboard on to the top of weight (3) through the distance d feet, say, and the deviations of the plumb-lines are noted when the ship settles down at a steady angle.

(*c*) The weight (2) is shifted from port to starboard on to the top of weight (4) through the distance d feet, and the deviations of the plumb-line noted.

(*d*) The weights (1) and (2) are replaced in their original positions, when the vessel should again resume her upright position.

[1] If two positions are taken, one is forward and the other aft. If three positions are taken, one is forward, one aft, and one amidships.

(*e*) The weight (3) is moved from starboard to port, and the deviations of the plumb-lines noted.

(*f*) The weight (4) is moved from starboard to port, and the deviations of the plumb-lines noted.

With the above method of conducting the experiment,[1] and using two plumb-lines, we obtain eight readings, and if three plumb-lines were used we should obtain twelve readings. It is important that such checks should be obtained, as a single result might be rendered quite incorrect, owing to the influence of the hawsers, etc. A specimen experiment is given on p. 119, in which two plumb-lines were used. The deviations obtained

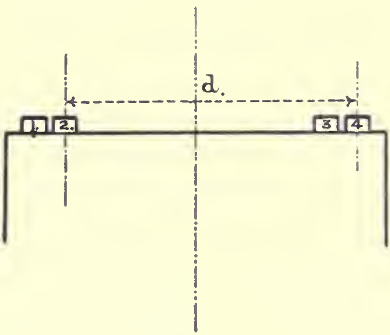

FIG. 59.

are set out in detail, the mean deviation for a shift of $12\frac{1}{2}$ tons through 36 feet being $5\frac{5}{32}$ inches, or the mean deviation for a shift of 25 tons through 36 feet is $10\frac{5}{16}$ inches.

Precautions to be taken when performing an Inclining Experiment.—A rough estimate should be made of the GM expected at the time of the experiment; the weight of ballast can then be determined which will give an inclination of about 4° or 5° when one-half is moved a known distance across the deck. The weight of ballast thus found can then be got ready for the experiment.

A *personal* inspection should be made to see that all weights likely to shift are efficiently secured, the ship cleared of all

[1] There is a slight rise of G, the centre of gravity of the ship, in this method; but the error involved is inappreciable.

free water, and boilers either emptied or run up quite full. Any floating stages should be released or secured by very slack painters.

If possible a fine day should be chosen, with the water calm and little wind. All men not actually employed on the experiment should be sent ashore. Saturday afternoon or a dinner hour is found a convenient time, since then the majority of the workmen employed finishing the ship are likely to be away.

The ship should be hauled head or stern on to the wind, if any, and secured by hawsers at the bow and stern. When taking the readings, these hawsers should be slacked out, so as to ensure that they do not influence the reading. The ship should be plumbed upright before commencing.

An account should be taken, with positions of all weights to be placed on board to complete, of all weights to be removed, such as yard plant, etc., and all weights that have to be shifted.

The following is a specimen report of an inclining experiment :—

Report on Inclining Experiment performed on "———" on ———, 189–, at ———. Density of water — cubic feet to the ton.

Draught of water	16′ 9″ forward.
,, ,, ...	22′ 10″ aft.
Displacement in tons at this draught	5372

The wind was slight, and the ship was kept head to wind during the experiment. Ballast used for inclining, 50 tons. Lengths of pendulums, two in number, 15 feet. Shift of ballast across deck, 36 feet.

	Deviation of pendulum in 15 feet.	
	Forward.	Aft.
Experiment 1, 12½ tons port to starboard	$5\frac{1}{8}''$	$5\frac{1}{8}''$
,, 2, 12½ ,, ,,	$10\frac{1}{4}''$	$10\frac{3}{8}''$
Ballast replaced, zero checked	right	right
Experiment 3, 12½ tons starboard to port	$5\frac{1}{4}''$	$5\frac{1}{8}''$
,, 4, 12½ ,, ,,	$10\frac{3}{8}''$	$10\frac{1}{4}''$

The condition of the ship at the time of inclining is as defined below :—

Bilges dry.
Water-tanks empty.

No water in boilers, feed-tanks, condensers, distillers, cisterns, etc.
Workmen on board, 66.
Tools on board, 5 tons.
Masts and spars complete.
No boats on board.
Bunkers full.
Anchors and cables, complete and stowed.
No provisions or stores on board.
Engineers' stores, half on board.
Hull complete.

The mean deviation in 15 feet for a shift of 25 tons through 36 feet is $10\frac{5}{16}$ inches = 10·312 inches.

$$\therefore \text{GM} = \frac{25 \times 36 \times 15 \times 12}{10·312 \times 5372} = 2·92 \text{ feet}$$

The ship being in an incomplete condition at the time of the inclining experiment, it was necessary to take an accurate account of all weights that had to go on board to complete, with their positions in the ship, together with an account of all weights that had to be removed, with their positions. The total weights were then obtained, together with the position of their final centre of gravity, both in a longitudinal and vertical direction. For the ship of which the inclining experiment is given above, it was found that to fully complete her a total weight of 595 tons had to be placed on board, having its centre of gravity 11 feet before the midship ordinate, and 3·05 feet below the designed L.W.L. Also 63 tons of yard plant, men, etc., had to be removed, with centre of gravity 14 feet abaft the midship ordinate, and 15 feet above the designed L.W.L. The centre of buoyancy of the ship at the experimental water-line was 10·8 feet abaft the midship ordinate, and the transverse metacentre at this line was calculated at 3·14 feet above the designed L.W.L.

We may now calculate the final position of the centre of gravity of the completed ship as follows, remembering that in the experimental condition the centre of gravity must be in the same vertical line as the centre of buoyancy. The vertical position of G in the experimental condition is found by subtracting the experimental GM, viz. 2·92 feet, from the height of the metacentre above the L.W.L. as given above, viz. 3·14 feet.

	Tons.	Above L.W.L.		Below L.W.L.		Abaft amidships.		Before amidships.	
		Lever.	Moment.	Lever.	Moment.	Lever.	Moment.	Lever.	Moment.
Weight of ship at time of experiment ...	5372	0·22	1182	—	—	10·8	58,017	—	—
Weight to go on board to complete	595	—	—	3·05	1813	—	—	11	6545
	5967		1182		1813		58,017		6545
Weight to be taken from ship	63	15	945	—	—	14·0	882	—	—
	5904		237		1813		57,135		6545
					237		6,545		
					1576		50,590		

The final position of the centre of gravity of the ship is therefore—

$$\frac{1576}{5904} = 0{\cdot}266 \text{ feet below the L.W.L.}$$
$$\frac{50590}{5904} = 8{\cdot}57 \text{ feet abaft amidships}$$

the final displacement being 5904 tons.

The mean draught corresponding to the displacement can be found by the methods we have already dealt with, and corresponding to this draught, we can find on the metacentric diagram the position of the transverse metacentre. In this case the metacentre was 2·73 feet above the L.W.L., and consequently the value of GM for the completed condition was—

$$2{\cdot}73 + 0{\cdot}266 = 2{\cdot}996 \text{ feet}$$

or say, for all practical purposes, that the transverse metacentric height in the completed condition was 3 feet.

It is also possible to ascertain what the draughts forward and aft will be in the completed condition, as we shall see in the next chapter.

Values of GM, the "Metacentric Height."—We have discussed in this chapter the methods adopted to find for a given ship the value of the transverse metacentric height

GM. This distance depends upon two things: the position of G, the centre of gravity of the ship; and the position of M, the transverse metacentre. The first is dependent on the vertical distribution of the weights forming the structure and lading of the ship, and its position in the ship must vary with differences in the disposition of the cargo carried. The transverse metacentre depends solely upon the form of the ship, and its position can be completely determined for any given draught of water when we have the sheer drawing of the vessel. There are two steps to be taken in finding its position for any given ship floating at a certain water-line.

1. We must find the vertical position of the centre of buoyancy, the methods adopted being explained in Chapter II.

2. We then find the distance separating the centre of buoyancy and the transverse metacentre, or BM, as explained in the present chapter.

By this means we determine the position of M in the ship.

The methods of estimating the position of G, the centre of gravity for a new ship, will be dealt with separately in Chapter VI.; but we have already seen how the position of G can be determined for a given ship by means of the inclining experiment. Having thus obtained the position of M and G in the ship, we get the distance GM, or the metacentric height.

The following table gives the values of the metacentric height in certain classes of ships. For fuller information reference must be made to the works quoted at the end of the book.

Type of ship.	Values of GM.
Harbour vessels, as tugs, etc.	15 to 18 inches
Modern protected cruisers	2 to $2\frac{1}{2}$ feet
Modern British battleships	$3\frac{1}{2}$ feet
Older central citadel armourclads ...	4 to 8 feet
Shallow-draught gunboats for river service	12 feet
Merchant steamers (varying according to the nature and distribution of the cargo)	1 to 3 feet
Sailing-vessels	3 to $3\frac{1}{2}$ feet

The amount of metacentric height given to a vessel is based largely upon experience with successful ships. In order that a vessel may be "*stiff*," that is, difficult to incline by external forces—as, for example, by the pressure of the wind on the sails—the metacentric height must be large. This is seen by reference to the expression for the moment of statical stability at small angles of inclination from the upright, viz.—

$$W \times GM \sin \theta \text{ (see p. 94)}$$

W being the weight of the ship in tons; θ being the angle of inclination, supposed small. This, being the moment tending to right the ship, is directly dependent on GM. A "*crank*" ship is a ship very easily inclined, and in such a ship the metacentric height is small. For steadiness in a seaway the metacentric height must be small.

There are thus two opposing conditions to fulfil—

1. The metacentric height GM must be enough to enable the ship to resist inclination by external forces. This is especially the case in sailing-ships, in order that they may be able to stand up under canvas without heeling too much. In the case of the older battleships with short armour belts and unprotected ends, sufficient metacentric height had to be provided to allow of the ends being riddled, and the consequent reduction of the moment of inertia of the water-plane.

2. The metacentric height must be moderate enough (if this can be done consistently with other conditions being satisfied) to make the vessel steady in a seaway. A ship which has a very large GM comes back to the upright very suddenly after being inclined, and consequently a vessel with small GM is much more comfortable at sea, and, in the case of a man-of-war, affords a much steadier gun platform.

In the case of sailing-ships, a metacentric height of from 3 to $3\frac{1}{2}$ feet is provided under ordinary conditions of service, in order to allow the vessel to stand up under her canvas. It is, however, quite possible that, when loaded with homogeneous cargoes, as wool, etc., this amount cannot be obtained, on account of the centre of gravity of the cargo being high up in the ship. In this case, it would be advisable to take in water or other ballast in order to lower the centre of gravity, and thus increase the metacentric height.

In merchant steamers the conditions continually vary on account of the varying nature and distribution of the cargo carried, and it is probable that a GM of 1 foot should be the minimum provided when carrying a homogeneous cargo (consistently with satisfactory stability being obtained at large inclinations). There are, however, cases on record of vessels going long voyages with a metacentric height of less than 1 foot, and being reported as comfortable and seaworthy. Mr. Denny (*Transactions of the Institution of Naval Architects*, 1896) mentioned a case of a merchant steamer, 320 feet long (carrying a homogeneous cargo), which sailed habitually with a metacentric height of 0·6 of a foot, the captain reporting her behaviour as admirable in a seaway, and in every way comfortable and safe.

Effect on Initial Stability due to the Presence of Free Water in a Ship.—On reference to p. 118, where the inclining experiment for obtaining the vertical position of the centre of gravity of a ship is explained, it will be noticed that special attention is drawn to the necessity for ascertaining that no free water is allowed to remain in the ship while the experiment is being performed. By free water is meant water having a free surface. In the case of the boilers, for instance, they should either be emptied or run up quite full. We now proceed to ascertain the necessity for taking this precaution. If a compartment, such as a ballast tank in the double bottom, or a boiler, is run up quite full, it is evident that the water will have precisely the same effect on the ship as if it were a solid body having the same weight and position of its centre of gravity as the water, and this can be allowed for with very little difficulty. Suppose, however, that we have on board in a compartment, such as a ballast tank in the double bottom, a quantity of water, and the water does not completely fill the tank, but has a free surface, as wl, Fig. 60.[1] If the ship is heeled over to a small angle θ, the water in the tank must adjust itself so that its surface $w'l'$ is parallel to the level waterline W'L'. Let the volume of either of the small wedges wsw', lsl' be v_0, and g, g' the positions of their centres of gravity, b, b'

[1] Fig. 60 is drawn out of proportion for the sake of clearness.

being the centres of gravity of the whole volume of water in the upright and inclined positions respectively. Then, if V_0 be the total volume of water in the tank, we have—

$$V_0 \times bb' = v_0 \times gg'$$

$$\text{and } bb' = \frac{v_0}{V_0} \times gg'$$

and bb' is parallel to gg'. Now, in precisely the same way as we

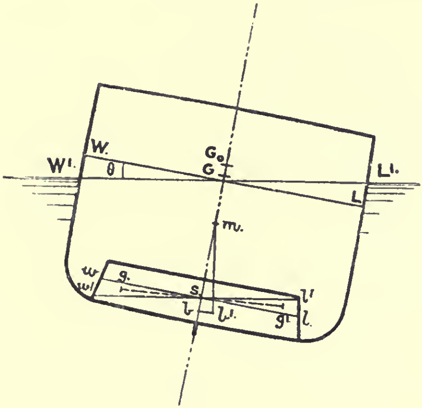

FIG. 60.

found the moment of transference of the wedges WSW′, LSL′, in Fig. 45, we can find the moment of transference of the small wedges wsw', lsl', viz.—

$$v_0 \times gg' = i \times \theta$$

where i is the moment of inertia of the free surface of the water in the tank about a fore-and-aft axis through s; and θ is the circular measure of the angle of inclination.

Substituting this value for $v_0 \times gg'$, we have—

$$bb' = \frac{i \times \theta}{V_0}$$

Draw the new vertical through b', meeting the middle line in m; then—

$$bb' = bm \times \theta$$

and consequently—

$$bm \times \theta = \frac{i \times \theta}{V_0}$$

$$\text{and } bm = \frac{i}{V_0}$$

Now, if the water were solid its centre of gravity would be at b both in the upright and inclined conditions, but the weight of the water now acts through the point b' in the line $b'm$, and its effect on the ship is just the same as if it were a solid weight concentrated at the point m. So that, although b is the *actual centre of gravity* of the water, its effect on the ship, when inclined through ever so small an angle, is the same as though it were at the point m, and in consequence of this the point m is termed the *virtual centre of gravity* of the water.[1] This may be made clearer by the following illustrations :—

1. Suppose that one instant the water is solid, with its centre of gravity at b, and the following instant it became water. Then, for small angles of inclination, its effect on the ship would be the same as if we had raised its weight through a vertical distance bm from its actual to its virtual centre of gravity.

2. Imagine a pendulum suspended at m, with its bob at b. On the ship being inclined to the small angle θ, the pendulum will take up the position mb', and this corresponds exactly to the action of the water.

We thus see that the centre of gravity of the ship cannot be regarded as being at G, but as having risen to G_0, and if W_0 be the weight of water in tons $= \frac{V_0}{35}$ (the water being supposed salt), we have—

$$W \times GG_0 = W_0 \times bm$$
$$= \frac{V_0}{35} \times bm$$

and therefore—

[1] See a paper by Mr. W. Hök, at the Institution of Naval Architects, 1895, on "The Transverse Stability of Floating Vessels containing Liquids, with Special Reference to Ships carrying Oil in Bulk." See also a paper in the "Transactions of the Institution of Engineers and Shipbuilders in Scotland for 1889," by the late Professor Jenkins, on the stability of vessels carrying oil in bulk.

$$GG_0 = \frac{V_0 \times bm}{35W} = \frac{V_0}{V} \times bm \quad (V = \text{volume of displacement})$$

But we have seen that—

$$bm = \frac{i}{V_0}$$

and therefore—

$$GG_0 = \frac{V_0}{V} \times \frac{i}{V_0} = \frac{i}{V}$$

The new moment of stability at the angle θ is—

$$W \times G_0M \times \sin \theta = W \times (GM - GG_0) \sin \theta$$
$$= W \times \left(GM - \frac{i}{V} \right) \sin \theta$$

the metacentric height being reduced by the simple expression $\frac{i}{V}$. We notice here that the *amount* of water does not effect the result, but only the *moment of inertia of the free surface.* The necessity for the precaution of clearing all free water out of a ship on inclining is now apparent. A small quantity of water will have as much effect on the position of the centre of gravity, and therefore on the trustworthiness of the result obtained, as a large quantity of water, provided it has the same form of free surface. If a small quantity of water has a large free surface, it will have more effect than a very large quantity of water having a smaller free surface.

Example.—A vessel has a compartment of the double bottom at the middle line, 60 feet long and 30 feet broad, partially filled with salt water. The total displacement is 9100 tons, and centre of gravity of the ship and water is 0·26 feet below the water-line. Find the loss of metacentric height due to the water having a free surface.

We have here given the position of the centre of gravity of the ship and the water. The rise of this centre of gravity due to the mobility of the water is, using the above notation—

$$\frac{i}{V}$$

$$\text{and } i = \tfrac{1}{12}(60 \times 30) \times (30)^2$$
$$= 5 \times (30)^3$$

Since the free surface is a rectangle 60 feet long and 30 broad

and $V = 9100 \times 35$ cubic feet

therefore the loss in metacentric height $= \frac{5 \times 30^3}{9100 \times 35} = 0·424$ feet

EXAMPLES TO CHAPTER III.

1. Find the circular measure of $5\frac{1}{2}°$, $10\frac{1}{4}°$, $15\frac{3}{4}°$.

Ans. 0·09599; 0·17889; 0·27489.

2. Show that sin 10° is one-half per cent. less in value than the circular measure of 10°, and that tan 10° is one per cent. greater in value than the circular measure of 10°.

3. A cylinder weighing 500 lbs., whose centre of gravity is 2 feet from the axis, is placed on a smooth table and takes up a position of stable equilibrium. It is rolled along parallel to itself through an angle of 60°. What will be the tendency then to return to the original position?

Ans. 866 foot-lbs.

4. Find the moment of inertia about the longest axis through the centre of gravity, of a figure formed of a square of side $2a$, having a semicircle at each end.

$$Ans. \left(\frac{16 + 3\pi}{12}\right)a^4.$$

5. Find the moment of inertia of a square of side $2a$ about a diagonal.

Ans. $\frac{4}{3}a^4$.

6. A square has a similar square cut out of its centre such that the moment of inertia (about a liue through the centre parallel to one side) of the small square and of the portion remaining is the same. What proportion of the area of the original square is cut out?

Ans. 0·71 nearly.

7. A vessel of rectangular cross-section throughout floats at a constant draught of 10 feet, and has its centre of gravity in the load water-plane. The successive half-ordinates of the load water-plane in feet are 0·5, 6, 12, 16, 15, 9, 0; and the common interval 20 feet. Find the transverse metacentric height.

Ans. 8 inches.

8. A log of fir, specific gravity 0·5, is 12 feet long, and the section is 2 feet square. What is its transverse metacentric height when floating in equilibrium in fresh water?

Ans. 0·47 foot.

9. The semi-ordinates of a water-plane 34 feet apart are 0·4, 13·7, 25·4, 32·1, 34·6, 35·0, 34·9, 34·2, 32·1, 23·9, 6·9 feet respectively. Find its moment of inertia about the centre line.

Ans. 6,012,862.

10. The semi-ordinates of the load water-plane of a vessel are 0, 3·35, 6·41, 8·63, 9·93, 10·44, 10·37, 9·94, 8·96, 7·16, and 2·5 feet respectively. These ordinates being 21 feet apart, find—

(1) The tons per inch immersion.

(2) The distance between the centre of buoyancy and the transverse metacentre, the load displacement being 484 tons.

Ans. (1) 7·73 tons; (2) 5·2 feet nearly.

11. The semi-ordinates, 16·6 feet apart, of a vessel's water-plane are 0·2, 2·3, 6·4, 9·9, 12·3, 13·5, 13·8, 13·7, 12·8, 10·6, 6·4, 1·9, 0·2 feet respectively, and the displacement up to this water-plane is 220 tons. Find the length of the transverse BM.

Ans. 20·6 feet.

12. A vessel of 613 tons displacement was inclined by moving 30 cwt. of rivets across the deck through a distance of 22′ 6″. The end of a plumb-

line 10 feet long moved through $2\frac{1}{4}$ inches. What was the metacentric height at the time of the experiment?

Ans. 2·93 feet.

13. The semi-ordinates of a ship's water-plane 35 feet apart are, commencing from forward, 0·4, 7·12, 15·28, 21·88, 25·62, 26·9, 26·32, 24·42, 20·8, 15·15. 6·39 feet respectively. There is an after appendage of 116 square feet, with its centre of gravity 180 feet abaft the midship ordinate. Find—

(1) The area of the water-plane.
(2) The tons per inch immersion.
(3) The distance of the centre of flotation abaft amidships.
(4) The position of the transverse metacentre above the L.W.L., taking the displacement up to the above line as 5372 tons, and the centre of buoyancy of this displacement 8·61 feet below the L.W.L.

Ans. (1) 13,292 square feet; (2) 31·6 tons; (3) 14·65 feet; (4) 3·34 feet.

14. A ship displacing 9972 tons is inclined by moving 40 tons 54 feet across the deck, and a mean deviation of $9\frac{1}{3}$ inches is obtained by pendulums 15 feet long. Find the metacentric height at the time of the operation.

Ans. 4·18 feet.

15. A ship weighing 10,333 tons was inclined by shifting 40 tons 52 feet across the deck. The tangent of the angle of inclination caused was found to be 0·05. If the transverse metacentre was 4·75 feet above the designed L.W.L., what was the position of the centre of gravity of the ship at the time of the experiment?

Ans. 0·73 foot above the L.W.L.

16. A vessel of 26 feet draught has the moment of inertia of the L.W.P. about a longitudinal axis through its centre of gravity 6,500,000 in foot-units. The area of the L.W.P. is 20,000 square feet, the volume of displacement 400,000 cubic feet, and the centre of gravity of the ship may be taken in the L.W.P. Approximate to the metacentric height.

Ans. $5\frac{1}{4}$ feet.

17. Prove the rule given on p. 60 for the distance of the centre of gravity of a semicircle of radius a from the diameter, viz. $\frac{4}{3\pi}a$, by finding the transverse BM of a pontoon of circular section floating with its axis in the surface of the water.

(M in this case is in the centre of section.)

18. Take a body shaped as in Kirk's analysis, p. 80, of length 140 feet; length of parallel middle body, 100 feet; extreme breadth, 30 feet; draught, 12 feet. Find the transverse BM.

Ans. 5·7 feet.

19. A vessel of 1792 tons displacement is inclined by shifting 5 tons already on board transversely across the deck through 20 feet. The end of a plumb-line 15 feet long moves through $5\frac{1}{4}$ inches. Determine the metacentric height at the time of the experiment.

Ans. 1·91 feet.

20. A vessel of displacement 1722 tons is inclined by shifting 6 tons of ballast across the deck through $22\frac{1}{2}$ feet. A mean deviation of $10\frac{1}{2}$ inches is obtained with pendulums 15 feet long. The transverse metacentre is 15·28 feet above the keel. Find the position of the centre of gravity of the ship with reference to the keel.

Ans. 13·95 feet.

21. The ship in the previous question has 169 tons to go on board at 10 feet above keel, and 32 tons to come out at 20 feet above keel. Find the metacentric height when completed, the transverse metacentre at the displacement of 1859 tons being 15·3 feet above keel.

Ans. 1·8 feet.

22. A vessel of 7000 tons displacement has a weight of 30 tons moved transversely across the deck through a distance of 50 feet, and a plumb-bob hung down a hatchway shows a deviation of 12 inches in 15 feet. What was the metacentric height at the time of the operation?

Ans. 3·21 feet.

23. A box is 200 feet long, 30 feet broad, and weighs 2000 tons. Find the height of the transverse metacentre above the bottom when the box is floating in salt water on an even keel. *Ans.* 12·26 feet.

24. Show that for a rectangular box floating at a uniform draught of d feet, the breadth being 12 feet, the distance of the transverse metacentre above the bottom is given by $\frac{d^2 + 24}{2d}$ feet, and thus the transverse metacentre is in the water-line when the draught is 4·9 feet.

25. A floating body has a constant triangular section. If the breadth at the water-line is $\sqrt{2}$ times the draught, show that the curve of metacentres in the metacentric diagram lies along the line drawn from zero draught at 45° to the horizontal, and therefore the metacentre is in the water-line for all draughts.

26. A floating body has a square section with one side horizontal. Show that the transverse metacentre lies above the centre of the square so long as the draught does not much exceed 21 per cent. of the depth of the square. Also show that as the draught gets beyond 21 per cent. of the depth, the metacentre falls below the centre and remains below until the draught reaches 79 per cent. of the depth; it then rises again above the centre of the square, and continues to rise as long as any part of the square is out of the water.

(This may be done by constructing a metacentric diagram, or by using the methods of algebra, in which case a quadratic equation has to be solved.)

27. Show that a square log of timber of 12 inches side, 10 feet long, and weighing 320 lbs., must be loaded so that its centre of gravity is more than 1 inch below the centre in order that it may float with a side horizontal in water of which 35 cubic feet weigh 1 ton.

28. A prismatic vessel is 70 feet long. The section is formed at the lower part by an isosceles triangle, vertex downwards, the base being 20 feet, and the height 5 feet; above this is a rectangle 20 feet wide and 5 feet high. Construct to scale the metacentric diagram for all drafts.

29. A vessel's load water-plane is 380 feet long, and 75 feet broad, and its moment of inertia in foot-units about the centre line works out to 8,000,000 about. State whether you consider this a reasonable result to obtain, the water-plane not being very fine.

30. Find the value of the coefficient a in the formula $\mathrm{BM} = a\frac{\mathrm{B}^2}{\mathrm{D}}$ referred to on p. 107, for floating bodies having the following sections throughout their length :—

(*a*) Rectangular cross-section.
(*b*) Triangular cross-section, vertex down.
(*c*) Vertical-sided for one half the draught, the lower half of the section being in the form of a triangle.

Ans. (*a*) 0·08; (*b*) 0·16; (*c*) 0·11.

For ordinary ships the value of a will lie between the first and last of these.

31. A lighter in the form of a box is 100 feet long, 20 feet broad, and floats at a constant draught of 4 feet. The metacentric height when empty is 6 feet. Two bulkheads are built 10 feet from either end. Show that a small quantity of water introduced into the central compartment will render the lighter unstable in the upright condition.

32. At one time, in ships which were found to possess insufficient stability, girdling was secured to the ship in the neighbourhood of the water-line. Indicate how far the stability would be influenced by this means.

33. A floating body has a constant triangular section. If the breadth at the water-line is equal to the draught, show that the locus of metacentres in the metacentric diagram makes an angle with the horizontal of about 40°.

34. A cylinder is placed into water with its axis vertical. Show that if the centre of gravity is in the water-plane, the cylinder will float upright if the radius ÷ the draught is greater than $\sqrt{2}$.

35. In a wholly submerged body show that for stable equilibrium the centre of gravity must lie below the centre of buoyancy.

36. A floating body has a constant triangular section, vertex downwards, and has a constant draught of 12 feet, the breadth at the water-line being 24 feet. The keel just touches a quantity of mud, specific gravity 2. The water-level now falls 6 feet: find the amount by which the metacentric height is diminished due to this.

Ans. $2\frac{3}{4}$ feet about.

37. A floating body of circular section 6 feet in diameter has a metacentric height of 1·27 feet. Show that the centre of buoyancy and centre of gravity coincide, when the body is floating with the axis in the surface.

38. It is desired to increase the metacentric height of a vessel which is being taken in hand for a complete overhaul. Discuss the three following methods of doing this, assuming the ship has a metacentric diagram as in Fig. 56, the extreme load draught being 15 feet:—

(1) Placing ballast in the bottom.
(2) Removing top weight.
(3) Placing a girdling round the ship in the neighbourhood of the water-line.

39. Show that the angle θ in Fig. 56 is between 29° and 30° for a vessel whose coefficient of L.W.P. is 0·75, and whose block coefficient of displacement is 0·55. In any case, if these coefficients are denoted by n and k respectively, show that $\tan\theta = \frac{1}{3} + \frac{n}{6k}$ approximately (use Normand's formula, p. 63).

CHAPTER IV.

LONGITUDINAL METACENTRE, LONGITUDINAL BM, CHANGE OF TRIM.

Longitudinal Metacentre.—We now have to deal with inclinations in a fore-and-aft or longitudinal direction. We do not have the same difficulty in fixing on the fore-and-aft position of the centre of gravity of a ship as we have in fixing its vertical position, because we know that if a ship is floating steadily at a given water-line, the centre of gravity must be in the same vertical line as the centre of buoyancy, by the conditions of equilibrium laid down on p. 89. It is simply a matter of calculation to find the longitudinal position of the centre of buoyancy of a ship when floating at a certain water-line, if we have the form of the ship given, and thus the fore-and-aft position of the centre of gravity is determined.

We have already dealt with the inclination of a ship in a transverse direction, caused by shifting weights already on board across the deck; and in a precisely similar manner we can incline a ship in a longitudinal or fore-and-aft direction by shifting weights along the deck in the line of the keel. The *trim* of a ship is the difference between the draughts of water forward and aft. Thus a ship designed to float at a draught forward of 12 feet, and a draft aft of 15 feet, is said to trim 3 feet by the stern.

We have, on p. 93, considered the definition of the transverse metacentre, and the definition of the longitudinal metacentre is precisely analogous.

For a given water-line WL of a vessel, let B be the centre of buoyancy (see Fig. 61), and BM the vertical through it.

Suppose the trim of the vessel to change slightly,[1] the vessel retaining the same volume of displacement, B′ being the new centre of buoyancy, and B′M the vertical through it, meeting

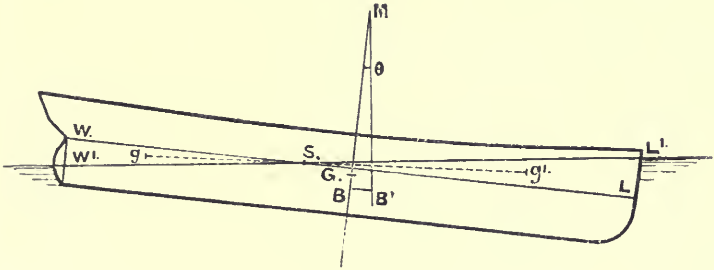

FIG. 61.

BM in M. Then the point M is termed the *longitudinal metacentre.*

The distance between G, the centre of gravity of the ship, and M, the longitudinal metacentre, is termed the *longitudinal metacentric height.*

Formula for finding the Distance of the Longitudinal Metacentre above the Centre of Buoyancy.—Let Fig. 62 represent the profile of a ship floating at the water-line W′L′, the original water-line being WL. The original trim was AW − BL; the new trim is AW′ − BL′. The change of trim is—

$$(AW - BL) - (AW' - BL') = WW' + LL'$$

i.e. *the change of trim is the sum of the changes of draughts forward and aft.* This change, we may suppose, has been caused by the shifting of weights from aft to forward. The inclination being regarded as small, and the displacement remaining constant, the line of intersection of the water-planes WL, W′L′ must pass through the centre of gravity of the water-plane WL, or, as we have termed it, the centre of flotation, in accordance with the principle laid down on p. 94. This centre of flotation will usually be abaft the middle of length, and this introduces a complication which makes the calculation for the longitudinal metacentre more difficult than the corre-

[1] Much exaggerated in the figure.

sponding calculation for the transverse metacentre. In this latter case, it will be remembered that the centre of flotation is in the middle line of the water-plane.

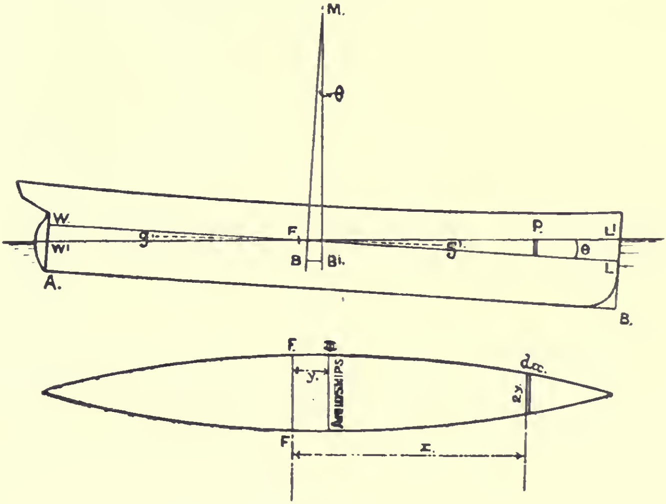

Fig. 62.

In Fig. 62—

Let B be the centre of buoyancy when floating at the water-line WL;

B′, the centre of buoyancy when floating at the water-line W′L′;

FF, the intersection of the water-planes WL, W′L′;

v, the volume of either the immersed wedge FLL′ or the emerged wedge FWW′;

g, g', the centres of gravity of the wedges WFW′, LFL′ respectively;

V, the volume of displacement in cubic feet;

θ, the angle between the water-lines WL, W′L′, which is the same as the angle between BM and B′M (this angle is supposed very small).

We have, using the principle laid down on p. 96—

$$v \times gg' = V \times BB'$$

$$\text{or BB}' = \frac{v \times gg'}{\text{V}}$$

$$\text{But BB}' = \text{BM} \times \theta \text{ } (\theta \text{ is in circular measure})$$

$$\therefore \text{BM} \times \theta = \frac{v \times gg'}{\text{V}}$$

The part of this expression that we do not know is $v \times gg'$, or the moment of transference of the wedges. At P take a small transverse slice of the wedge FLL′, of breadth in a fore-and-aft direction, dx; length across, $2y$; and distance from F, x. Then the depth of the slice is—

$$x \times \theta$$

$$\text{and the volume is } 2y \times x\theta \times dx$$

This is an elementary volume, analogous to the elementary area $y \cdot dx$ used in finding a large area. The moment of this elementary volume about the transverse line FF is—

$$2yx \cdot \theta \cdot dx \times x$$

$$\text{or } 2yx^2 \cdot \theta \cdot dx$$

If we summed all such moments as this for the length FL, we should get the moment $v \times \text{F}g'$, and for the length FW, $v \times \text{F}g$, or for the whole length, $v \times gg'$; therefore, using our ordinary notation—

$$v \times gg' = \int 2yx^2 \cdot \theta \cdot dx$$

$$= 2\theta \int yx^2 \cdot dx \text{ } (\theta \text{ being constant})$$

We therefore have—

$$\text{BM} \times \theta = \frac{2\theta \int yx^2 \cdot dx}{\text{V}}$$

$$\text{or BM} = \frac{2\int yx^2 \cdot dx}{\text{V}}$$

Referring to p. 99, it will be seen that we defined the moment of inertia of an area about a given axis as—

$$\int d\text{A} \times y^2$$

where dA is a small elementary area;

y its distance from the given axis.

Consider, now, the expression obtained, $2\int yx^2 \cdot dx$. The elementary area is $2y \cdot dx$, and x is its distance from a transverse axis passing through the centre of flotation. We may therefore say—

$$BM = \frac{I_0}{V}$$

where I_0 is the moment of inertia of the water-plane about a transverse axis passing through the centre of flotation. It will be seen at once that this is the same form of expression as for the transverse BM.

The method usually adopted for finding the moment of inertia of a water-plane about a transverse axis through the centre of flotation is as follows[1] :—

We first find the moment of inertia about the ordinary midship ordinate. If we call this I, and y the distance of the centre of flotation from the midship ordinate, we have, using the principle given on p. 100—

$$I = I_0 + Ay^2$$
$$\text{or } I_0 = I - Ay^2$$

The method actually adopted in practice will be best understood by working the following example.

Numbers of ordinates.	Semi-ordinates of L.W.P.	Simpson's multipliers.	Products for area.	Multipliers for moment.	Products for moment.	Multipliers for moment of inertia.	Products for moment of inertia.
1	0·0	½	0·0	5	0·0	5	0·0
1½	1·37	2	2·74	4½	12·33	4½	55·49
2	2·67	1½	4·01	4	16·04	4	64·16
3	4·87	4	19·48	3	58·44	3	175·32
4	6·31	2	12·62	2	25·24	2	50·48
5	6·85	4	27·40	1	27·40	1	27·40
6	7·21	2	14·42	0	139·45	0	—
7	7·15	4	28·60	1	28·60	1	28·60
8	6·87	2	13·74	2	27·48	2	54·96
9	6·33	4	25·32	3	75·96	3	227·88
10	5·08	1½	7·62	4	30·48	4	121·92
10½	3·56	2	7·12	4½	32·04	4½	144·18
11	0·71	½	0·35	5	1·75	5	8·75
			163·42		196·31		959·14
					139·45		
					56·86		

[1] This calculation for the L.W.P. is usually performed on the displacement sheet.

In column 2 of the table are given the lengths of semi-ordinates of a load water-plane corresponding to the numbers of the ordinates in column 1. The ordinates are 7·1 feet apart. It is required to find the longitudinal BM, the displacement being 91·6 tons in salt water.

The distance apart of the ordinates being 7·1 feet, we have—

$$\text{Area} = 163{\cdot}42 \times (\tfrac{1}{3} \times 7{\cdot}1) \times 2$$
$$= 773{\cdot}5 \text{ square feet}$$

$$\left.\begin{array}{r}\text{Distance of centre of gravity of}\\ \text{water-plane abaft No. 6 ordinate}\end{array}\right\} = \frac{56{\cdot}86 \times 7{\cdot}1}{163{\cdot}42} = 2{\cdot}47 \text{ feet}$$

(the stations are numbered from forward).

The calculation up to now has been the ordinary one for finding the area and position of the centre of gravity. Column 4 is the calculation indicated by the formula—

$$\text{Area} = 2\int y \,.\, dx$$

Column 6 is the calculation indicated by the formula—

$$\text{Moment} = 2\int yx \,.\, dx$$

It will be remembered that in column 5 we do not put down the actual distances of the ordinates from No. 6 ordinate, but the number of intervals away; the distance apart of the ordinates being introduced at the end. By this means the result is obtained with much less labour than if column 5 contained the actual distances. The formula we have for the moment of inertia is $2\int y \,.\, x^2 \,.\, dx$. We follow a similar process to that indicated above; we do not multiply the ordinates by the square of the actual distances, but by the square of the number of intervals away, leaving to the end the multiplication by the square of the interval. Thus for ordinate No. 2 the actual distance from No. 6 is $4 \times 7{\cdot}1 = 28{\cdot}4$ feet. The square of this is $(4)^2 \times (7{\cdot}1)^2$. For ordinate No. 4 the square of the distance is $(2)^2 \times (7{\cdot}1)^2$. The multiplication by $(7{\cdot}1)^2$ can be done at the end. In column 7 is placed the number of intervals from No. 6, as in column 5; and if the products in column 6 are multiplied successively by the numbers in column 7, we shall obtain in column 8 the ordinates put through Simpson's rule, and also multiplied by the square of the number of intervals from No. 6 ordinate. The whole of column 8 is added up, giving a result 959·14. To obtain the moment of inertia about No. 6 ordinate, this has to be multiplied as follows :—

(*a*) By one-third the common interval to complete Simpson's rule, or $\frac{1}{3} \times 7 \cdot 1$.

(*b*) By the square of the common interval, for the reasons fully explained above.

(*c*) By two for both sides.

We therefore have the moment of inertia of the water-plane about No. 6 ordinate—

$$959 \cdot 14 \times (\tfrac{1}{3} \times 7 \cdot 1) \times (7 \cdot 1)^2 \times 2 = 228{,}858$$

The moment of inertia about a transverse axis through the centre of flotation will be less than this by considering the formula $I = I_0 + Ay^2$, where I is the value found above about No. 6 ordinate, and I_0 is the moment of inertia we want. We found above that the area $A = 773 \cdot 5$ square feet, and $y = 2 \cdot 47$ feet ;

$$\therefore I_0 = 228{,}858 - (773 \cdot 5 \times 2 \cdot 47^2)$$
$$= 224{,}139$$

The displacement up to this water-plane is 91·6 tons, and the volume of displacement is—

$$91 \cdot 6 \times 35 = 3206 \text{ cubic feet}$$

$$\text{The longitudinal BM} = \frac{I_0}{V}$$

$$= \frac{224139}{3206} = 69 \cdot 9 \text{ feet}$$

Approximate Formula for the Height of the Longitudinal Metacentre above the Centre of Buoyancy.—The following formula is due to M. J. A. Normand, M.I.N.A.,[1] and is found to give exceedingly good results in practice :—

Let L be the length on the load water-line in feet ;

B, the breadth amidships in feet ;

[1] See "Transactions of the Institution of Naval Architects," 1882.

V, the volume of displacement in cubic feet;

A, the area of the load water-plane in square feet.

Then the height of the longitudinal metacentre above the centre of buoyancy—

$$H = 0{\cdot}0735\frac{A^2 \times L}{B \times V}$$

In the example worked above, the breadth amidships was 14·42 feet; and using the formula, we find—

$$H = 67{\cdot}5 \text{ feet nearly}$$

This compares favourably with the actual result of 69·9 feet. The quantities required for the use of the formula would all be known at a very early stage of a design and a close approximation to the height H can thus very readily be obtained. A formula such as this is useful as a check on the result of the calculation for the longitudinal BM.

We may also obtain an approximate formula in the same manner as was done for the transverse BM on p. 107. Using a similar system of notation, we may say—

$$\left.\begin{array}{l}\text{Moment of inertia of L.W.P. about a trans-}\\ \text{verse axis through the centre of flotation}\end{array}\right\} = n' \times L^3 \times B$$

n' being a coefficient of a similar nature to n used on p. 103.

$$\text{Volume of displacement} = k \times L \times B \times D$$

$$\therefore H = \frac{n' \times L^3 \times B}{k \times L \times B \times D}$$

$$= b\frac{L^2}{D}$$

where b is a coefficient obtained from the coefficients n' and k. Sir William White, in the "Manual of Naval Architecture," says, with reference to the value of b, that "the value 0·075 may be used as a rough approximation in most cases; but there are many exceptions to its use." If this approximation be applied to the example we have worked, the mean moulded draught being 5·8 feet—

$$\text{The value of } H = 65 \text{ feet}$$

This formula shows very clearly that the length of a ship is more effective than the draught in determining the value of the longitudinal BM in any given case. For vessels which have an unusual proportion of length to draught, the values of the longitudinal BM found by using this formula will not be trustworthy.

To estimate the Displacement of a Vessel when floating out of the Designed Trim.—The following method is found useful when it is not desired to actually calculate the displacement from the drawings, and a close approximation is sufficiently accurate. Take a ship floating parallel to her designed L.W.L.; we can at once determine the displacement when floating at such a water-line from the curve of displacement (see p. 23). If now a weight already on board is shifted aft, say, the ship will change trim, and she will trim more by the stern than designed. The new water-plane must pass through the centre of gravity of the original water-plane, or, as we have termed it, the centre of flotation, and

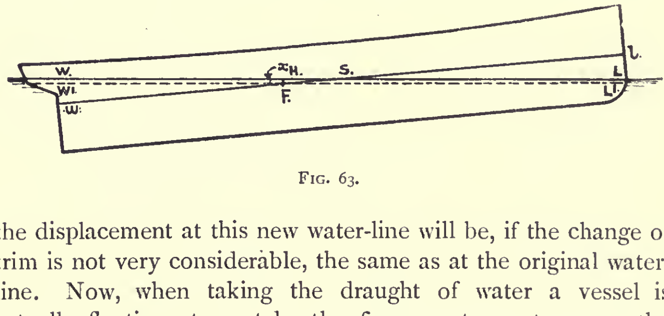

Fig. 63.

the displacement at this new water-line will be, if the change of trim is not very considerable, the same as at the original water-line. Now, when taking the draught of water a vessel is actually floating at, we take the figures set up at or near the forward and after perpendiculars. These draught-marks should be either at the perpendiculars or equally distant from them. The draughts thus obtained are added together and divided by two, giving us the mean draught. Now run a line parallel to the designed water-line at this mean draught, as in Fig. 63, where WL represents the actual water-line, and *wl* the line just drawn. It will not be true that the displacement of the ship is the same as that given by the water-line *wl*. Let F be the centre of flotation of the water-line wl, and draw W′L′ through F parallel to WL. Then the actual displacement will be that up to W′L′, which is nearly the same as that up to wl, with the displacement of the layer WW′L′L added. The displacement up to wl is found at once from the curve of displacement. Let T be the tons per inch at wl, and therefore very nearly the tons per inch at W′L′ and WL. SF, the distance the centre of flotation of the water-plane wl is abaft the middle of length, is supposed known, and equals d inches, say. Now, the angle between wl and WL is given by—

$$\tan\theta = \frac{w\text{W} + l\text{L}}{\text{length of ship}}$$
$$= \frac{\text{amount out of trim}}{\text{length of ship}}$$

But if x is the thickness of layer in inches between W′L′ and WL, we also have in the triangle SFH—

$$\tan\theta = \frac{x}{d} \text{ very nearly (for small angles } \tan\theta = \sin\theta \text{ very nearly)}$$

and accordingly x may be determined. This, multiplied by the tons per inch T, will give the displacement of the layer.

The following example will illustrate the above :—

Example.—A vessel floats at a draught of 16′ 5½″ forward, 23′ 1½″ aft, the normal trim being 2 feet by the stern. At a draught of 19′ 9½″, her displacement, measured from the curve of displacement, is 5380 tons, the tons per inch is 31·1 tons, and the centre of flotation is 12·9 feet abaft amidships. Estimate the ship's displacement.

The difference in draught is 23′ 1½″ − 16′ 5½″ = 6′ 8″, or 4′ 8″ out of trim. The distance between the draught-marks is 335 feet, and we therefore have for the thickness of the layer—

$$12 \times 12{\cdot}9 \times \frac{56}{335 \times 12} = 2{\cdot}15 \text{ inches}$$

The displacement of the layer is therefore—

$$2{\cdot}15 \times 31{\cdot}1 = 67 \text{ tons}$$

The displacement is therefore—

$$5380 + 67 = 5447 \text{ tons nearly}$$

Change of Trim due to Longitudinal Shift of Weights already on Board.—We have seen that change of trim is the sum of the change of draughts forward and aft, and that change of trim can be caused by the shift of weights on board in a fore-and-aft direction. We have here an analogous case to the inclining experiment in which heeling is caused by shifting weights in a transverse direction. In Fig. 64, let w be

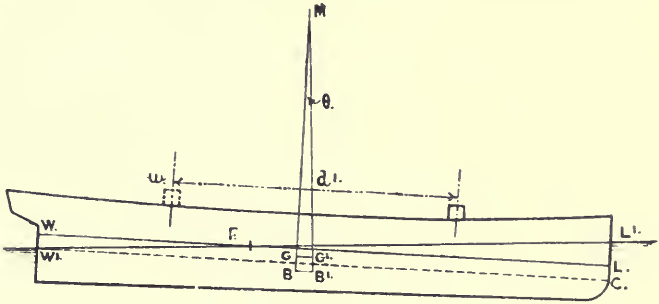

FIG. 64.

a weight on the deck when the vessel is floating at the water-line WL, G being the position of the centre of gravity. Now suppose the weight w to be shifted forward a distance of d feet. G will, in consequence of this, move forward parallel to the line joining the original and final positions of w, and if W be the displacement of the ship in tons, G will move to G′ such that—

$$GG' = \frac{w \times d}{W}$$

Now, under these circumstances, the condition of equilibrium is not fulfilled if the water-line remains the same, viz. that the centre of gravity and the centre of buoyancy must be in the same vertical line, because G has shifted to G′. The ship must therefore adjust herself till the centre of gravity and the centre of buoyancy are in the same vertical line, when she will float at a new water-line, W′L′, the new centre of buoyancy being B′. The original vertical through G and B meets the new vertical through G′ and B′ in the point M, and this point will be the longitudinal metacentre, supposing the change of trim to be small, and GM will be the longitudinal metacentric height. Draw W′C parallel to the original water-line WL, meeting the forward perpendicular in C. Then, since CL = W′W, the change of trim WW′ + LL′ = CL′ = x, say. The angle of inclination of W′L′ to WL is the same as the angle between W′L′ and W′C = θ, say, and—

$$\tan\theta = \frac{\text{CL}'}{\text{length}} = \frac{x}{\text{L}}$$

But we also have—

$$\tan\theta = \frac{\text{GG}'}{\text{GM}}$$

therefore, equating these two values for tan θ, we have—

$$\frac{x}{\text{L}} = \frac{\text{GG}'}{\text{GM}}$$

$$= \frac{w \times d}{\text{W} \times \text{GM}}$$

using the value obtained above for GG′; or—

$$\left.\begin{array}{l} x\text{, the change of trim due to the} \\ \text{moment of transference of the} \\ \text{weight } w \text{ through the distance } d\text{,} \end{array}\right\} = \frac{w \times d}{\text{W} \times \text{GM}} \times \text{L feet}$$

or—

$$\text{The change of trim in inches} = \frac{12 \times w \times d \times \text{L}}{\text{W} \times \text{GM}}$$

and the *moment to change trim* 1 *inch is*—

$$w \times d = \frac{\text{W} \times \text{GM}}{12 \times \text{L}} \text{ foot-tons}$$

To determine this expression, we must know the vertical position of the centre of gravity and the position of the longitudinal metacentre. The vertical position of the centre of gravity will be estimated in a design when dealing with the metacentric height necessary, and the distance between the centre of buoyancy and the centre of gravity is then subtracted from the value of the longitudinal BM found by one of the methods already explained. The distance BG is, however, small compared with either of the distances BM or GM that any small error in estimating the position of the centre of gravity cannot appreciably affect the value of the moment to change trim one inch. In many ships BM approximately equals the length of the ship, and therefore GM also; we may therefore say that in such ships the moment to change trim 1 inch = $\frac{1}{12}$ the displacement in tons. For ships that are long in proportion to the draught, the moment to change trim 1 inch is greater than would be given by this approximate rule.

In the ship for which the value of the longitudinal BM was calculated on p. 136, the centre of buoyancy was $2\frac{1}{4}$ feet below the L.W.L., the centre of gravity was estimated at $1\frac{1}{4}$ feet below the L.W.L.; and the length between perpendiculars was 75 feet.

$$\therefore \text{GM} = 69{\cdot}9 - 1$$
$$= 68{\cdot}9 \text{ feet}$$

and the moment to change trim 1 inch $= \dfrac{91{\cdot}6 \times 68{\cdot}9}{12 \times 75}$

$$= 7{\cdot}01 \text{ foot-tons}$$

the draughts being taken at the perpendiculars.

Example.—A vessel 300 feet long and 2200 tons displacement has a longitudinal metacentric height of 490 feet. Find the change of trim caused by moving a weight of 5 tons already on board through a distance of 200 feet from forward to aft.

Here the moment to change trim 1 inch is—

$$\frac{2200 \times 490}{12 \times 300} = 300 \text{ foot-tons nearly}$$

The moment aft due to the shift of the weight is—

$$5 \times 200 = 1000 \text{ foot-tons}$$

and consequently the change of trim *aft* is—

$$\frac{1000}{300} = 3\tfrac{1}{3} \text{ inches}$$

Approximate Formula for the Moment to change Trim 1 inch.—Assuming Normand's approximate formula for the height of the longitudinal metacentre above the centre of buoyancy given on p. 139—

$$\text{H} = 0{\cdot}0735 \frac{\text{A}^2 \times \text{L}}{\text{B} \times \text{V}}$$

we may construct an approximate formula for the moment to change trim 1 inch as follows.

We have seen that the moment to change trim 1 inch is—

$$\frac{\text{W} \times \text{GM}}{12 \times \text{L}}$$

We can write $W = \frac{V}{35}$ and assume that, for all practical purposes, $BM = GM = 0{\cdot}0735\frac{A^2 \times L}{B \times V}$

Substituting this in the above formula, we have—

$$\left.\begin{array}{l}\text{Moment to change}\\ \text{trim 1 inch}\end{array}\right\} = \frac{V}{35 \times 12 \times L} \times \left(0{\cdot}0735\frac{A^2 \times L}{B \times V}\right)$$

$$\text{or } 0{\cdot}000175\frac{A^2}{B}$$

For further approximations, see Example 18, p. 157.

Applying this to the case worked out in detail on p. 136—

$$\text{Area of L.W.P.} = A = 773{\cdot}5 \text{ square feet}$$
$$\text{Breadth} = B = 14{\cdot}42 \text{ feet}$$

so that the moment to change trim 1 inch approximately should equal—

$$0{\cdot}000175\frac{(773{\cdot}5)^2}{14{\cdot}42} = 7{\cdot}26 \text{ foot-tons}$$

the exact value, as calculated on p. 144, being 7·01 foot-tons.

It is generally sufficiently accurate to assume that one-half the change of trim is forward, and the other half is aft. In the example on p. 144, if the ship floated at a draught of 12′ 3″ forward and 14′ 9″ aft, the new draught forward would be—

$$12' \ 3'' - 1\tfrac{2}{3}'' = 12' \ 1\tfrac{1}{3}''$$

and the new draught aft would be—

$$14' \ 9'' + 1\tfrac{2}{3}' = 14' \ 10\tfrac{2}{3}''$$

Referring, however, to Fig. 64, it will be seen that when, as is usually the case, the centre of flotation is not at the middle of the length, WW′ is not equal to LL′, so that, strictly speaking, the total change of trim should not be divided by 2, and one-half taken forward and the other half aft. Consider the triangles FWW′, FLL′; these triangles are similar to one another, and the corresponding sides are proportional, so that—

$$\frac{WW'}{WF} = \frac{LL'}{LF}$$

and both these triangles are similar to the triangle W′CL′. Consequently—

$$\frac{WW'}{WF} = \frac{LL'}{LF} = \frac{CL'}{W'C} = \frac{\text{change of trim}}{\text{length}}$$

$$\therefore\ WW' = \frac{WF}{\text{length}} \times \text{change of trim}$$

$$\text{and } LL' = \frac{LF}{\text{length}} \times \text{change of trim}$$

that is to say, the proportion of the change of trim either aft or forward, is the proportion the length of the vessel abaft or forward of the centre of flotation bears to the length of the vessel. Where the change of trim is small, this makes no appreciable difference in the result, but there is a difference when large changes of trim are under consideration.

For example, in the case worked out on p. 144, suppose a weight of 50 tons is moved through 100 feet from forward to aft; the change of trim caused would be—

$$\frac{5000}{300} = 16\frac{2}{3} \text{ inches}$$

The centre of flotation was 12 feet abaft the middle of length. The portion of the length abaft the centre of flotation is therefore $\frac{138}{300}$ of the length. The increase of draught aft is therefore—

$$\frac{138}{300} \times \frac{50}{3} = 7\frac{2}{3} \text{ inches}$$

and the decrease of draught forward is—

$$\frac{162}{300} \times \frac{50}{3} = 9 \text{ inches}$$

instead of $8\frac{1}{3}$ inches both forward and aft. The draught forward is therefore—

$$12'\ 3'' - 9'' = 11'\ 6''$$

and the draught aft—

$$14'\ 9'' + 7\tfrac{2}{3}'' = 15'\ 4\tfrac{2}{3}''$$

It will be noticed that the mean draught is not the same as before the shifting, but two-thirds of an inch less, while the displacement remains the same. This is due to the fact that, as the ship increases her draught aft and decreases it forward, a fuller portion of the ship goes into the water and a finer portion comes out.

Effect on the Trim of a Ship due to adding a Weight of Moderate Amount.—If we wish to place a weight on board a ship so that the vessel will not change trim, we must place it so that the upward force of the added buoyancy will act in the same line as the downward force of the added weight. Take a ship floating at a certain water-line, and imagine her to sink down a small amount, so that the new waterplane is parallel to the original water-plane. The added buoyancy is formed of a layer of parallel thickness, and having very nearly the shape of the original water-plane. The upward force of this added buoyancy will act through the centre of gravity of the layer, which will be very nearly vertically over the centre of gravity of the original water-plane, or, as we have termed it, the centre of flotation. We therefore see that to place a weight of moderate amount on a ship so that no change of trim takes place, we must place it vertically over or under the centre of flotation. The ship will then sink to a new water-line parallel to the original water-line, and the distance she will sink is known at once, if we know the tons per inch at the original water-line. Thus a ship is floating at a draught of 13 feet forward and 15 feet aft, and the tons per inch immersion is 20 tons. If a weight of 55 tons be placed over or under the centre of flotation, she will sink $\frac{55}{20}$ inches, or $2\frac{3}{4}$ inches, and the new draught will be $13'\ 2\frac{3}{4}''$ forward and $15'\ 2\frac{3}{4}''$ aft.

It will be noticed that we have made two assumptions, both of which are rendered admissible by considering that the weight is of moderate amount. First, that the tons per inch does not change appreciably as the draught increases, and this is, for all practical purposes, the case in ordinary ships. Second, that the centre of gravity of the parallel layer of added buoyancy is in the same section as the centre of flotation. This latter assumption may be taken as true for small changes in draught caused by the addition of weights of moderate amount; but for large changes it will not be reasonable, because the centres of gravity of the water-planes are not all in the same section, but vary for each water-plane. As a rule, water-planes are fuller aft than forward near the L.W.P., and this more so as the draught increases; and so, if we draw on the profile of the sheer drawing a curve through the centres of gravity of water-planes parallel to the L.W.P., we should obtain a curve which slopes somewhat aft as the draught increases. We shall discuss further the methods which have to be adopted when the weights added are too large for the above assumptions to be accepted.

We see, therefore, that if we place a weight of moderate amount on board a ship at any other place than over the centre of flotation, she will not sink in the water to a water-line parallel to the original water-line, but she will change trim as well as sink bodily in the water. The change of trim will be forward or aft according as the weight is placed forward or aft of the centre of flotation.

In determining the new draught of water, we proceed in two steps:—

1. Imagine the weight placed over the centre of flotation.

2. Then imagine the weight shifted either forward or aft to the assigned position. This shift will produce a certain moment forward or aft, as the case may be, equal to the weight multiplied by its longitudinal distance from the centre of flotation. This moment divided by the moment to change trim 1 inch as calculated for the original water-plane will give the change of trim.

The steps will be best illustrated by the following example:—

A vessel is floating at a draught of 12′ 3″ forward and 14′ 6″ aft. The tons per inch immersion is 20; length, 300 feet; centre of flotation, 12 feet abaft the middle of length; moment to change trim 1 inch, 300 foot-tons. A weight of 30 tons is placed 20 feet from the forward end of the ship. What will be the new draught of water?

The first step is to see the sinkage caused by placing the weight over the centre of flotation. This sinkage is $1\frac{1}{2}$ inches, and the draughts would then be—

12′ $4\frac{1}{2}$″ forward, 14′ $7\frac{1}{2}$″ aft

Now, the shift from the centre of flotation to the given position is 142 feet, so that the moment forward is 30 × 142 foot-tons, and the change of trim by the bow is—

$$\frac{30 \times 142}{300}, \text{ or } 14\frac{1}{4} \text{ inches nearly}$$

This has to be divided up in the ratio of 138 : 162, because the centre of flotation is 12 feet abaft the middle of length. We therefore have—

$$\text{Increase of draught forward } \tfrac{162}{300} \times 14\tfrac{1}{4}'' = 7\tfrac{3}{4}'' \text{ say}$$
$$\text{Decrease of draught aft } \tfrac{138}{300} \times 14\tfrac{1}{4}'' = 6\tfrac{1}{2}'' \text{ say}$$

The final draughts will therefore be—

$$\text{Forward, } 12'\ 4\tfrac{1}{2}'' + 7\tfrac{3}{4}'' = 13'\ 0\tfrac{1}{4}''$$
$$\text{Aft, } 14'\ 7\tfrac{1}{2}'' - 6\tfrac{1}{2}'' = 14'\ 1''$$

Effect on the Trim of a Ship due to adding a Weight of Considerable Amount.—In this case the assumptions made in the previous investigation will no longer hold, and we must allow for the following :—

1. Variation of the tons per inch immersion as the ship sinks deeper in the water.

2. The centre of flotation does not remain in the same transverse section.

3. The addition of a large weight will alter the position of G, the centre of gravity of the ship.

4. The different form of the volume of displacement will alter the position of B, the centre of buoyancy of the ship, and also the value of BM.

5. Items 3 and 4 will alter the value of the moment to change trim 1 inch.

As regards 1, we can obtain first an approximation to the sinkage by dividing the added weight by the tons per inch immersion at the original water-line. The curve of tons per inch immersion will give the tons per inch at this new draught. The mean between this latter value and the original tons per inch, divided into the added weight, will give a very close approximation to the increased draught. Thus, a vessel floats at a constant draught of 22′ 2″, the tons per inch immersion being 44·5. It is required to find the draught after adding a weight of 750 tons. The first approximation to the increase of draught is $\frac{750}{44\cdot5} = 17$ inches nearly. At a draught of 23′ 7″ it is found that the tons per inch immersion is 45·7. The mean tons per inch is therefore $\frac{1}{2}(44\cdot5 + 45\cdot7) = 45\cdot1$, and the increase in draught is therefore $\frac{750}{45\cdot1} = 16\cdot63$, or $16\frac{5}{8}$ inches nearly. This assumes that the ship sinks to a water-plane parallel to the first water-plane. In order that this can be the case, the weight must have been placed in the same transverse section as the centre of gravity of the layer of displacement between the two water-planes. We know that the weight and buoyancy of the ship must act in the same vertical line, and therefore, for the vessel to sink down without change of trim, the added weight must act in the same vertical line as the added buoyancy. We can approximate very closely to the centre of gravity of the layer as follows: Find the centre of flotation of the original W.P. and that of the parallel W.P. to which the vessel is supposed to sink. Put these points on the profile drawing at the respective water-lines. Draw a line joining them, and bisect this line. Then this point will be a very close approximation to the centre of gravity of the layer. A weight of 750 tons placed as above, with its centre of gravity in the transverse section containing this point, will cause the ship to take up a new draught of 23′ 6⅝″ with no change of trim.

We can very readily find the new position of G, the centre of gravity of the ship due to the addition of the weight. Thus, suppose the weight of 750 tons in the above example is placed with its centre of gravity 16 feet below the C.G. of the ship; then, supposing the displacement before adding the weight to be 9500 tons, we have—

$$\begin{aligned}\text{Lowering of G} &= \frac{750 \times 16}{10250} \\ &= 1\cdot17 \text{ feet}\end{aligned}$$

We also have to take account of 4. In the case we have taken, the new C.B. below the *original water-line* was 9·7 feet, as against 10·5 feet in the original condition, or a rise of 0·8 foot.

For the new water-plane we have a different longitudinal BM, and, knowing the new position of B and of G, we can determine the new longitudinal metacentric height. From this we can obtain the new *moment to change trim* 1 *inch*, using, of course, the new displacement. In the above case this works out to 950 foot-tons.

Now we must suppose that the weight is shifted from the assumed position in the same vertical line as the centre of gravity of the layer to its given position, and this distance must be found. The weight multiplied by the longitudinal shift will give the moment changing the trim either aft or forward, as the case may be. Suppose, in the above case, this distance is 50 feet forward. Then the moment changing trim by the bow is—

$$750 \times 50 = 37{,}500 \text{ foot-tons}$$

and the approximate change of trim is—

$$37{,}500 \div 950 = 39\tfrac{1}{2} \text{ inches}$$

This change of trim has to be divided up in the ordinary way for the change of draught aft and forward. In this case we have—

$$\text{Increase of draught forward} = \tfrac{217}{400} \times 39\tfrac{1}{2} = 21\tfrac{1}{2} \text{ inches say}$$
$$\text{Decrease of draught aft} = \tfrac{183}{400} \times 39\tfrac{1}{2} = 18 \text{ inches say}$$

We therefore have for our new draughts—

$$\text{Draught aft, } 25' \ 2'' + 16\tfrac{5}{8}'' - 18'' = 22' \ 0\tfrac{5}{8}''$$
$$\text{Draught forward, } 22' \ 2'' + 16\tfrac{5}{8}'' + 21\tfrac{1}{2}'' = 25' \ 4\tfrac{1}{8}''$$

For all ordinary purposes this would be sufficiently accurate; but it is evidently still an approximation, because we do not take account of the new GM for the final water-line, and the consequent new moment to change trim 1 inch. These can be calculated if desired, and corrections made where necessary.

To determine the Position of a Weight on Board a Ship such that the Draught aft shall remain constant whether the Weight is or is not on Board.—Take a ship floating at the water-line WL, as in Fig. 65. If a weight w be placed with its centre of gravity in the transverse section that contains the centre of flotation, the vessel will very nearly sink to a parallel water-line W′L′.[1] This, however, is not what is required, because the draught aft is the distance WW′ greater than it should be. The weight will have to be

[1] Strictly speaking, the weight should be placed with its centre of gravity in the transverse section that contains the centre of gravity of the zone between the water-lines WL and W′L′.

moved forward sufficient to cause a change of trim forward of WW′ + LL′, and then the draught aft will be the same as it originally was, and the draught forward will increase by the amount WW′ + LL′. This will be more clearly seen, perhaps, by working the following example:—

It is desired that the draught of water aft in a steamship (particulars given below) shall be constant, whether the coals

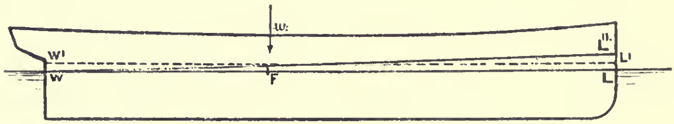

FIG. 65.

are in or out of the ship. Find the approximate position of the centre of gravity of the coals in order that the desired condition may be fulfilled: Length of ship, 205 feet; displacement, 522 tons (no coals on board); centre of flotation from after perpendicular, 104·3 feet; longitudinal BM, 664 feet; longitudinal GM, 661·5 feet; tons per inch, 11·4; weight of coals, 57 tons.

From the particulars given, we find that—

$$\left.\begin{array}{l}\text{Moment to change}\\ \text{trim 1 inch}\end{array}\right\} = \frac{661\cdot5 \times 522}{12 \times 205} = 140 \text{ foot-tons}$$

The bodily sinkage, supposing the coals placed with the centre of gravity in the transverse section containing the centre of flotation, will be $\frac{57}{11\cdot4} = 5$ inches. Therefore the coals must be shifted forward from this position through such a distance that a change of trim of 10 inches forward is produced. Accordingly, a forward moment of—

$$140 \times 10 = 1400 \text{ foot-tons}$$

is required, and the distance forward of the centre of flotation the coals require shifting is—

$$\frac{1400}{57} = 24\cdot6 \text{ feet}$$

Therefore, if the coals are placed—

$$104{\cdot}3 + 24{\cdot}6 = 128{\cdot}9 \text{ feet}$$

forward of the after perpendicular, the draught aft will remain very approximately the same as before.

Change of Trim caused by a Compartment being open to the Sea.—The principles involved in dealing with a problem of this character will be best understood by working out the following example:—

A rectangular-shaped lighter, 100 feet long, 40 feet broad, 10 feet deep, floating in salt water at 3 feet level draught, has a collision bulkhead 6 feet from the forward end. If the side is broached before this bulkhead below water, what would be the trim in the damaged condition?

Let ABCD, Fig. 66, be the elevation of the lighter, with a

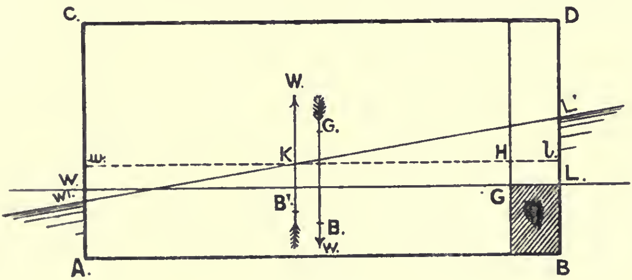

FIG. 66.

collision bulkhead 6 feet from the forward end, and floating at the level water-line WL. It is well to do this problem in two stages—

1. Determine the amount of mean sinkage due to the loss of buoyancy.

2. Determine the change of trim caused.

1. The lighter, due to the damage, loses an amount of buoyancy which is represented by the shaded part GB, and if we assume that she sinks down parallel, she will settle down at a water-line wl such that volume wG = volume GB. This will determine the distance x between wl and WL.

$$\text{For the volume } wG = wH \times 40 \text{ feet} \times x$$
$$\text{and the volume } GB = GL \times 40 \text{ feet} \times 3 \text{ feet}$$
$$\therefore x = \frac{40 \times 6 \times 3}{94 \times 40} = \tfrac{18}{94} \text{ feet}$$
$$= 2\tfrac{1}{4} \text{ inches nearly}$$

2. We now deal with the change of trim caused.

$$\text{The volume of displacement} = 100 \times 40 \times 3 \text{ cubic feet}$$
$$\text{The weight of the lighter} = \frac{100 \times 40 \times 3}{35} = \tfrac{2400}{7} \text{ tons}$$

and this weight acts down through G, the centre of gravity, which is at 50 feet from either end.

But we have lost the buoyancy due to the part forward of bulkhead EF, and the centre of buoyancy has now shifted back to B′ such that the distance of B′ from the after end is 47 feet. Therefore we have W, the weight of lighter, acting down through G, and W, the upward force of buoyancy, acting through B′. These form a couple of magnitude—

$$W \times 3 \text{ feet} = \tfrac{2400}{7} \times 3 = \tfrac{7200}{7} \text{ foot-tons}$$

tending to trim the ship forward.

To find the amount of this trim, we must find the moment to change trim 1 inch—

$$= \frac{W \times GM}{12 \times L}$$

using the ordinary notation.

Now, GM very nearly equals BM;

$$\therefore \text{moment to change trim 1 inch} = \frac{\frac{2400}{7}}{12 \times 100} \times BM$$
$$= \tfrac{2}{7} \times BM$$
$$BM = \frac{I_0}{V}$$

where I_0 = the moment of inertia of the intact water-plane about a transverse axis through its centre of gravity;

and V = volume of displacement in cubic feet.

$$I = \tfrac{1}{12}(94 \times 40) \times (94)^2$$
$$V = 12{,}000$$
$$\therefore BM = \frac{40 \times (94)^3}{144000}$$

and moment to alter trim 1 inch $= \dfrac{2 \times 40 \times (94)^3}{7 \times 144000}$

$= 66$ foot-tons nearly

$\therefore$ the change of trim $= \frac{7200}{7} \div 66$

$= 15\frac{1}{2}$ inches

The new water-line W′L′ will pass through the centre of gravity of the water-line *wl* at K, and the change of trim aft and forward must be in the ratio 47 : 53; or—

Decrease of draught aft $= \frac{47}{100} \times 15\frac{1}{2} = 7\frac{1}{4}$ inches

Increase of draught forward $= \frac{53}{100} \times 15\frac{1}{2} = 8\frac{1}{4}$ inches

therefore the new draught aft is given by—

$$3'\ 0'' + 2\tfrac{1}{4}'' - 7\tfrac{1}{4}'' = 2'\ 7''$$

and the new draught forward by—

$$3'\ 0'' + 2\tfrac{1}{4}'' + 8\tfrac{1}{4}'' = 3'\ 10\tfrac{1}{2}''$$

The same result would be obtained by considering the weight of water in the compartment GB acting downwards, and taking its moment about the centre of flotation K of the intact part of the water-line *wl*. This gives a moment forward of—

$$\left(\frac{6 \times 40 \times 3}{35}\right) \times 50 \text{ foot-tons} = \tfrac{7200}{7} \text{ foot-tons}$$

as obtained above.

It will be noticed that we have assumed that the moment to change trim for the water-plane *wl* remains constant as the vessel changes trim. The slight alteration can be allowed for, if thought desirable, by taking the mean between the moment to change trim for the water-planes *wl* and W′L′, and using that to determine the change of trim.

EXAMPLES TO CHAPTER IV.

1. A ship is floating at a draught of 20 feet forward and 22 feet aft, when the following weights are placed on board in the positions named :—

Weight in tons.			Distance from C.G. of water-plane in feet.	
20	...	...	100	before
45	...	...	80	before
60	...	...	50	abaft
30	...	...	10	abaft

What will be the new draught forward and aft, the moment to change trim 1 inch being 800 foot-tons, and the tons per inch = 35 ?

Ans. 20′ $5\frac{3}{4}$″ forward, 22′ 3″ aft.

2. A vessel 300 feet long, designed to float with a trim of 3 feet by the stern, owing to consumption of coal and stores, floats at a draught of 9′ 3″ forward, and 14′ 3″ aft. The load displacement at a mean draught of 13′ 6″ is 2140 tons; tons per inch, $18\frac{1}{4}$; centre of flotation, $12\frac{1}{2}$ feet abaft the middle of length. Approximate as closely as you can to the displacement.

Ans. 1775 tons.

3. A vessel is 300 feet long and 36 feet beam. Approximate to the moment to change trim 1 inch, the coefficient of fineness of the L.W.P. being 0·75.

Ans. 319 foot-tons.

4. A light-draught stern-wheel steamer is very approximately of the form of a rectangular box of 120 feet length and 20 feet breadth. When fully laden, the draught is 18 inches, and the centre of gravity of vessel and lading is 8 feet above the water-line. Find the transverse and longitudinal metacentric heights, and also the moment to change trim one inch.

Ans. 13·47 feet, $791\frac{1}{4}$ feet ; $56\frac{1}{2}$ foot-tons.

5. A vessel is floating at a draught of 12′ 3″ forward and 14′ 6″ aft. The tons per inch immersion is 20 ; length, 300 feet ; centre of flotation, 12 feet abaft amidships ; moment to change trim 1 inch, 300 foot-tons. Where should a weight of 60 tons be placed on this vessel to bring her to an even keel.

Ans. 123 feet forward of amidships.

6. What weight placed 13 feet forward of amidships will have the same effect on the trim of a vessel as a weight of 5 tons placed 10 feet abaft the forward end, the length of the ship being 300 feet, and the centre of flotation 12 feet abaft amidships.

Ans. 30·4 tons.

7. A right circular pontoon 50 feet long and 16 feet in diameter is just half immersed on an even keel. The centre of gravity is 4 feet above the bottom. Calculate and state in degrees the transverse heel that would be produced by shifting 10 tons 3 feet across the vessel. State, in inches, the change of trim produced by shifting 10 tons longitudinally through 20 feet.

Ans. 3 degrees nearly ; 25 inches nearly.

8. Show why it is that many ships floating on an even keel will increase the draught forward, and decrease the draught aft, or, as it is termed, go down by the head, if a weight is placed at the middle of the length.

9. Show that for vessels having the ratio of the length to the draught about 13, the longitudinal B.M. is approximately equal to the length. Why should a shallow draught river steamer have a longitudinal B.M. much greater than the length ? What type of vessel would have a longitudinal B.M. less than the length ?

10. Find the moment to change trim 1 inch of a vessel 400 feet long, having given the following particulars: Longitudinal metacentre above centre of buoyancy, 446 feet; distance between centre of gravity and centre of buoyancy, 14 feet; displacement, 15,000 tons.

Ans. 1350 foot-tons.

11. The moment of inertia of a water-plane of 22,500 square feet about a transverse axis 20 feet forward of the centre of flotation, is found to be 254,000,000 in foot-units. The displacement of the vessel being 14,000 tons, determine the distance between the centre of buoyancy and the longitudinal metacentre.

Ans. 500 feet.

12. In the preceding question, if the length of the ship is 405 feet, and the distance between the centre of buoyancy and the centre of gravity is 13 feet, determine the change of trim caused by the longitudinal transfer of 150 tons through 50 feet.

Ans. $5\frac{3}{8}$ inches nearly.

13. A water-plane has an area of 13,200 square feet, and its moment of inertia about a transverse axis $14\frac{1}{4}$ feet forward of its centre of gravity works out to 84,539,575 in foot-units. The vessel is 350 feet long, and has a displacement to the above water-line of 5600 tons. Determine the moment to change trim 1 inch, the distance between the centre of gravity and the centre of buoyancy being estimated at 8 feet.

Ans. 546 foot-tons.

14. The semi-ordinates of a water-plane of a ship 20 feet apart are as follows: 0·4, 7·5, 14·5, 21·0, 26·6, 30·9, 34·0, 36·0, 37·0, 37·3, 37·3, 37·3, 37·3, 37·2, 37·1, 36·8, 35·8, 33·4, 28·8, 21 7, 11·5 feet respectively. The after appendage, whole area 214 square feet, has its centre of gravity 6·2 feet abaft the last ordinate. Calculate—

(1) Area of the water-plane.

(2) Position of C.G. of water-plane.

(3) Transverse B.M.

(4) Longitudinal B.M.

(Volume of displacement up to the water-plane 525,304 cubic feet.)

Ans. (1) 24,015 square feet; (2) 18·2 feet abaft middle ordinate; (3) 17·16 feet; (4) 447·6 feet.

15. The semi-ordinates of the L.W.P. of a vessel $15\frac{1}{4}$ feet apart are, commencing from forward, 0·1, 2·5, 5·3, 8·1, 10·8, 13·1, 15·0, 16·4, 17·6, 18·3, 18·5, 18·5, 18·4, 18·1, 17·5, 16·6, 15·3, 13·3, 10·8, 7·6, 3·8 feet respectively. Abaft the last ordinate there is a portion of the water-plane, the half-area being 27 square feet, having its centre of gravity 4 feet abaft the last ordinate. Calculate the distance of the longitudinal metacentre above the centre of buoyancy, the displacement being 2206 tons.

Ans. 534 feet.

16. State the conditions that must hold in order that a vessel shall not change trim in passing from river water to salt water.

17. A log of fir, specific gravity 0·5, is 12 feet long, and the section is 2 feet square. What is its longitudinal metacentric height when floating in stable equilibrium?

Ans. 16·5 feet nearly.

18. Using the approximate formula for the moment to change trim 1 inch given on p. 145, show that this moment will be very nearly given by

$30 \cdot \frac{T^2}{B}$, where T is the tons per inch immersion, and B is the breadth.

Show also that in ships of ordinary form, the moment to change trim 1 inch approximately equals $\frac{1}{10000} \cdot L^2B$.

CHAPTER V.

STATICAL STABILITY, CURVES OF STABILITY, CALCULATIONS FOR CURVES OF STABILITY, INTEGRATOR, DYNAMICAL STABILITY.

Statical Stability at Large Angles of Inclination. Atwood's Formula.—We have up to the present only dealt with the stability of a ship at small angles of inclination, and within these limits we can determine what the statical stability is by using the metacentric method as explained on p. 94. We must now, however, investigate how the statical stability of a ship can be determined for large angles of inclination, because in service it is certain that she will be heeled over to much larger angles than 10° to 15°, which are the limits beyond which we cannot employ the metacentric method.

Let Fig. 67 represent the cross-section of a ship inclined to a large angle θ. WL is the position on the ship of the original water-line, and B the original position of the centre of buoyancy. In the inclined position she floats at the water-line W′L′, which intersects WL in the point S, which for large angles will not usually be in the middle line of the ship. The volume SWW′ is termed, as before, the "*emerged wedge*," and the volume SLL′ the "*immersed wedge*," and g, g' are the positions of the centres of gravity of the emerged and immersed wedges respectively. The volume of displacement remains the same, and consequently these wedges are equal in volume. Let this volume be denoted by v. The centre of buoyancy of the vessel when floating at the water-line W′L′ is at B′, and the upward support of the buoyancy acts through B′; the downward force of the weight acts through G, the centre of gravity of the ship. Draw GZ and BR perpendicular to the vertical through B′, and gh, $g'h'$ perpendicular to the new water-line W′L′. Then the moment of the couple tending to right the ship is W × GZ, or, as we term it, the *moment of statical stability*. Now—

$$\begin{aligned} GZ &= BR - BP \\ &= BR - BG \sin \theta \end{aligned}$$

so that the moment of statical stability at the angle θ is—

$$W(BR - BG . \sin \theta)$$

The length BR is the only term in this expression that we do not know, and it is obtained in the following manner. The

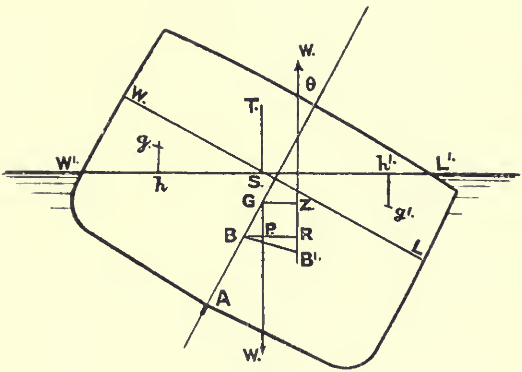

FIG. 67.

new volume of displacement W′AL′ is obtained from the old volume WAL by shifting the volume WSW′ to the position LSL′, through a *horizontal* distance *hh*′. Therefore the *horizontal* shift of the centre of gravity of the immersed volume from its original position at B, or BR, is given by—

$$BR = \frac{v \times hh'}{V}$$

(using the principle discussed on p. 96). Therefore the moment of statical stability at the angle θ is—

$$W\left(\frac{v \times hh'}{V} - BG . \sin \theta\right) \text{ foot-tons}$$

This is known as "Atwood's formula."

$$\text{The righting arm or lever} = \frac{v \times hh'}{V} - BG \cdot \sin\theta$$

If we want to find the length of the righting arm or lever at a given angle of heel θ, we must therefore know—

(1) The position of the centre of buoyancy B in the upright condition.

(2) The position of the centre of gravity G of the ship.

(3) The volume of displacement V.

(4) The value of the moment of transference of the wedges parallel to the new water-line, viz. $v \times hh'$.

This last expression involves a considerable amount of calculation, as the form of a ship is an irregular one. The methods adopted will be fully explained later, but for the present we will suppose that it can be obtained when the form of the ship is given.

Curve of Statical Stability.—The lengths of GZ thus obtained from Atwood's formula will vary as the angle of heel increases, and usually GZ gradually increases until an angle is reached when it obtains a maximum value. On further inclination, an angle will be reached when GZ becomes zero, and, further than this, GZ becomes negative when the couple $W \times GZ$ is no longer a couple tending to right the ship, but is an upsetting couple tending to incline the ship still further. Take H.M.S. *Captain*[1] as an example. The lengths of the lever GZ, as calculated for this ship, were as follows:—

At 7	degrees,	GZ =	$4\frac{1}{4}$	inches
„ 14	„	„ =	$8\frac{1}{2}$	„
„ 21	„	„ =	$10\frac{3}{4}$	„
„ 28	„	„ =	10	„
„ 35	„	„ =	$7\frac{3}{4}$	„
„ 42	„	„ =	$5\frac{1}{4}$	„
„ 49	„	„ =	2	„
„ $54\frac{1}{2}$	„	„ =	*nil*	

Now set along a base-line a scale of degrees on a con-

[1] The *Captain* was a rigged turret-ship which foundered in the Bay of Biscay. A discussion of her stability will be found in "Naval Science," vol. i.

venient scale (say $\frac{1}{4}$ inch = 1 degree), and erect ordinates at the above angles of the respective lengths given. If now we pass a curve through the tops of these ordinates, we shall obtain what is termed a "*curve of statical stability*," from which we can obtain the length of GZ for any angle by drawing the ordinate to the curve at that angle. The curve A, in Fig. 68, is the curve so constructed for the *Captain*. The angle

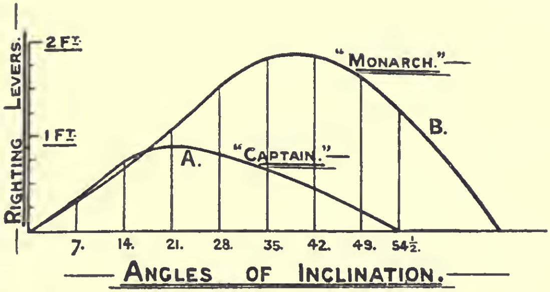

FIG. 68.

at which GZ obtains its maximum value is termed the "*angle of maximum stability*," and the angle at which the curve crosses the base-line is termed the "*angle of vanishing stability*," and the number of degrees at which this occurs is termed the "*range of stability*." If a ship is forced over beyond the angle of vanishing stability, she cannot right herself; GZ having a negative value, the couple operating on the ship is an upsetting couple.

In striking contrast to the curve of stability of the *Captain* is the curve as constructed for H.M.S. *Monarch*.[1] The lengths of the righting levers at different angles were calculated as follows:—

At 7	degrees,	GZ =	4	inches
„ 14	„	„ =	$8\frac{1}{4}$	„
„ 21	„	„ =	$12\frac{1}{4}$	„
„ 28	„	„ =	$18\frac{1}{4}$	„
„ 35	„	„ =	$21\frac{3}{4}$	„

[1] The *Monarch* was a rigged ship built about the same time as the *Captain*, but differing from the *Captain* in having greater freeboard. See also the volume of "Naval Science" above referred to.

M

At 42 degrees,	GZ	= 22	inches
" 49 "	"	= 20	"
" $54\frac{1}{2}$ "	"	= $17\frac{1}{2}$	"
" $69\frac{1}{2}$ "	"	= *nil*	

The curve for this ship, using the above values for GZ, is given by B, Fig. 68. The righting lever goes on lengthening in the *Monarch's* case up to the large angle of 40°, and then shortens but slowly; that of the *Captain* begins to shorten at about 21° of inclination, and disappears altogether at $54\frac{1}{2}°$, an angle at which the *Monarch* still possesses a large righting lever.

Referring to Atwood's formula for the lever of statical stability at the angle θ, viz.—

$$GZ = \frac{v \times hh'}{V} - BG \cdot \sin \theta$$

we see that the expression consists of two parts. The first part is purely geometrical, depending solely upon the *form* of the ship; the second part, BG . sin θ, brings in the influence of the position of the *centre of gravity of the ship*, and this depends on the distribution of the weights forming the structure and lading of the ship. We shall deal with these two parts separately.

(1) Influence of *form* on curves of stability.

(2) Influence of *position of centre of gravity* on curves of stability.

(1) We have here to take account of the form of the ship above water, as well as the form of the ship below water. The three elements of form we shall consider are draught, beam, and freeboard. These are, of course, relative; for convenience we shall keep the draught constant, and see what variation is caused by altering the beam and freeboard. For the sake of simplicity, let us take floating bodies in the form of boxes.[1] The position of the centre of gravity is taken as constant. Take the standard form to be a box :—

Draught	21 feet.
Beam	$50\frac{1}{2}$ "
Freeboard	$6\frac{1}{2}$ "

[1] These illustrations are taken from a paper read at the Institution of Naval Architects by Sir N. Barnaby in 1871.

The curve of statical stability is shown in Fig. 69 by the curve A. The deck-edge becomes immersed at an inclination of $14\frac{1}{2}^\circ$, and from this angle the curve increases less rapidly than before, and, having reached a maximum value, decreases, the angle of vanishing stability being reached at about 38°.

Now consider the effect of adding $4\frac{1}{2}$ feet to the beam, thus making the box—

Draught	21	feet.
Beam	55	,,
Freeboard	$6\frac{1}{2}$	,,

The curve is now given by B, Fig. 69, the angle of vanishing stability being increased to about 45°. Although the

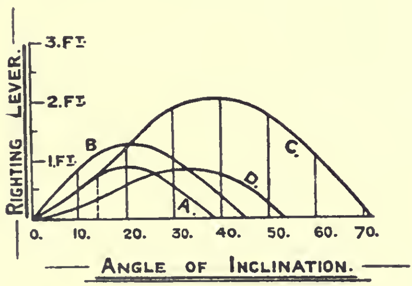

Fig. 69.

position of the centre of gravity has remained unaltered, the increase of beam has caused an increase of GM, the metacentric height, because the transverse metacentre has gone up. We know that for small angles the lever of statical stability is given by GM . sin θ, and consequently we should expect the curve B to start as shown, steeper than the curve A, because GM is greater. There is a very important connection between the metacentric height and the slope of the curve of statical stability at the start, to which we shall refer hereafter.

Now consider the effect of adding $4\frac{1}{2}$ feet to the freeboard of the original form, thus making the dimensions—

Draught	21	feet.
Beam	$50\frac{1}{2}$	,,
Freeboard	11	,,

The curve is now given by C, Fig. 69, which is in striking contrast to both A and B. The angle of vanishing stability is now 72°. The curves A and C coincide up to the angle at which the deck-edge of A is immersed, viz. $14\frac{1}{2}$°, and then, owing to the freeboard still being maintained, the curve C leaves the curve A, and does not commence to decrease until 40°.

These curves are very instructive in showing the influence of beam and freeboard on stability at large angles. We see—

(*a*) An increase of beam increases the initial stability, and therefore the slope of the curve near the origin, but does not greatly influence the area enclosed by the curve or the range.

(*b*) An increase of freeboard has no effect on initial stability (supposing the increase of freeboard does not affect the centre of gravity), but has a most important effect in lengthening out the curve and increasing its area. The two bodies whose curves of statical stability are given by A and C have the same GM, but the curves of statical stability are very different.

(2) We now have to consider the effect on the curve of statical stability of the position of the centre of gravity. If the centre of gravity G is *above* the centre of buoyancy B, as is usually the case, the righting lever is *less* than $\frac{v \times hh'}{V}$ by the expression BG . sin θ. Thus the deduction becomes greater as the angle of inclination increases, because sin θ increases as θ increases, reaching a maximum value of sin $\theta = 1$ when $\theta = 90°$; the deduction also increases as the C.G. rises in the ship. Thus, suppose, in the case C above, the centre of gravity is raised 2 feet. Then the ordinate of the curve C at any angle θ is diminished by $2 \times \sin\,\theta$. For 30°, sin $\theta = \frac{1}{2}$, and the deduction is there 1 foot. In this way we get the curve D, in which the range of stability is reduced from 72° to 53° owing to the 2-feet rise of the centre of gravity.

It is usual to construct these curves as indicated, the ordinates being righting levers, and not righting moments. The righting moment at any angle can be at once obtained by multiplying the lever by the constant displacement. The real curve of statical stability is of course a curve, the ordinates of which represent righting *moments*. This should not be lost sight of, as the following will show. In Fig. 70 are given the

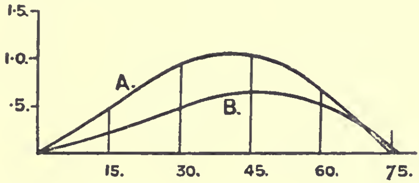

Fig. 70.

curves of righting levers for a merchant vessel in two given conditions, A for the light condition at a displacement of 1500 tons, and B for the load condition at a displacement of 3500 tons. Looking simply at these curves, it would be thought that the ship in the light condition had the better stability; but in Fig. 71, in which A represents the curve of

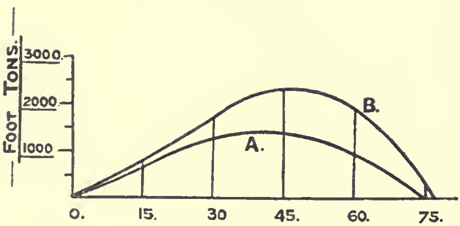

Fig. 71.

righting moments in the light condition, and curve B the curve of righting moments in the load condition, we see that the ship in the light condition has very much less stability than in the load condition.

We see that the following are the important features of a curve of statical stability :—

(*a*) Inclination the tangent to the curve at the origin has to the base-line;

(*b*) The angle at which the maximum value occurs, and the length of the righting lever at this angle;

(*c*) The range of stability.

The angle the tangent at the origin makes with the base-line can be found in a very simple manner as follows: At the angle whose circular measure is unity, viz. 57·3°, erect a perpendicular to the base, and make its length equal to the metacentric height GM, for the condition at which the curve has to be drawn, using the same scale as for the righting levers (see Fig. 72). Join the end of this line with the origin, and the curve as it approaches the origin will tend to lie along this curve.

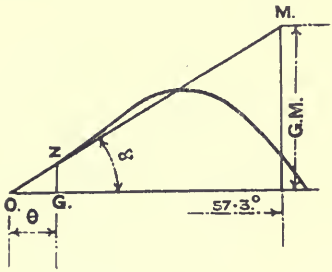

FIG. 72.

The proof of this is given below.[1]

Specimen Curves of Stability.—In Fig. 73 are given some specimen curves of stability for typical classes of ships.

A is the curve for a modern British battleship of about $3\frac{1}{2}$ feet metacentric height. The range is about 63°.

B is the curve for the American monitor *Miantonomoh*. This ship had a low freeboard, and to provide sufficient stability a very great metacentric height was provided. This is shown by the steepness of the curve at the start.

C is the curve for a merchant steamer carrying a miscellaneous cargo, with a metacentric height of about 2 feet. In

[1] For a small angle of inclination θ, we know that $GZ = GM \times \theta$, θ being in circular measure;

$$\text{or } \frac{GZ}{\theta} = \frac{GM}{1}$$

If now we express θ in degrees, say $\theta = \phi°$, then—

$$\frac{GZ}{\phi°} = \frac{GM}{\text{angle whose circular measure is 1}}$$

$$\text{or } \frac{GZ}{\phi°} = \frac{GM}{57·3°}$$

If α is the angle OM makes with the base, then—

$$\tan \alpha = \frac{GM}{57·3°} = \frac{GZ}{\phi°}$$

and thus the line OM lies along the curve near the origin.

this ship there is a large righting lever even at 90°. It must be stated that, although this curve is typical for many ships, yet the forms of the curves of stability for merchant steamers must vary considerably, owing to the many different types of ships and the variation in loading. The curves for a number of

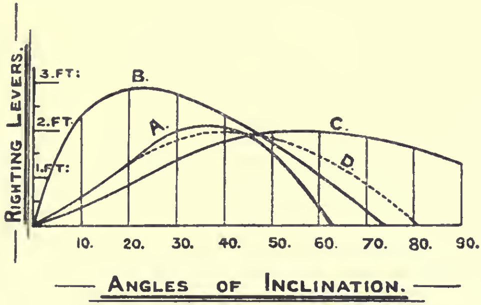

FIG. 73.

merchant steamers are given in the "Manual of Naval Architecture," by Sir W. H. White, and the work on "Stability," by Sir E. J. Reed.

D is the curve of stability for a sailing-ship having a metacentric height of $3\frac{1}{2}$ feet. For further examples see the works referred to above.

Fig. 74 gives an interesting curve of stability for a vessel

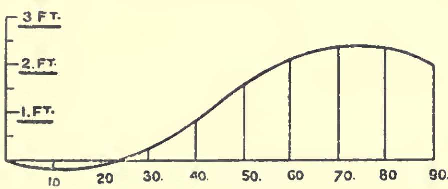

FIG. 74.

which is unstable in the upright condition, but stable at a moderate angle of heel. This vessel has a negative metacentric height, and would not remain in the upright position, but on heeling to an angle of 25° she will resist further inclination, and consequently, if left to herself, the vessel will loll over to this angle, and there be perfectly stable. Such a condition is quite likely to occur in a steamship which starts on a voyage with a small metacentric height, loaded with a homogeneous cargo. Towards the end of the voyage the coal is nearly all burnt out abreast the boilers, and this weight, low down in the ship, being removed, causes the C.G. of the ship to rise, and thus possibly be above the transverse metacentre. The ship is then unstable in the upright condition, but would incline over to the angle at which the curve of stability crosses the base-line.

The curve of stability for a floating body of circular form is very readily obtainable, because the section is such that the upward force of the buoyancy always acts through the centre of the section, as shown in Fig. 75. The righting lever at any angle θ is GM . sin θ, where G is the centre of gravity, and M the centre of the section. Taking the GM as two feet, then the ordinates of the curve of stability are 0, 1·0, 1·73, 2·0, 1·73, 1·0, 0 at intervals of 30°. The maximum occurs at 90°, and the range is 180°. The curve is shown in Fig. 76.

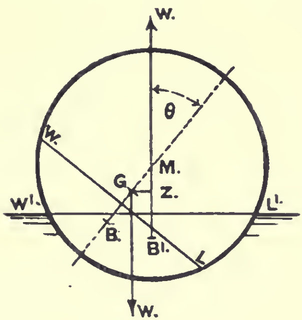

FIG. 75.

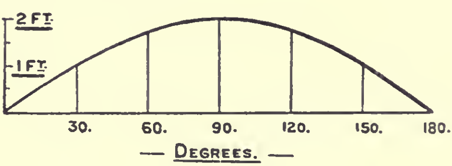

FIG. 76.

Calculations for Curves of Stability.—We now proceed to investigate methods that are or have been adopted in practice to determine for any given ship the curve of righting levers. The use of the integrator is now very general for doing this, and it saves an enormous amount of work; but, in order to get a proper grasp of the subject, it is advisable to understand the methods that were in use previous to the introduction of the integrator.

In constructing and using curves of stability, certain assumptions have to be made. These may be stated as follows :—

1. The sides and deck are assumed to be water-tight for the range over which the curve is drawn.

2. The C.G. is taken in the same position in the ship, and consequently we assume that no weights shift their position throughout the inclination.

3. The trim is assumed to be unchanged, that is, the ship is supposed to be constrained to move about a horizontal longitudinal axis fixed in direction only, and to adjust herself to the required displacement without change of trim.

It is not possible in this work to deal with all the systems of calculation that have been employed; a selection only will be given in this chapter. For further information the student is referred to the *Transactions of the Institution of Naval Architects*, and to the work by Sir E. J. Reed on the "Stability of Ships." The following are the methods that will be discussed :—

1. Blom's mechanical method.

2. Barnes' method.

3. Direct method (sometimes employed as a check on other methods).

4. By Amsler's Integrator and Cross-curves of stability.

1. **Blom's Mechanical Method.**—Take a sheet of drawing-paper, and prick off from the body-plan the shape of each equidistant section[1] (*i.e.* the ordinary sections for displacement), and cut these sections out up to the water-line at which the curve of stability is required, marking on each section the middle line. Now secure all these sections together in their proper relative positions by the smallest possible use of gum.

[1] In settling the sections to be used for calculating stability by any of the methods, regard must be had to the existence of a poop or forecastle the ends of which are watertight, and the ends of these should as nearly as possible be made stop points in the Simpson's rule.

The weight of these represents the displacement of the ship. Next cut out sections of the ship for the angle at which the stability is required, taking care to cut them rather above the real water-line, and gum together in a similar manner to the first set. Then balance these sections against the first set, and cut the sections down parallel to the inclined water-line until the weight equals that of the first set. When this is the case, we can say that at the inclined water-line the displacement is the same as at the original water-line in the upright condition. This must, of course, be the case as the vessel heels over. On reference to Fig. 67, it will be seen that what we want to find is the line through the centre of buoyancy for the inclined position, perpendicular to the inclined water-line, so that if we can find B′ for the inclined position, we can completely determine the stability. This is done graphically by finding the centre of gravity of the sections we have gummed together, and the point thus found will give us the position of the centre of buoyancy for the inclined condition. This is done by successively suspending the sections and noting where the plumb-lines cross, as explained on p. 49. Having then the centre of buoyancy, we can draw through it a line perpendicular to the inclined water-line, and if we then spot off the position of the centre of gravity, we can at once measure off the righting lever GZ. A similar set of sections must be made for each angle about 10° apart, and thus the curve of stability can be constructed.

2. **Barnes's Method of calculating Statical Stability.**—In this method a series of tables are employed, called Preliminary and Combination Tables, in which the work is set out in tabulated form. Take the section in Fig. 77 to represent the ship, WL being the upright water-line for the condition at which the curve of stability is required. Now, for a small transverse angle of inclination it is true that the new water-plane for the same displacement will pass through the centre line of the original water-plane WL, but as the angle of inclination increases, a plane drawn through S will cut off a volume of displacement sometimes greater and sometimes less than the original volume, and the actual water-line will take up some

such position as W′L′, Fig. 77, supposing too great a volume to be cut off by the plane through S. Now, we cannot say straight off where the water-line W′L′ will come. What we have to do is this: Assume a water-line *wl* passing through S; find the volume of the assumed immersed wedge *l*SL, the volume of the assumed emerged wedge *w*SW, and the area of the assumed water-plane *wl*. Then the difference of the volumes of the wedges divided by the area of the water-plane will give the thickness of the layer between *wl* and the correct water-plane, supposing the difference of the volumes is not too great. If this is the case, the area of the new water-plane is

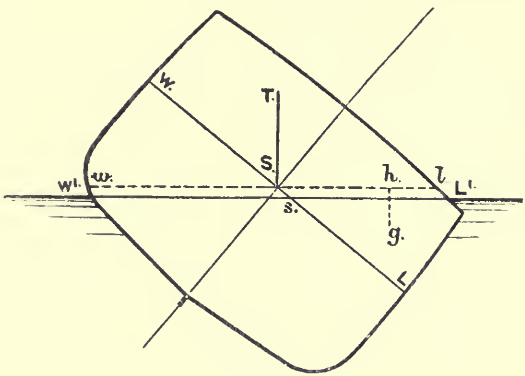

Fig. 77.

found, and a mean taken between it and the original. In this way the thickness of the layer can be correctly found. If the immersed wedge is in excess, the layer has to be deducted; if the emerged wedge is in excess, the layer has to be added.

To get the volumes of either of the wedges, we have to proceed as follows: Take radial planes a convenient angular interval apart, and perform for each plane the operation symbolized by $\frac{1}{2}\int y^2 \cdot dx$, *i.e.* the half-squares of the ordinates are put through Simpson's rule in a fore-and-aft direction for each of the planes. Then put the results through Simpson's rule, using the circular measure of the angular interval. The result will be the volume of the wedge at the particular angle. For proof of this see below.[1]

The results being obtained for the immersed and emerged wedges, we can now determine the thickness of the layer. This work is arranged as follows : The preliminary table, one table for each angle, consists of two parts, one for the immersed wedge, one for the emerged wedge. A specimen table is given on p. 174 for 30°. The lengths of the ordinates of each radial plane are set down in the ordinary way, and operated on by Simpson's multipliers, giving us a function of the area on the immersed side of 550·1, and on the emerged side of 477·4. We then put down the squares of the ordinates, and put them through the Simpson's multipliers, giving us a result for the immersed side of 17,888, and for the emerged side 14,250. The remainder of the work on the preliminary table will be described later.

We now proceed to the combination table for 30° (see p. 175), there being one table for each angle. The functions of squares of ordinates are put down opposite their respective angles, both for the immersed wedge and the emerged wedge, up to and including 30°, and these are put through Simpson's multipliers. In this case the immersed wedge is in excess, and so we find the volume of the layer to be taken off to be 7836 cubic feet, obtaining this by using the proper multipliers. At the bottom is placed the work necessary for finding the thickness of the layer. We have the area of the whole plane 20,540 square feet, and this divided into the excess volume of the immersed wedge, 7836 cubic feet, gives the thickness of the layer to take off, viz. 0·382 foot, to get the true water-line.

We now have to find the moment of transference of the

[1] The area of the section S*l*L is given by $\frac{1}{2}\int y^2 . d\theta$, as on p. 15, and the volume of the wedge is found by integrating these areas right fore and aft, or—

$$\tfrac{1}{2}\int\int y^2 . d\theta . dx$$

which can be written—

$$\tfrac{1}{2}\int\int y^2 . dx . d\theta$$
$$\text{or } \int(\tfrac{1}{2}\int y^2 . dx)d\theta$$

i.e. $\frac{1}{2}\int y^2 . dx$ is found for each radial plane, and integrated with respect to the angular interval.

wedges, $v \times hh'$ in Atwood's formula, and this is done by using the assumed wedges and finding their moments about the line ST, and then making at the end the correction rendered necessary by the layer. To find these moments we proceed as follows: In the preliminary table are placed the *cubes* of the ordinates of the radial plane, and these are put through Simpson's rule; the addition for the emerged and immersed sides are added together, giving us for the 30° radial plane 1,053,633. These sums of functions of cubes are put in the combination table for each radial plane up to and including 30°, and they are put through Simpson's rule, and then respectively multiplied by the cosine of the angle made by each radial plane with the extreme radial plane at 30°. The sum of these products gives us a function of the *sum of the moments* of the assumed immersed and emerged wedges about ST. The multiplier for the particular case given is 0·3878, so that the uncorrected moment of the wedges is 3,391,662,[1] in foot-units, *i.e.* cubic feet, multiplied by feet.

[1] The proof of the process is as follows: Take a section of the wedge S*l*L, Fig. 78, and draw ST perpendicular to S*l*. Then what is required is the moment of the section about ST, and this integrated throughout the length. Take P and P′ on the curved boundary, very close together, and join SP, SP′; call the angle P′Sl, θ, and the angle PSP′, $d\theta$.

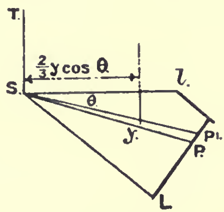

Fig. 78.

Then the area PSP′ $= \frac{1}{2}y^2 . d\theta$ SP $= y$

The centre of gravity of SPP′ is distant from ST, $\frac{2}{3}y . \cos\theta$, and the moment of SPP′ about ST is—

$$(\tfrac{1}{2}y^2 . d\theta) \times (\tfrac{2}{3}y . \cos\theta)$$
$$\text{or } \tfrac{1}{3}y^3 . \cos\theta . d\theta$$

We therefore have the moment of *l*SL about ST—

$$\tfrac{1}{3}\int y^3 . \cos\theta . d\theta$$

and therefore the moment of the wedge about ST is—

$$\int(\tfrac{1}{3}\int y^3 . \cos\theta . d\theta)dx$$
$$\text{or } \tfrac{1}{3}\iint y^3 . \cos\theta . dx . d\theta$$

i.e. find the value of $\frac{1}{3}\int y^3 . \cos\theta . dx$ for radial planes up to and including the angle, and then integrate with respect to the angular interval. It will be seen that the process described above corresponds with this formula.

WATER SECTION INCLINED AT 30°.

Number of section.	Ordinates.	Multipliers.[1]	Functions of ordinates.	Squares of ordinates.	Multipliers.[1]	Functions of squares.	Cubes of ordinates.	Multipliers.[1]	Functions of cubes.
				IMMERSED WEDGE.					
1	4·5	¼	1·1	20	¼	5	91	¼	23
1½	11·3	1	11·3	128	1	128	1,443	1	1,443
2	18·4	½	9·2	339	½	169	6,230	½	3,115
2½	24·4	1	24·4	595	1	595	14,527	1	14,527
3	28·6	¾	21·2	818	¾	613	23,394	¾	17,546
4	33·8	2	67·6	1,142	2	2,284	38,614	2	77,228
5	35·5	1	35·5	1,260	1	1,260	44,739	1	44,739
6	35·6	2	71·2	1,267	2	2,534	45,118	2	90,236
7	35·6	1	35·6	1,267	1	1,267	45,118	1	45,118
8	35·6	2	71·2	1,267	2	2,534	45,118	2	90,236
9	35·6	1	35·6	1,267	1	1,267	45,118	1	45,118
10	35·6	2	71·2	1,267	2	2,534	45,118	2	90,236
11	33·8	¾	25·3	1,142	¾	857	38,614	¾	28,961
11½	31·0	1	31·0	961	1	961	29,791	1	29,791
12	26·9	½	13·5	724	½	362	19,465	½	9,732
12½	21·6	1	21·6	467	1	467	10,078	1	10,078
13	14·3	¼	3·6	205	¼	51	2,924	¼	731
			550·1			17,888			598,858
				EMERGED WEDGE.					
1	4·7	¼	1·2	22	¼	5	104	¼	26
1½	9·3	1	9·3	86	1	86	804	1	804
2	13·8	½	6·9	190	½	95	2,628	½	1,314
2½	17·8	1	17·8	317	1	317	5,640	1	5,640
3	21·4	¾	16·1	458	¾	343	9,800	¾	7,350
4	27·3	2	54·6	745	2	1,490	20,346	2	40,692
5	32·8	1	32·8	1,076	1	1,076	35,288	1	35,288
6	36·7	2	73·4	1,347	2	2,694	49,431	2	98,862
7	38·2	1	38·2	1,459	1	1,459	55,743	1	55,743
8	36·5	2	73·0	1,332	2	2,664	48,627	2	97,254
9	33·6	1	33·6	1,129	1	1,129	37,933	1	37,933
10	29·3	2	58·6	858	2	1,716	25,154	2	50,308
11	23·8	¾	17·8	566	¾	425	13,481	¾	10,111
11½	20·6	1	20·6	424	1	424	8,742	1	8,742
12	16·9	½	8·5	286	½	143	4,827	½	2,413
12½	12·9	1	12·9	166	1	166	2,147	1	2,147
13	8·4	¼	2·1	71	¼	18	593	¼	148
			477·4			14,250	Emerged		454,775
							Immersed		598,858
									1,053,633

[1] The multipliers used here are half the ordinary Simpson's multipliers; the results are multiplied at the end by two to allow for this.

CALCULATION FOR GZ AT 30°.

Inclinations of radial planes.	Immersed Wedge. Functions of ordinates of radial planes.	Functions of squares of ordinates.	Multipliers.	Functions of squares of ordinates for volumes of wedges.	Sums of functions of cubes of ordinates for both sides.	Multipliers.	Products of sums of functions of cubes for both sides.	Cosines of inclinations of radial planes.	Functions of cubes for moments of wedges.
0	—	15,340	1	15,340	974,388	1	974,388	0·8660	843,820
10	—	15,760	4	63,040	990,153	4	3,960,612	0·9397	3,721,787
20	—	16,840	1½	25,260	1,034,251	1½	1,551,377	0·9848	1,527,796
25	—	17,701	2	35,402	1,066,771	2	2,133,542	0·9962	2,125,434
30	550	17,888	½	8,944	1,053,633	½	526,816	1·0000	526,816

Immersed wedge	147,986
Emerged wedge	134,522
	13,464
Multiplier *	0·582
Volume of layer	7,836 cubic feet

	8,745,653
Multiplier †	0·3878
Uncorrected moment	3,391,662
Correction for layer	13,502
Corrected moment	3,378,160
Volume of displacement	398,090
BR	8·485
BG × sin 30°	5·950
GZ	2·535

Longitudinal interval = 30 feet
Circular measure 10° = 0·1745

* Multiplier $= \frac{1}{3} \times 2 \times (\frac{1}{3} \times 30) \times (\frac{1}{3} \times 0{\cdot}1745) = 0{\cdot}582$
† Multiplier $= \frac{1}{3} \times 2 \times (\frac{1}{3} \times 30) \times (\frac{1}{3} \times 0{\cdot}1745) = 0{\cdot}3878$
$BG = 11{\cdot}90$; $\sin 30° = 0{\cdot}5$; $BG \sin \theta = 5{\cdot}95$ feet

Emerged Wedge.				
0	—	15,340	1	15,340
10	—	15,157	4	60,628
20	—	14,766	1½	22,149
25	—	14,640	2	29,280
30	477	14,250	½	7,125
				134,522

Area and Position of C.G. of Radial Plane.

	Functions of area.	Functions of moment.
Immersed wedge	550	17,888
Emerged wedge	477	14,250
	1,027	3,538

$$\text{Area} = 1027 \times 2 \times (\tfrac{1}{3} \times 30) = 20{,}540 \text{ square feet}$$

$$\text{C.G. of radial plane on immersed side} = \frac{3538}{1027} \times \frac{1}{2} = 1{\cdot}723 \text{ feet}$$

$$\text{Thickness of layer} = \frac{7836}{20540} = 0{\cdot}382 \text{ feet}$$

We now have to make the correction for the layer. We already have the volume of the layer, and whether it has to be added or subtracted, and we can readily find the position of the centre of gravity of the radial plane. This is done at the bottom of the combination table from information obtained on the preliminary table. We assume that the centre of gravity of the layer is the same distance from ST as the centre of gravity of the radial plane, which may be taken as the case, unless the thickness of the layer is too great. If the layer is thick, a new water-line is put in at thickness found, and the area and C.G. of this water-line found. The mean between the result of this and of the original plane can then be used. The volume of the layer, 7836 cubic feet, is multiplied by the distance of its centre of gravity from ST, viz. 1·723 feet, giving a result of 13,502 in foot-units, *i.e.* cubic feet multiplied by feet. The correction for the layer is added to or subtracted from the uncorrected moment in accordance with the following rules :—

If the *immersed* wedge is in excess, and the centre of gravity of the layer is on the *immersed* side, the correction for the layer has to be *subtracted.*

If the *immersed* wedge is in excess, and the centre of gravity of the layer is on the *emerged* side, the correction for the layer has to be *added.*

If the *emerged* wedge is in excess, and the centre of gravity of the layer is on the *emerged* side, the correction for the layer has to be *subtracted.*

If the *emerged* wedge is in excess, and the centre of gravity of the layer is on the *immersed* side, the correction for the layer has to be *added.*

We, in this case, *subtract* the correction for the layer, obtaining the true moment of transference of the wedges as 3,378,160, or $v \times hh'$ in Atwood's formula. The volume of displacement is 398,090 cubic feet; BG is 11·90 feet; sin 30° = 0·5. So we can fill in all the items in Atwood's formula—

$$GZ = \frac{v \times hh'}{V} - BG \sin \theta$$

$$\text{or } GZ = 2·535 \text{ feet}$$

In arranging the radial planes, it is best to arrange that the deck edge comes at a stop point in Simpson's first rule, because there is a sudden change of ordinate as the deck edge is passed, and for the same reason additional intermediate radial planes are introduced near the deck edge. In the case we have been considering, the deck edge came at about 30°. The radial planes that were used were accordingly at—

0°, 10°, 20°, 25°, 30°, 35°, 40°, 50°, 60°, 70°, 80°, 90°

Barnes's method of calculating stability has been very largely employed. It was introduced by Mr. F. K. Barnes at the Institution of Naval Architects in 1861, and in 1871 a paper was read at the Institution by Sir W. H. White and the late Mr. John, giving an account of the extensions of the system, with specimen calculations. For further information the student is referred to these papers, and also to the work on "Stability," by Sir E. J. Reed. At the present time it is not used to any large extent, owing to the introduction of the integrator, which gives the results by a mechanical process in much less time. It will be seen that in using this method to find the stability at a given angle, we have to use all the angles up to and including that angle at which the stability is required. Thus a mistake made in the table at any of the smaller angles is repeated right through, and affects the accuracy of the calculation at the larger angles. In order to obtain an independent check at any required angle, we can proceed as follows:—

3. **Triangular or Direct Method of calculating Stability.**—Take the body-plan, and draw on the trial plane through the centre of the upright water-line at the required angle. This may or may not cut off the required displacement. We then, by the ordinary rules of mensuration, discussed in Chapter I., find the area of all such portions as S*l*L, Fig. 77, for all the sections,[1] and also the position of the centre of gravity, *g*, for each section, thus obtaining the distance S*h*.

[1] The sections are made into simple figures, as triangles and trapeziums, in order to obtain the area and position of C.G. of each.

This is done for both the immersed and emerged wedges. The work can then be arranged in tabular form thus :

Number of section.	Areas.	Simpson's multipliers.	Products for volume.	Levers as S*h*.	Products for moment about ST.
1 2 etc.	A_1 A_2 etc.	1 4 etc.	A_1 $4A_2$ etc.	x_1 x_2 etc.	A_1x_1 $4A_2x_2$ etc.
			S_1		M_1

The volume of the wedge $= S_1 \times \frac{1}{3}$ common interval
The moment of wedge about ST $= M_1 \times \frac{1}{3}$ common interval

This being done for both wedges, and calculating the area of the radial plane, we can find the volume of the layer and the uncorrected moment of the wedges. The correction for the layer is added or subtracted from this, exactly as in Barnes's method, and the remainder of the work follows exactly the methods described above for Barnes's method.

There is the disadvantage about the methods we have hitherto described, that we obtain a curve of stability for one particular displacement, but it is often necessary to know the stability of a ship at very different displacements to the ordinary load displacement, as, for example, in the light condition, or the launching condition. The method we are now about to investigate enables us to determine at once the curve of stability at any given displacement and any assumed position of the centre of gravity.

4. **Amsler's Integrator. Cross-curves of Stability.** —The Integrator is an extension of the instrument we have described on p. 77, known as the planimeter. A diagram of one form of the integrator is given in Fig. 79. A bar, BB, has a groove in it, and the instrument has two wheels which run in this groove. W is a balance weight to make the instrument run smoothly. There are also three small wheels that run on the paper, and a pointer as in the planimeter. By passing the pointer round an area, we can find—

(1) A number which is proportional to the *area*, *i.e.* a function of the area.

(2) A function of the *moment of the area* about the axis the bar is set to.

(3) A function of the *moment of inertia of the area* about the same axis.

The bar is set parallel to the axis about which moments are required, by means of distance pieces.

(1) is given by the reading indicated by the wheel marked A.

(2) is given by the reading indicated by the wheel marked M.

(3) is given by the reading indicated by the wheel marked I.

The finding of the moment of inertia is not required in our present calculation.

Now let M'LMW represent the body-plan[1] of a vessel inclined to an angle of 30°; then, as the instrument is set, the

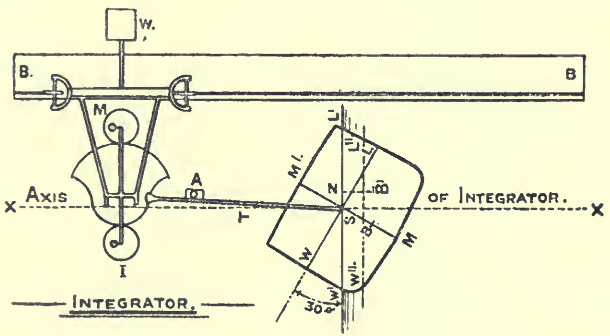

Fig. 79.

axis of moments is the line through S perpendicular to the inclined water-line, and is what we have termed ST. What we want to find is a line through the centre of buoyancy in the inclined position perpendicular to the inclined water-line. By passing the pointer of the instrument round a section, as W'L'M, we can determine its area, and also its moment about the axis ST by using the multipliers; and doing this for all the sections in the body, we can determine the displacement and also the moment of the displacement about ST.[2] Dividing the

[1] The body-plan is drawn for both sides of the ship—the fore-body in black say, and the after-body in red.

[2] This is the simplest method, and it is the best for beginners to employ; but certain modifications suggest themselves after experience with the instrument.

moment by the displacement, we obtain at once the distance of the centre of buoyancy in the inclined condition from the axis ST. It is convenient in practice to arrange the work in a similar manner to that described for the planimeter, p. 79, and the following specimen calculation for an angle of 30° will illustrate the method employed. Every instrument has multipliers for converting the readings of the wheel A into areas, and those of the wheel M into moments. The multipliers must also take account of the scale used.

30°

Sections.	Areas.				Moments.			
	Readings.	Differences.	Simpson's multipliers.	Products.	Readings.	Differences.	Simpson's multipliers.	Products.
Initial readings	3,146	—	—	—	3900	—	—	—
1 and 17	3,210	64	1	64	3910	− 10	1	− 10
2, 4, 6, 8, 10, 12, 14, and 16	8,859	5649	4	22,596	3124	+ 786	4	3144
3, 5, 7, 9, 11, 13, and 15	14,345	5486	2	10,972	2381	+ 743	2	1486
				33,632				4620

Multiplier for displacement = 0·02

Multiplier for moment = 0·2133

$$\text{Displacement} = 33{,}632 \times 0{\cdot}02 = 672{\cdot}6 \text{ tons}$$

$$= 4620 \times 0{\cdot}2133 = 985 \text{ foot-tons}$$

$$GZ = \frac{985}{672{\cdot}6} = 1{\cdot}46 \text{ feet}$$

In this case the length of the ship was divided into sixteen equal parts, and accordingly Simpson's first rule can be employed. The common interval was 8·75 feet. The multiplier for the instrument was $\frac{15}{1000}$ for the areas, and $\frac{40}{1000}$ for the moments, and, the drawing being on the scale of $\frac{1}{4}$ inch = 1 foot, the readings for areas had to be multiplied by $(4)^2 = 16$, and for moments by $(4)^3 = 64$. The multiplier for displacement in tons is therefore—

$$\frac{15}{1000} \times 16 \times (\tfrac{1}{3} \times 8{\cdot}75) \times \tfrac{1}{35} = 0{\cdot}02$$

and for the moment in foot-tons is—

$$\frac{40}{1000} \times 64 \times (\tfrac{1}{3} \times 8{\cdot}75) \times \tfrac{1}{35} = 0{\cdot}2133$$

We therefore have, assuming that the centre of gravity is at S—

$$GZ = \frac{985}{672{\cdot}6} = 1{\cdot}46 \text{ feet}$$

Now, this 672·6 tons is not the displacement up to the original water-line WL, and we now have to consider a new conception, viz. *cross-curves of stability*. These are the converse of the ordinary curves of stability we have been considering. In these we have the righting levers at a constant displacement and varying angles. In a cross-curve we have the righting levers for a constant angle, but varying displacement. Thus in Fig. 79, draw a water-line W″L″ parallel to

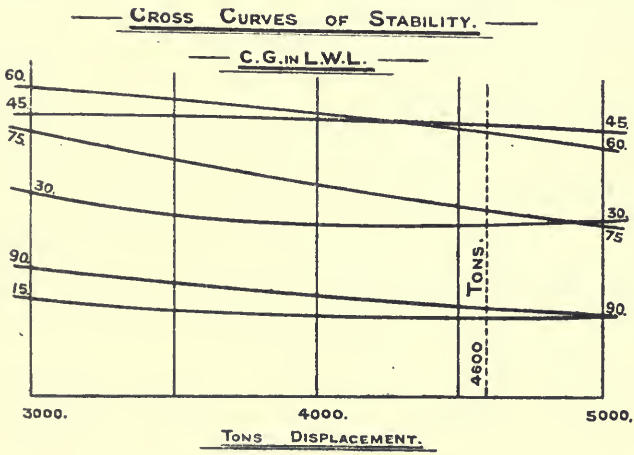

FIG. 80.

W′L′, and for the volume represented by W″ML″ find the displacement and position of the centre of buoyancy in exactly the same way as we have found it for the volume W′ML′. The distance which this centre of buoyancy is from the axis gives us the value of GZ at this displacement, supposing the centre of gravity is at S. The same process is gone through for one or two more water-lines, and we shall have values of GZ at varying displacements at a constant angle. These can be set off as ordinates of a curve, the abscissæ being the displacements in tons. Such a curve is termed the "cross-curve of stability" at 30°, and for any intermediate displacement we can find the value of GZ at 30° by drawing the ordinate to the curve at this displacement. A similar process is gone through for each angle, the same position for the centre of gravity being assumed all through, and a series of cross-curves obtained. Such a set of cross-curves is shown in Fig. 80 for displacements between 3000 and 5000 tons at angles of 15°, 30°, 45°, 60°, 75°, and 90°. At any intermediate displacement, say at 4600 tons, we can draw the ordinate and measure off the values of GZ, and so obtain the ordinates necessary to construct the ordinary curve of stability at that displacement and assumed position of the centre of gravity. The relation between the cross-curves and the ordinary curves of stability is clearly shown in Fig. 81. We have four curves

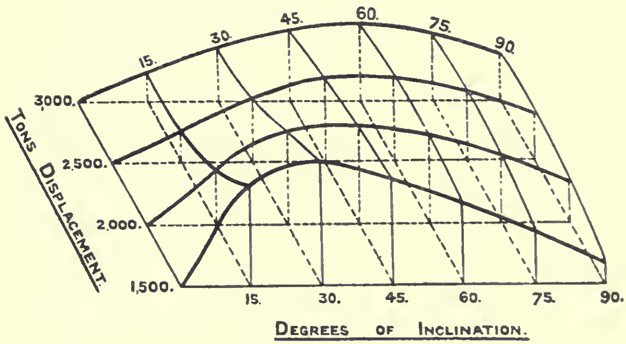

Fig. 81.

of stability for a vessel at displacements of 1500, 2000, 2500, and 3000 tons. These are placed as shown in perspective. Now, through the tops of the ordinates at any given angle we can draw a curve, and this will be the cross-curve of stability at that angle.

It will have been noticed that throughout our calculation we have assumed that the centre of gravity is always at the point S, and the position of this point should be clearly stated on the cross-curves. It is evident that the centre of gravity cannot always remain in this position, which has only been assumed for convenience. The correction necessary can readily be made as follows: If G, the centre of gravity, is below the assumed position S, then $GZ = SZ + SG \cdot \sin \theta$, and if G is above S, then $GZ = SZ - SG \cdot \sin \theta$ for any angle θ. Thus the ordinates are measured from the cross-curves at the required displacement, and then, SG being known, SG sin 15°, SG sin 30°, etc., can be found, and the correct values of GZ determined for every angle.

Dynamical Stability.—The amount of *work* done by a force acting through a given distance is measured by the product of the force and the distance through which it acts. Thus, a horse exerting a pull of 30,000 lbs. for a mile does—

$$30,000 \times 1760 \times 3 = 158,400,000 \text{ foot-lbs. of work}$$

Similarly, if a weight is lifted, the work done is the product of the weight and the distance it is lifted. In the case of a ship being inclined, work has to be done on the ship by some external forces, and it is not always possible to measure the work done by reference to these forces, but we can do so by reference to the ship herself. When the ship is at rest, we have seen that the vertical forces that act upon the ship are—

(1) The weight of the ship acting vertically downwards through the centre of gravity;

(2) The buoyancy acting vertically upwards through the centre of buoyancy;

these two forces being equal in magnitude. When the ship is inclined, they act throughout the whole of the inclination. The centre of gravity is raised, and the centre of buoyancy is lowered. The weight of the ship has been made to move upwards the distance the centre of gravity has been raised, and the force of the buoyancy has been made to move downwards the distance the centre of buoyancy has been lowered. The work done on the ship is equal to the weight multiplied by the rise of the centre of gravity added to the force of the buoyancy multiplied by the depression of the centre of buoyancy; or—

Work done on the ship = weight of the ship multiplied by the vertical separation of the centre of gravity and the centre of buoyancy.

This calculated for any given angle of inclination is termed "*the dynamical stability*" at that angle, and is the work that has to be expended on the ship in heeling her over to the given angle.

Moseley's Formula for the Dynamical Stability at any Given Angle of Inclination.—Let Fig. 67, p. 159, represent a vessel heeled over by some external force to the angle θ; g, g' being the centres of gravity of the emerged and immersed wedges; gh, $g'h'$ being drawn perpendicular to the new water-line W'L'. The other points in the figure have their usual meaning, BR and GZ being drawn perpendicular to the vertical through B'.

The vertical distance between the centres of gravity and buoyancy when inclined at the angle θ is B'Z.

The original vertical distance when the vessel is upright is BG.

Therefore the vertical separation is—

$$B'Z - BG$$

and according to the definition above—

$$\text{Dynamical stability} = W(B'Z - BG)$$

where W = the weight of the ship in tons.

$$\text{Now, } B'Z = B'R + RZ = B'R + BG \cdot \cos\theta$$

Now, using v for the volumes of either the immersed wedge or the emerged wedge, and V for the volume of displacement of the ship, and using the principle given on p. 96, we have—

$$v \times (gh + g'h') = V \times B'R$$

$$\text{or } B'R = \frac{v \times (gh + g'h')}{V}$$

Substituting the above value for BZ, we have—

$$B'Z = \frac{v \times (gh + g'h')}{V} + BG \cos\theta$$

$$\therefore \left.\begin{array}{l}\text{the dynamical}\\ \text{stability}\end{array}\right\} = W\left[\frac{v \times (gh + g'h')}{V} + BG(1 - \cos\theta)\right]$$

which is known as *Moseley's formula.*

It will be seen that this formula is very similar to Atwood's formula (p. 159), and it is possible to calculate it out for varying angles by using the tables in Barnes's method of calculating stability. It is possible, however, to find the dynamical stability of a ship at any angle much more readily if the curve of statical stability has been constructed, and the method adopted, if the dynamical stability is required, is as follows :—

The dynamical stability of a ship at any given angle is equal to the area of the curve of statical stability up to that angle (the ordinates of this curve being the actual righting moments).

As the demonstration of this is somewhat difficult, it is given in Appendix A, p. 247.

To illustrate this principle, take the case of a floating body whose section is in the form of a circle, and which floats with its centre in the surface of the water. The transverse metacentre of this body must be at the centre of the circular section. Let the centre of gravity of the vessel be at G, and the centre of buoyancy be at B. Then for an inclination through 90° G will rise till it is in the surface of the water, but the centre of buoyancy will always remain at the same level, so that the dynamical stability at 90° = W × GM.

Now take the curve of statical stability for such a vessel. The ordinate of this curve at any angle $\theta = W \times GM . \sin\theta$, and consequently the ordinates at angles 15° apart will be W . GM . sin 0°, W . GM . sin 15°, and so on; or, 0, 0·258 W . GM, 0·5 W . GM, 0·707 W . GM, 0·866 W . GM, 0·965 W . GM, and W . GM. If this curve is set out, and its area calculated, it will be found that its area is W × GM, which is the same as the dynamical stability up to 90°, as found above. It should be noticed that the angular interval should not be taken as degrees, but should be measured in circular measure (see p. 86). The circular measure of 15° is 0·2618.

The dynamical stability at any angle depends, therefore, on the area of the curve of statical stability up to that angle; and thus we see that the *area* of the curve of stability is of importance as well as the *angle* at which the ship becomes unstable, because it is the dynamical stability that tells us the work that has to be expended to force the ship over. For full information on this subject, the student is referred to the "Manual of Naval Architecture," by Sir W. H. White, and Sir E. J. Reed's work on the "Stability of Ships."

Examples to Chapter V.

1. A two-masted cruiser of 5000 tons displacement has its centre of gravity at two feet above the water-line. It is decided to add a military top to each mast. Assuming the weight of each military top with its guns, men, and ready-ammunition supply to be 12 tons, with its centre of gravity 70 feet above the water-line, what will be the effect of this change on—

(1) The metacentric height of the vessel?

(2) The maximum range of stability, assuming the present maximum range is 90°, and the tangent to the curve at this point inclined at 45° to the base-line?

(Scale used, $\frac{1}{4}$ inch = 1°, $\frac{1}{4}$ inch = $\frac{1}{10}$ foot GZ.)

Ans. (1) Reduce 0·325 foot, assuming metacentric curve horizontal; (2) reduce range to about 86$\frac{3}{4}$°, assuming no change in cross-curves from 5000 to 5024 tons.

2. The curve of statical stability of a vessel has the following values of GZ at angular intervals of 15°: 0, 0·55, 1·03, 0·99, 0·66, 0·24, and − 0·20 feet. Determine the loss in the range of stability if the C.G. of the ship were raised 6 inches.

Ans. 16°.

3. Obtain, by direct application of Atwood's formula, the moment of stability in foot-tons at angles of 30°, 60°, and 90°, in the case of a prismatic vessel 140 feet long and 40 feet square in section, when floating with sides vertical at a draught of 20 feet, the metacentric height being 2 feet.

4. A body of square section of 20 feet side and 100 feet long floats with one face horizontal in salt water at a draught of 10 feet, the metacentric height being 4 inches. Find the dynamical stability at 45°.

Ans. 171 foot-tons.

5. Indicate how far a vessel having high bulwarks is benefited by them as regards her stability. What precautions should be taken in their construction to prevent them becoming a source of danger rather than of safety?

6. Show from Atwood's formula that a ship is in stable, unstable, or neutral equilibrium according as the centre of gravity is below, above, or coincident with the transverse metacentre respectively.

7. A vessel in a given condition displaces 4600 tons, and has the C.G. in the 19-feet water-line. The ordinates of the cross-curves at this displacement, with the C.G. assumed in the 19-feet water-line, measure as follows: 0·63, 1·38, 2·15, 2·06, 1·37, 0·56 feet at angles of 15°, 30°, 45°, 60°, 75° and 90° respectively. The metacentric height is 2·4 feet.

Draw out the curve of stability, and state (1) the angle of maximum stability, (2) the angle of vanishing stability, and (3) find the dynamical stability at 45° and 90°.

Ans. (1) $50\frac{3}{4}°$; (2) $100\frac{1}{2}°$; (3) 3694, 9650 foot-tons.

8. A vessel has a metacentric height of 3·4 feet, and the curve of stability has ordinates at 15°, 30°, $37\frac{1}{2}°$, 45°, and 60° of 0·9, 1·92, 2·02, 1·65, and − 0·075 feet respectively. Draw out this curve, and state the angle of maximum stability and the angle at which the stability vanishes.

Ans. $35\frac{1}{2}°$, $59\frac{1}{2}°$.

9. A vessel's curve of stability has the following ordinates at angles of 15°, 30°, 45°, 60°, and 75°, viz. 0·51, 0·97, 0·90, 0·53, and 0·08 feet respectively. Estimate the influence on the range of stability caused by lifting the centre of gravity of the ship 0·2 feet.

Ans. Reduce nearly 6°.

10. A square box of 18 feet side floats at a constant draught of 6 feet, the centre of gravity being in the water-line. Obtain, by direct drawing or otherwise, the value of GZ up to 90° at say 6 angles. Draw in the curve of statical stability, and check it by finding its area and comparing that with the dynamical stability of the box at 90°.

(Dynamical stability at 90° = 3 × weight of box.)

11. A vessel fully loaded with timber, some on the upper deck, starts from the St. Lawrence River with a list. She has two cross-bunkers extending to the upper deck. She reaches a British port safely, with cargo undisturbed, but is now upright. State your opinion as to the cause of this.

12. Show by reference to the curves of stability of box-shaped vessels on p. 163 that at the angle at which the deck edge enters the water the tangent to the curve makes the maximum angle with the base-line.

CHAPTER VI.

CALCULATION OF WEIGHTS AND STRENGTH OF BUTT CONNECTIONS. STRAINS EXPERIENCED BY SHIPS.

Calculations of Weights. — We have discussed in Chapter I. the ordinary rules of mensuration employed in finding the areas we deal with in ship calculations. For any given uniform plate we can at once determine the weight if the weight per square foot is given. For iron and steel plates of varying thicknesses, the weight per square foot is given on p. 36. For iron and steel angles and T bars of varying sizes and thicknesses tables are calculated, giving the weight per lineal foot. Such a table is given on p. 189 for steel angles, etc., the thicknesses being in $\frac{1}{20}$ths of an inch. It is the Admiralty practice to specify angles, bars, etc., not in thickness, but in weight per lineal foot. Thus an angle bar $3'' \times 3''$ is specified to weigh 7 lbs. per lineal foot, and a Z bar $6'' \times 3\frac{1}{2}'' \times 3''$ is specified to weigh 15 lbs. per lineal foot. When the bars are specified in this way, reference to tables is unnecessary. The same practice is employed with regard to plates, the thickness being specified as so many pounds to the square foot.

If we have given the size of an angle bar and its thickness, we can determine its weight per foot as follows: Assume the bar has square corners, and is square at the root, then, if a and b are the breadth of the flanges in inches, and t is the thickness in inches, the length of material t inches thick in the section is $(a + b - t)$ inches, or $\frac{a + b - t}{12}$ feet; and if the bar is of iron, the weight per lineal foot is—

$$\left(\frac{a + b - t}{12}\right) \times 40 \times t \text{ lbs.}$$

WEIGHT OF STEEL ANGLES IN POUNDS PER FOOT.

Sum of flanges in inches.	Thickness in Twentieths of an Inch.																			
	$\frac{1}{20}$	$\frac{2}{20}$	$\frac{3}{20}$	$\frac{4}{20}$	$\frac{5}{20}$	$\frac{6}{20}$	$\frac{7}{20}$	$\frac{8}{20}$	$\frac{9}{20}$	$\frac{10}{20}$	$\frac{11}{20}$	$\frac{12}{20}$	$\frac{13}{20}$	$\frac{14}{20}$	$\frac{15}{20}$	$\frac{16}{20}$	$\frac{17}{20}$	$\frac{18}{20}$	$\frac{19}{20}$	$\frac{20}{20}$
2	0·332	0·645	0·943	1·22	1·49	1·73	1·96	2·18												
2½	0·417	0·816	1·20	1·56	1·91	2·24	2·56	2·86	3·14											
3	0·501	0·986	1·45	1·91	2·34	2·75	3·15	3·54	3·90	4·25										
3½	0·587	1·15	1·71	2·24	2·76	3·26	3·75	4·22	4·67	5·10	5·51									
4	0·672	1·33	1·96	2·58	3·19	3·77	4·34	4·90	5·43	5·95	6·45	6·94								
4½	0·757	1·50	2·22	2·92	3·61	4·28	4·94	5·58	6·20	6·80	7·39	7·96	8·51							
5	0·842	1·67	2·47	3·26	4·04	4·79	5·53	6·26	6·96	7·65	8·32	8·97	9·62	10·23						
5½	0·927	1·84	2·73	3·60	4·46	5·30	6·13	6·94	7·73	8·50	9·26	10·00	10·72	11·42						
6			2·98	3·94	4·88	5·81	6·72	7·62	8·49	9·35	10·19	11·02	11·83	12·61	13·39					
6½			3·24	4·28	5·31	6·32	7·32	8·30	9·26	10·20	11·13	12·04	12·93	13·81	14·66					
7			3·49	4·62	5·74	6·83	7·91	8·98	10·02	11·05	12·06	13·05	14·04	14·99	15·94	16·86				
7½			3·75	4·96	6·16	7·34	8·51	9·66	10·79	11·90	13·00	14·08	15·14	16·18	17·21	18·22				
8			4·00	5·30	6·59	7·85	9·10	10·34	11·55	12·75	13·93	15·09	16·25	17·37	18·49	19·58	20·66			
8½			4·26	5·64	7·01	8·36	9·70	11·02	12·32	13·60	14·87	16·11	17·35	18·56	19·76	20·94	22·10			
9			4·51	5·98	7·44	8·87	10·29	11·70	13·08	14·45	15·80	17·13	18·46	19·75	21·04	22·30	23·55	24·79		
9½			4·77	6·32	7·86	9·38	10·89	12·38	13·85	15·30	16·74	18·16	19·56	20·95	22·31	23·66	25·00	26·32		
10			5·02	6·66	8·28	9·89	11·48	13·06	14·61	16·15	17·67	19·18	20·67	22·13	23·59	25·02	26·44	27·84	29·24	
10½			5·28	7·00	8·71	10·40	12·08	13·74	15·38	17·00	18·61	20·20	21·77	23·32	24·86	26·38	27·88	29·38	30·85	
11			5·53	7·34	9·13	10·91	12·67	14·41	16·14	17·85	19·54	21·21	22·88	24·51	26·14	27·74	29·33	30·90	32·47	34·00
11½			5·79	7·68	9·56	11·42	13·27	15·10	16·91	18·70	20·48	22·24	23·98	25·70	27·41	29·10	30·77	32·43	34·08	35·70
12			6·04	8·02	9·99	11·93	13·86	15·78	17·67	19·55	21·41	23·26	25·09	26·89	28·69	30·46	32·22	33·97	35·70	37·40

If the bar is of steel, the weight per lineal foot is—

$$\left(\frac{a + b - t}{12}\right) \times 40{\cdot}8 \times t \text{ lbs.}$$

Thus a $3'' \times 3'' \times \frac{3}{8}''$ steel angle bar would weigh 7·17 lbs., and a steel angle bar $3'' \times 3''$ of 7 lbs. per foot would be slightly less than $\frac{3}{8}$ inch thick.

It is frequently necessary to calculate the weight of a portion of a ship's structure, having given the particulars of its construction; thus, for instance, a bulkhead, a deck, or the outer bottom plating. In any case, the first step must be to find the area of plating and the lengths of angle bars. The weight of the net area of the plating will not give us the total weight of the plating, because we have to allow for butt straps, laps, rivet-heads, and in certain cases liners. The method employed to find the allowance in any given case is to take a sample plate and find what percentage the additions come to that affect this plate, and to use this percentage as an addition to the net weight found for the whole. To illustrate this, take the following example :—

A deck surface of 10,335 square feet is to be covered with $\frac{5}{16}$-inch steel plating, worked flush, jointed with single-riveted edges and butts. Find the weight of the deck, allowing 3 per cent. for rivet-heads.

$\frac{5}{16}$-inch steel plates are 12·75 lbs. per square foot, so that the net weight is—

$$\frac{10{,}335 \times 12{\cdot}75}{2240} = 58{\cdot}8 \text{ tons}$$

Now, assume an average size for the plates, say $16' \times 4'$. $\frac{3}{4}$-inch rivets will probably be used, and the width of the edge strip and butt strap will be about 5 inches. The length round half the edge of the plate is 20 feet, and the area of the strap and lap belonging to this plate is—

$$20 \times \tfrac{5}{12} = 8{\cdot}33 \text{ square feet}$$

The percentage of the area of the plate is therefore—

$$\frac{8{\cdot}33}{64} \times 100 = 13 \text{ per cent.}$$

Adding 3 per cent. for rivet-heads, the percentage to add to the net weight is 16 per cent., or 9·4 tons. The total weight is therefore 68·2 tons.

It is usual to add 3 per cent. to allow for the weight of rivet-heads. For lapped edges and butt straps, both double riveted, the percentage[1] comes to about 10 per cent. for laps, $5\frac{1}{2}$ per cent. for butt straps, and 3 per cent. for liners as ordinarily fitted to the raised strakes of plating. No definite rule can be laid down, because the percentage must vary according to the particular scantlings and method of working the plating, etc., specified.

The length of stiffeners or beams required for a given area can be very approximately determined by dividing the area in square feet by the spacing of the stiffeners or beams in feet. For wood decks, 3 per cent. can be added for fastenings.

Example.—The beams of a deck are 3 feet apart, and weigh 22 lbs. per foot run; the deck plating weighs 10 lbs. per square foot, and this is covered by teak planking 3 inches thick. Calculate the weight of a part 54 feet long by 10 feet wide of this structure, including fastenings.

(*S. and A. Exam.* 1897.)

Net area of deck = 54 × 10 = 540
Add for butts and laps 7 per cent. = 37·8

577·8

(Assume single-riveted butt straps and single-riveted laps.)

Weight of plating = 577·8 × 10
= 5778 lbs.
Running feet of beams = $\frac{540}{3}$ = 180
Weight of beams = 180 × 22
= 3960 lbs.[2]
Total weight of plating and beams = 9,738 lbs.
Add 3 per cent. for rivet-heads = 292 ,,

10,030 ,,

Weight of teak[3] = 540 × $\frac{50}{4}$ = 6750 lbs.
Add 3 per cent. for fastenings = 202 ,,

Weight of wood deck 6952 ,,

Summary.

Plating and beams	10,030 lbs.	
Wood deck	6,952 ,,	
Total ...	16,982 ,,	≐ 7·6 tons.

[1] A number of percentages worked out for various thicknesses, etc., will be found in Mr. Mackrow's "Pocket Book."

[2] No allowance made for beam arms, which should be done if a whole deck is calculated.

[3] Teak taken as 50 lbs. per cubic foot.

Use of Curves.—For determining the weight of some of the portions of a ship, the use of curves is found of very great assistance. Take, for instance, the transverse framing of a ship. For a certain length this framing will be of the same character, as, for example, in a battleship, within the double bottom, where the framing is fitted intercostally between the longitudinals. We take a convenient number of sections, say the sections on the sheer drawing, and calculate the weight of the complete frame at each section. Then along a base of length set up ordinates at the sections, of lengths to represent the calculated weights of the frames at the sections. Through the spots thus obtained draw a curve, which should be a fair line. The positions of the frames being placed on, the weight of each frame can be obtained by a simple measurement, and so the total weight of the framing determined. The curve AA in Fig. 82 gives a curve as constructed in this way for the transverse framing below armour in the double bottom of a battleship. Before and abaft the double bottom, where the character of the framing is different, curves are constructed in a similar manner.

Weight of Outer Bottom Plating.—The first step necessary is to determine the area we have to deal with. We

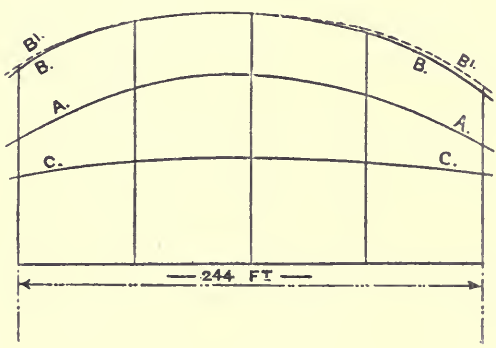

FIG. 82.

can construct a curve of girths, as BB, Fig. 82; but the area given by this curve will not give us the area of the plating, because although the surface is developed in a transverse direction there is no development in a longitudinal direction. (Strictly speaking, the bottom surface of a ship is an undevelopable surface.) The extra area due to the slope of the level lines is allowed for as follows: In plate I., between stations 3 and 4, a line *fg* is drawn representing the *mean slope* of all the level lines. Then the ordinate of the curve of girths midway between 3 and 4 stations is increased in the ratio *fg* : *h*. This done all along the curve will give us a new modified curve of girths, as B′B′, Fig. 82, and the area given by this curve will give a close approximation to the area of the outer bottom of the ship. This is, of course, a net area without allowing for butt, straps or laps. Having a modified curve of girths for the whole length, we can separate it into portions over which the character of the plating is the same. Thus, in a vessel built under Lloyd's rules, the plating is of certain thickness for one-half the length amidships, and the thickness is reduced before and abaft. Also, in a battleship, the thickness of plating is the same for the length of the double bottom, and is reduced forward and aft. The curves AA and BB, Fig. 82, are constructed as described above for a length of 244 feet.

Weight of Hull.—By the use of these various methods, it is possible to go right through a ship and calculate the weight of each portion of the structure. These calculable portions for a battleship are—

(1) Skin-plating and plating behind armour.

(2) Inner bottom plating.

(3) Framing within double bottom, below armour, behind armour, and above armour. Outside double bottom, below and above the protective deck.

(4) Steel and wood decks, platforms, beams.

(5) Bulkheads.

(6) Topsides.

There are, however, a large number of items that cannot be directly calculated, and their weights must be estimated by comparison with the weights of existing ships. Such items are stem and stern posts, shaft brackets, engine and boiler bearers, rudder, pumping and ventilation arrangements, pillars, paint, cement, fittings, etc.

It is, however, a very laborious calculation to determine the weight of the hull of a large ship by these means; and more often the weight is estimated by comparison with the ascertained weight of existing ships. The following is one method of obtaining the weight of steel which would be used in the construction of a vessel : The size of the vessel is denoted by the product of the length, breadth, and depth, and for known ships the weight of steel is found to be a certain proportion of this number, the proportion varying with the type of ship. The coefficients thus obtained are tabulated, and for a new ship the weight of steel can be estimated by using a coefficient which has been obtained for a similar type of ship. The weight of wood and outfit can be estimated in a similar manner.

Another method is described by Mr. J. Johnson, M.I.N.A., in the *Transactions of the Institution of Naval Architects* for 1897, in which the sizes of vessels are represented by *Lloyd's longitudinal number*,[1] modified as follows : In three-decked vessels, the girths and depths are measured to the upper deck

[1] *Lloyd's numbers*—

1. The scantlings and spacing of the frames, reversed frames, and floor-plates, the thickness of bulkheads and the diameter of pillars are regulated by numbers, which are produced as follows :—

2. For one and two decked vessels, the number is the sum of the measurements in feet arising from the addition of the half-moulded breadth of the vessel at the middle of the length, the depth from the upper part of the keel to the top of the upper-deck beams, with the normal round-up, and the girth of the half midship frame section of the vessel, measured from the centre line at the top of the keel to the upper-deck stringer plate.

3. For three-deck steam-vessels, the number is produced by the deduction of 7 feet from the sum of the measurements taken to the top of the upper-deck beams.

4. For spar-decked vessels and awning-decked steam-vessels, the number is the sum of the measurements in feet taken to the top of the main-deck beams, as described for vessels having one or two decks.

5. The scantlings of the keel, stem, stern-frame, keelson, and stringer plates, the thickness of the outside plating and deck ; also the scantlings of the angle bars on beam stringer plates, and keelson and stringer angles in hold, are governed by the *longitudinal number* obtained by multiplying that which regulates the size of the frames, etc., by the length of the vessel.

The measurements for regulating the above scantling numbers are taken as follows :—

1. The *length* is measured from the after part of the stem to the fore part of the stern-post on the range of the upper-deck beams in one, two, and three decked and spar-decked vessels, but on the range of main-deck beams in awning-decked vessels.

In vessels where the stem forms a cutwater, the length is measured from without deducting 7 feet. In spar and awning-deck vessels, the girths are measured to the spar or awning decks respectively. In one, two, and well-decked vessels, the girths and depths are taken in the usual way. Curves are drawn for each type of vessel, ordinates being the weight of iron or steel in tons for vessels built to the highest class at Lloyd's or Veritas, and abscissæ being Lloyd's longitudinal number modified as above. These curves being constructed for ships whose weights are known, it is a simple matter to determine the weight for a new ship of given dimensions. For further information the student is referred to the paper in volume 39 of the *Transactions*.

To calculate the Position of the Centre of Gravity of a Ship.—We have already seen in Chapter III. how to find the C.G. of a completed ship by means of the inclining experiment, and data obtained in this way are found very valuable in estimating the position of the C.G. of a ship that is being designed. It is evident that the C.G. of a ship when completed should be in such a position as to obtain the metacentric height considered necessary, and also to cause the ship to float correctly at her designed trim. Suppose, in a given ship, the C.G. of the naked hull has been obtained from the inclining experiment (that is, the weights on board at the time of the experiment that do not form part of the hull are set down and their positions determined, and then the weight and position of the C.G. of the hull determined by the rules we have dealt with in Chapter III.). The position of the C.G. of hull thus determined is placed on the midship section, and the ratio of the distance of the C.G. above the top of keel to the total depth from the top of keel to the top of the uppermost deck amidships will

the place where the upper-deck beam line would intersect the after edge of stem if it were produced in the same direction as the part below the cutwater.

2. The *breadth* in all cases is the greatest moulded breadth of the vessel.

3. The *depth* in one and two decked vessels is taken from the upper part of the keel to the top of the upper-deck beam at the middle of the length, assuming a normal round-up of beam of a quarter of an inch to a foot of breadth. In spar-decked vessels and awning-decked vessels, the depth is taken from the upper part of the keel to the top of the main-deck beam at the middle of the length, with the above normal round-up of beam.

give us a ratio that can be used in future ships of similar type for determining the position of the C.G. of the hull. Thus, in a certain ship the C.G. of hull was 20·3 feet above keel, the total depth being 34·4 feet. The above ratio in this case is therefore 0·59, and for a new ship of similar type, of depth 39·5 feet, the C.G. of hull would be estimated at 39·5 × 0·59, or 23·3 feet above the keel. For the fore-and-aft position, a similar ratio may be obtained between the distance of the C.G. abaft the middle of length and the length between perpendiculars. Information of this character tabulated for known ships is found of great value in rapidly estimating the position of the C.G. in a new design.

For a vessel of novel type, it is, however, necessary to calculate the position of the C.G., and this is done by combining together all the separate portions that go to form the hull. Each item is dealt with separately, and its C.G. estimated as closely as it is possible, both vertically and in a fore-and-aft direction. These are put down in tabular form, and the total weight and position of the C.G. determined.

In estimating the position of the C.G. of the bottom plating, we proceed as follows: First determine the position of the C.G. of the several curves forming the half-girth at the various stations. This is not generally at the half-girth up, but is somewhere inside or outside the line of the curve. Fig. 83 represents the section AB at a certain station. The curve is divided into four equal parts by dividers, and the C.G. of each of these parts is estimated as shown. The centres of the first two portions are joined, and the centres of the two top portions are joined as shown. The centres of these last-drawn lines, g_1, g_2, are joined, and the centre of the line g_1g_2, viz. G, is the C.G. of the line forming the curve AB, and GP is the distance from the L.W.L. This done for each of the sections will enable us to put a curve, CC in Fig. 82, of distances of C.G. of the half-girths from the L.W.L.[1] We then proceed to find the C.G. of the bottom plating as indicated in the following table. The area is obtained by putting the half-girths (modified as already

[1] This assumes the plating of constant thickness. Plates which are thicker, as at keel, bilge, and sheer, can be allowed for afterwards.

explained) through Simpson's rule. These products are then multiplied in the ordinary way to find the fore-and-aft position of the C.G. of the plating, and also by the distances of the C.G.

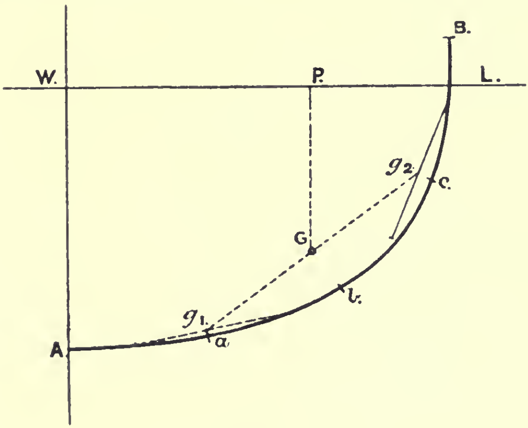

FIG. 83.[1]

of the sections below the L.W.L., which distances are measured off from the curve CC and are placed in column 6. The remainder of the work does not need any further explanation.

CALCULATION FOR AREA AND POSITION OF C.G. OF BOTTOM PLATING FOR A LENGTH OF 244 FEET.

Modified half-girths.	Simpson's multipliers.	Products.	Multiples for leverage.	Products.	C.G. from L.W.L.	Products.
41·5	1	41·5	2	83·0	18·1	751
51·1	4	204·4	1	204·4	21·6	4,415
53·0	2	106·0	0	287·4	22·2	2,354
49·6	4	198·4	1	198·4	21·3	4,226
37·5	1	37·5	2	75·0	19·0	712
		587·8		273·4		12,458
				14·0		

[1] The C.G. of wood sheathing, if fitted, can be obtained from this figure by setting off normally to the curve from G the half-thickness of the sheathing.

$$\text{Common interval} = 61 \text{ feet}$$

$$\text{Area both sides} = 587{\cdot}8 \times \tfrac{1}{3} \times 61 \times 2$$

$$= 23{,}904 \text{ square feet}$$

$$\text{C.G. abaft middle of length of plating} = \frac{14{\cdot}0}{587{\cdot}8} \times 61$$

$$= 1{\cdot}45 \text{ feet}$$

$$\text{C.G. below L.W.L.} = \frac{12{,}458}{587{\cdot}8} = 21{\cdot}2 \text{ feet}$$

CALCULATION FOR THE POSITION OF THE C.G. OF A VESSEL.

Items.	Tons.	From L.W.L.				From Middle of Length.			
		Below.		Above.		Before.		Abaft.	
		Lever.	Moment.	Lever.	Moment.	Lever.	Moment.	Lever.	Moment.
Equipment—									
Water	25	4·0	100	—	—	—	—	12·0	300
Provisions	30	4·5	135	—	—	25·0	750	—	—
Officers' stores ...	15	2·0	30	—	—	—	—	125·0	1,875
Officers, men, and effects	30	—	—	6·0	180	55·0	1650	—	—
Cables	30	4·0	120	—	—	85·0	2550	—	—
Anchors	10	—	—	15·0	150	90·0	900	—	—
Masts, yards, etc. ...	25	—	—	45·0	1125	—	—	7·0	175
Boats	10	—	—	21·0	210	—	—	20·0	200
Warrant officers' stores	20	1·5	30	—	—	65·0	1300	—	—
Armament	175	—	—	4·0	700	—	—	5·0	875
Machinery	450	4·0	1800	—	—	—	—	33·0	14,850
Engineers' stores ...	50	0·5	25	—	—	—	—	70·0	3,500
Coals	300	0·2	60	—	—	3·0	900	—	—
Protective deck	210	—	—	1·5	315	—	—	15·0	3,150
Hull	1250	—	—	1·5	1875	—	—	11·5	14,375
Total	2630 tons		2300		4555		8050		39,300
					2300				8,050
					2630)2255				2630)31,250
					0·857 ft. above L.W.L.				11·88 ft. abaft mid. length.

C.G. above L.W.L. = 0·857
Trans. met. " " = 2·97
Trans. met. above C.G = 2·97 − 0·857
= 2·113 feet.

Calculation for C.G. of a Completed Vessel.—By the use of the foregoing methods we can arrive at an estimate of the weight of hull, and also of the position of its C.G. relative to a horizontal plane, as the L.W.P., and to a vertical athwartship plane, as the midship section. To complete the ship for service, there has to be added the equipment, machinery, etc., and the weights of these are estimated, as also the positions of their centres of gravity. The whole is then combined in a table, and the position of the C.G. of the ship in the completed condition determined.

The preceding is such a table as would be prepared for a small protected cruiser. It should be stated that the table is not intended to represent any special ship, but only the type of calculation.

The total weight is 2630 tons, and the C.G. is 0·857 foot above the L.W.L. and 11·88 feet abaft the middle of length. The sheer drawing enables us to determine the position of the transverse metacentre, and the estimated G.M. is found to be 2·113 feet. The centre of buoyancy calculated from the sheer drawing should also be, if the ship is to trim correctly, at a distance of 11·88 feet abaft the middle of length.

Strength of Butt Fastenings.—Fig. 84 represents two plates connected together by an ordinary treble-riveted butt strap. The spacing of the rivets in the line of holes nearest the butt is such that the joint can be caulked and made watertight, and the alternate rivets are left out of the row of holes farthest from the butt. Such a connection as this could conceivably break in five distinct ways—

1. By the *whole of the rivets* on one side of the butt shearing.

2. By the *plate* breaking through the line of holes, AA, farthest from the butt.

3. By the *butt strap* breaking through the line of holes, BB, nearest the butt.

4. By the *plate* breaking through the middle row of holes, CC, and shearing the rivets in the line AA.

5. By the *strap* breaking through the middle row of holes, CC, and shearing the rivets in the line BB.

It is impossible to make such a connection as this equal to the strength of the unpunched plate, because, although we might

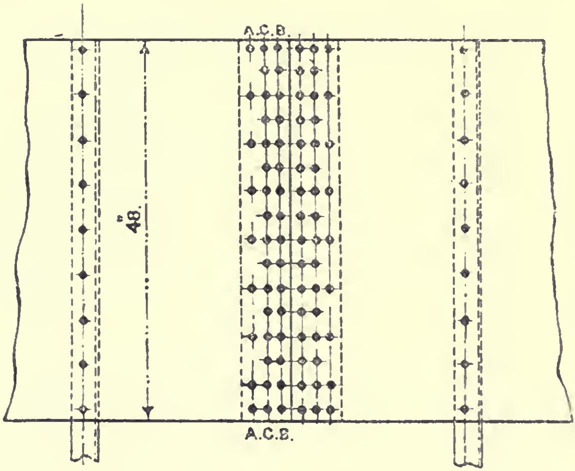

Fig. 84.

put in a larger number of rivets and thicken up the butt strap,

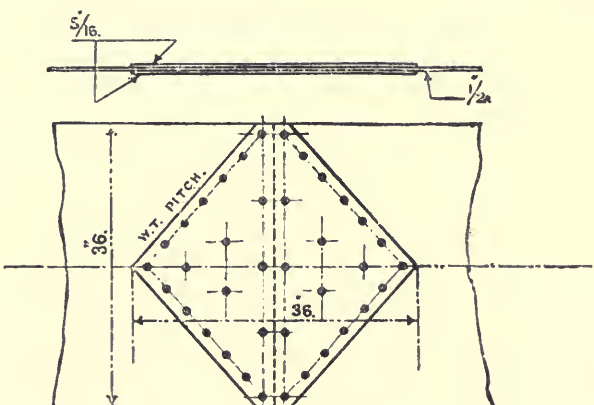

Fig. 85.

there would still remain the line of weakness of the plate through the line of holes, AA, farthest from the butt.

The most efficient form of strap to connect two plates together would be as shown in Fig. 85, of diamond shape. Here the plate is only weakened to the extent of one rivet-hole. Such an efficient connection as this is not required in ship construction, because in all the plating we have to deal with, such as stringers and outer bottom-plating, the plate is necessarily weakened by the holes required for its connection to the beam or frame, and it is unnecessary to make the connection stronger than the plate is at a line of holes for connecting it to the beam or frame. In calculating the strength of a butt connection, therefore, we take as the *standard strength* the strength through the line of holes at a beam or frame, and we so arrange the butt strap that the strength by any of the modes of fracture will at least equal this standard strength.

Experimental Data.—Before we can proceed to calculate the strength of these butt connections, we must have some experimental data as to the tensile strength of plates and the shearing strength of rivets. The results of a series of experiments were given by Mr. J. G. Wildish at the Institution of Naval Architects in 1885, and the following are some of the results given:—

SHEARING STRENGTH OF RIVETS IN TONS.

(*Pan heads and countersunk points.*)

	Single shear.	Double shear.
$\frac{3}{4}$ inch iron rivets in iron plates	10·0	18
$\frac{3}{4}$,, ,, steel ,, ...	8·4	—
$\frac{3}{4}$ inch steel ,, ,, ,, ...	11·5	21·2
$\frac{7}{8}$,, ,, ,, ,, ...	15·25	—
1·0 ,, ,, ,, ,, ...	20·25	—

It will be noticed that the shearing strength of the steel rivets of varying sizes is very nearly proportionate to the sectional area of the rivets. Taking the shearing strength of a $\frac{3}{4}$-inch steel rivet to be 11·5 tons, the strength proportionate to the area would be for a $\frac{7}{8}$-inch steel rivet, 15·6 tons, and for a 1-inch steel rivet 20·4 tons. Also, we see that the double shear of a rivet is about 1·8 times the single shear.

The following results were given as the results of tests of mild steel plates :—

Unpunched	$28\frac{1}{4}$ tons per square inch.
Holes *punched*	22 ,, ,, or a depreciation of 22 per cent.
Holes *drilled*	$29\frac{1}{2}$ tons per square inch.
Holes *punched* small, and the hole then *countersunk*	29 ,, ,,

The following give the strength of the material of the plates after being connected together by a butt strap :—

Holes *punched* the full size, the rivets having snap points	24·9 tons per square inch.
Holes *punched* small and then *countersunk*, the rivets being pan-head, with countersunk points	28·9 ,, ,,

It appears, from the above results, that if a plate has the holes drilled or has them punched and countersunk in the ordinary way as for flush riveting, the strength of the material is fully maintained. Also that, although punching holes in a plate reduces the strength from $28\frac{1}{4}$ to 22 tons per square inch, a reduction of 22 per cent., yet when connected by a butt strap, and riveted up, the strength rises to 24·9 tons per square inch, which is only 12 per cent. weaker than the unpunched plate, the process of riveting strengthening the plate.

In an ordinary butt-strap, with the holes spaced closely together in order to obtain a water-tight pitch for the rivets, it is found that the punching distresses the material in the neighbourhood of the holes, and the strength is materially reduced, as we have seen above. If, however, the butt strap is annealed after punching, the full strength of the material is restored. It is the practice, in ships built for the British Admiralty, for all butt straps of important structural plating to have the holes drilled or to be annealed after punching.[1] In either case the

[1] In ships built for the *British Admiralty*, for plating which forms an important feature in the general structural strength, such as the inner and outer bottom plating, deck plating, deck stringers, etc., the butt straps must have the holes drilled, or be annealed after the holes are punched.

strength is restored. For ships built to the rules of Lloyd's Register, butt straps above $\frac{10}{20}$ of an inch in thickness are annealed or the holes rimed after punching.[1]

In our calculations of the strength of butt straps, we therefore can assume that the strength of the material between the rivet-holes is the same as the strength of the material of the unpunched plate.

Again, the plating, in the cases we have to deal with, has the riveting flush on the outside, and the holes are made with a countersink for this purpose. Here also we can assume that the strength of the material is the same as the strength of the material of the unpunched plate.

The specified tests for the tensile strength of steel plates are as follows :—

For ships built for the British Admiralty, not less than 26 and not more than 30 tons per square inch of section.

For ships built to the rules of Lloyd's Register, not less than 28 and not more than 32 tons per square inch of section.

The plates tested above showed a tensile strength of $28\frac{1}{4}$ tons per square inch, or nearly midway between the limits laid down by the British Admiralty. It seems reasonable, therefore, in calculating the ultimate strength of riveted joints, to take as the strength of the material the *minimum* strength to which it has to be tested. Thus, in a ship built for the British Admiralty, we can use 26 tons as the strength per square inch of section, and in a ship built under Lloyd's rules, we can use 28 tons per square inch of section.

The following two examples will illustrate the methods adopted in calculating the strength of butt fastenings : [2]—

In such bottom plating, the countersunk holes must be punched about $\frac{1}{8}$ inch less in diameter than the rivets which are used, the enlargement of the holes being made in the countersinking, which must in all cases be carried through the whole thickness of the plates.

[1] In ships built to the rules of *Lloyd's Register*, stringer plates, sheerstrakes, garboard strakes, and all butt straps, when above $\frac{10}{20}$ of an inch in thickness, are carefully annealed, or the holes sufficiently rimed after punching, to remove the injurious effect of the punching.

[2] Admiralty tests, etc., adopted.

1. A steel stringer plate is 48 inches broad and $\frac{7}{16}$ inch thick. Sketch the fastenings in a beam and at a butt, and show by calculations that the butt connection is a good one.

(*S. and A. Exam.*, 1897.)

For a $\frac{7}{16}$-inch plate we shall require $\frac{3}{4}$-inch rivets, and setting these out at the beam, we require 9 rivets, as shown in Fig. 84. The effective breadth of the plate through this line of holes is therefore—

$$48 - 9(\tfrac{3}{4}) = 41\tfrac{1}{4} \text{ inches}$$

and the strength is—

$$41\tfrac{1}{4} \times \tfrac{7}{16} \times 26 = 470 \text{ tons}$$

and this is the standard strength that we have to aim at in designing the butt strap.

(1) As regards the number of rivets. The shearing strength of a $\frac{3}{4}$-inch rivet being 11·5 tons, the number of rivets necessary to equal the standard strength of 470 tons is—

$$\frac{470}{11 \cdot 5} = 40 \cdot 8, \text{ say 41 rivets}$$

If we set out the rivets in the strap as shown in Fig. 84, leaving the alternate rivets out in the line AA, it will be found that exactly 41 rivets is obtained, with a four-diameter pitch. So that, as regards the number of rivets the butt connection is a good one.

(2) The strength of the plate in the line AA is the same as at the beam, the same number of rivet-holes being punched in each case.

(3) If the strap is $\frac{7}{16}$ inch thick, the strength of the strap in the line BB is given by—

$$\{48 - 16(\tfrac{3}{4})\} \times \tfrac{7}{16} \times 26 = 410 \text{ tons}$$

This is not sufficient, and the strap must be thickened up. If made $\frac{1}{2}$ inch thick, the strength is—

$$\{48 - 16(\tfrac{3}{4})\} \times \tfrac{1}{2} \times 26 = 468$$

which is very nearly equal to the standard strength of 470 tons.

(4) The shear of the 9 rivets in the line AA is 103·5 tons, so that the strength of the plate through the line of holes CC and the shear of the rivets in the line AA are—

$$410 + 103 \cdot 5 = 513 \cdot 5 \text{ tons}$$

(5) Similarly, the strength of the strap through the line CC and the shear of the rivets in the line AA are—

$$468 + 184 = 652 \text{ tons}$$

The ultimate strengths of the butt connection in the five different ways it might break are therefore 471$\frac{1}{2}$, 470, 468, 513$\frac{1}{2}$, 652 tons respectively, and thus the standard strength of 470 tons is maintained for all practical purposes, and consequently the butt connection is a good one.

2. If it were required to so join two plates as to make the strength at the butt as nearly as possible equal to that of the unpierced plates, what kind of butt strap would you adopt?

Supposing the plates to be of mild steel 36 inches wide and $\frac{1}{2}$ inch thick, give the diameter, disposition, and pitch of rivets necessary in the strap.

(*S. and A. Exam.*, 1895.)

The first part of this question has been already dealt with on p. **201**. To lessen the number of rivets, it is best to use a double butt strap, as Fig. 85, so as to get a double shear of the rivets. Each of the butt straps should be slightly thicker than the half-thickness of the plate, say $\frac{5}{16}$ inch.

The standard strength to work up to is that of the plate through the single rivet-hole at the corner of the strap. $\frac{7}{8}$-inch rivets being used, the standard strength is—

$$(36 - \tfrac{7}{8}) \times \tfrac{1}{2} \times 26 = 457 \text{ tons}$$

The single shear of a $\frac{7}{8}$-inch rivet is $15\frac{1}{4}$ tons, and the double shear may be taken as—

$$15{\cdot}25 \times 1{\cdot}8 = 27\tfrac{1}{2} \text{ tons}$$

and consequently the least number of rivets required each side of the butt is—

$$\frac{457}{27{\cdot}5} = 16{\cdot}6, \text{ say } 17 \text{ rivets}$$

The strength of the plate along the slanting row of holes furthest from the butt must be looked into. The rivets here are made with a water-tight pitch, say from 4 to $4\frac{1}{2}$ diameters. If we set out the holes for a strap 2 feet wide, it will be found that the strength is below the standard. A strap 3 feet wide will, however, give a strength through this line of about 465 tons, which is very near the required 457 tons. There are 13 rivets along the edge of the strap, and the inside may be filled in as shown, giving a total number of rivets, each side of the butt, of 19.

Strains experienced by Ships.—The strains to which ships are subjected may be divided into two classes, viz.—

1. *Structural strains*, *i.e.* strains which affect the structure of the ship considered as a whole.

2. *Local strains*, *i.e.* strains which affect particular portions of the ship.

1. *Structural Strains.* — These may be classified as follows:—

(*a*) Strains tending to cause the ship to bend in a fore-and-aft direction.

(*b*) Strains tending to change the transverse form of the ship.

(*c*) Strains due to the propulsion of the vessel, either by steam or sails.

2. *Local Strains.*—These may be classified as follows:—

(*a*) Panting strains.

(*b*) Strains due to heavy local weights, as masts, engines, armour, guns, etc.

(*c*) Strains caused by the thrust of the propellers.

(*d*) Strains caused by the attachment of rigging.

(*e*) Strains due to grounding.

We will now deal with some of these various strains to which a ship may be subjected in a little more detail.

Longitudinal Bending Strains.—A ship may be regarded as a large beam or girder, subject to bending in a fore-and-aft direction. The support of the buoyancy and the distribution of weight vary considerably along the length of a ship, even when floating in still water. Take a ship and imagine she is cut by a number of transverse sections, as in Fig. 86. Each

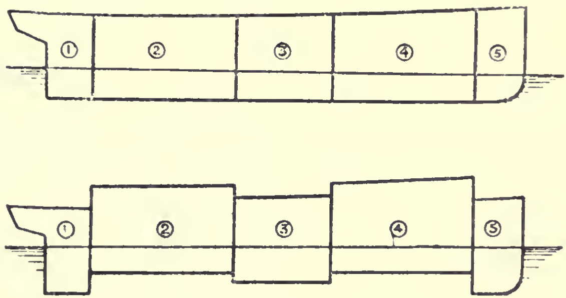

FIG. 86.

of the portions has its weight, and each has an upward support of buoyancy. But in some of the portions the weight exceeds the buoyancy, and in others the buoyancy exceeds the weight. The total buoyancy of all the sections must, of course, equal the total weight. Now imagine that there is a water-tight bulkhead at each end of each of these portions, and the ship is actually cut at these sections. Then the end portions (1) and (5) have considerable weight but small displacement, and consequently they would sink deeper in the water if left to themselves.[1] In the portions (2) and (4), on the other hand, the buoyancy might exceed the weight (suppose these are the fore-and-aft holds, and the ship is light), and if left to themselves they would rise. The

[1] Strictly speaking, each portion would change trim if left to itself, but we suppose that the various portions are attached, but free to move in a vertical direction.

midship portion (3) has a large amount of buoyancy, but also a large weight of engines and boilers, and this portion might very well have to sink a small amount if left to itself. In any actual ship, of course, it is a matter of calculation to find how the weight and buoyancy vary throughout the length. This case is somewhat analogous to the case of a beam supported and loaded as shown in Fig. 87. At each point along the

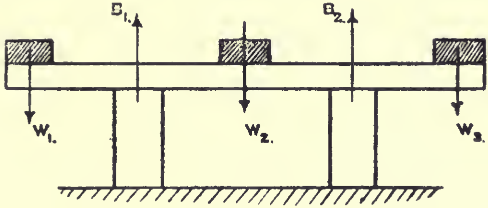

FIG. 87.

beam there is a tendency to bend, caused by the way the beam is loaded and supported, and the beam must be made sufficiently strong to withstand this bending tendency. In the same way, the ship must be constructed in such a manner as to effectually resist the bending strains that are brought to bear upon the structure.

When a vessel passes out of still water and encounters waves at sea, the strains to which she is subjected must differ very much from those we have been considering above. Suppose the ship to be end on to a series of waves having lengths

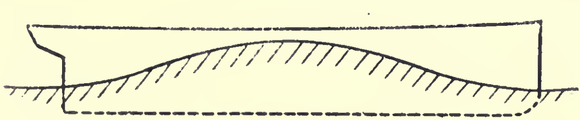

FIG. 88.

from crest to crest or from trough to trough equal to the length of the ship. We will take the two extremes.

(1) The ship is supposed to have the *crest* of the wave amidships.

(2) The ship is supposed to have the *trough* of the wave amidships.

(1) This is indicated in Fig. 88. At this instant there is an excess of weight at the ends, and an excess of buoyancy amidships. The ship may be roughly compared to a beam supported at the middle, with weights at the end, as in Fig. 89. The con-

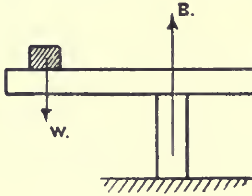

FIG. 89.

sequence is that there is a tendency for the ends to droop relatively to the middle. This is termed *hogging*.

(2) This is indicated in Fig. 90. At this instant there is an

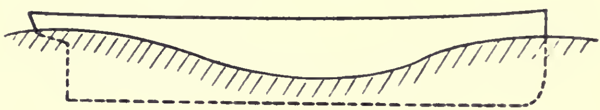

FIG. 90.

excess of weight amidships, and an excess of buoyancy at the ends, and the ship may be roughly compared to a beam supported at the ends and loaded in the middle, as Fig. 91. The

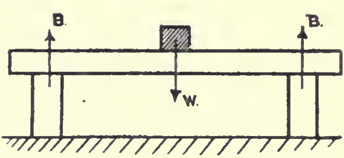

FIG. 91.

consequence is, there is a tendency for the middle to droop relatively to the ends. This is termed *sagging*.

We have seen above how the ship may be compared to a beam, and in order to understand how the material should be disposed in order best to withstand the bending strains, we will consider briefly some points in connection with ordinary beams.[1]

Take a beam supported at the ends and loaded at the middle. It will bend as shown exaggerated in Fig. 92. The

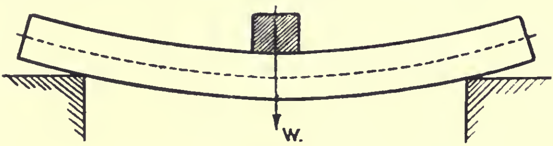

FIG. 92.

resistance the beam will offer to bending will depend on the form of the section of the beam. Take a beam having a sectional area of 16 square inches. We can dispose the material in many different ways. Take the following :—

(*a*) 8 inches wide, 2 inches deep (*a*, Fig. 93).

(*b*) 4 inches wide, 4 inches deep (*b*, Fig. 93).

(*c*) 2 inches wide, 8 inches deep (*c*, Fig. 93).

(*d*) 8 inches deep, with top and bottom flanges 5 inches wide and 1 inch thick (*d*, Fig. 93).

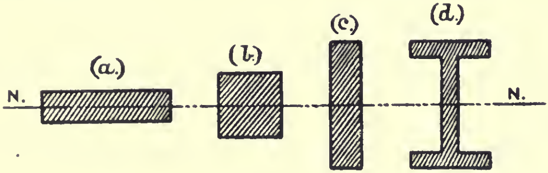

FIG. 93.

Then the resistances of these various sections to bending compare as follows :—

If (*a*) is taken as 1, then (*b*) is 2, (*c*) is 4, and (*d*) is $6\frac{5}{8}$.

We thus see that we can make the beam stronger to resist bending by disposing the material far away from the centre. The beam (*d*) is $6\frac{5}{8}$ times the strength of (*a*) against bending, although it has precisely the same sectional area. A line

[1] The subject of beams will be found fully discussed in works on Applied Mechanics.

drawn transversely through the centre of gravity of the section of a beam is termed the neutral axis.

These principles apply equally to the case of a ship, and we thus see that to resist bending strains the material of the structure should be disposed far away from the neutral axis.[1] In large vessels, and those of large proportion of length to breadth or length to depth, Lloyd's rules require that partial or complete steel decks shall be fitted on the upper decks, the upper-deck stringer made wider and thickened up, the sheer strake doubled or made thicker, the plating at the bilge thickened up or doubled, and the keelsons increased in strength. These are all portions of the structure farthest away from the neutral axis.

For hogging strains, the upper portions of the vessel are in tension and the lower portions are in compression. For sagging strains, the upper portions are in compression and the lower portions are in tension. Thus the portions of the structure that are useful in resisting these hogging and sagging strains are the upper and main decks and stringers, sheer-strake and plating below, plating at and below the bilge, both of the inner and outer bottom, keel, keelsons, and longitudinal framing.

Strains tending to change the Transverse Form of the Ship.—Strains of this character are set up in a ship rolling heavily. Take a square framework jointed at the corners, and imagine it to be rapidly moved backwards and forwards as a ship does when she rolls. The framework will not break, but will distort, as shown in Fig. 94. There is a tendency to distort in a similar way in a ship rolling heavily, and the connections of the beams to the sides, and the transverse structure of the ship, must be made sufficiently strong to prevent any of this racking taking place. Transverse bulkheads are valuable in resisting the tendency to change the transverse form. In ships built to Lloyd's Register, the ordinary depth of beam arms is $2\frac{1}{2}$ times the depth of the beam; but in sailing-ships, which only have one transverse bulkhead, the collision bulkhead, when the length of

[1] There are other strains, viz. shearing strains, which are of importance (see "Applied Mechanics," by Professor Cotterill, and a paper read at the Institution of Naval Architects in 1890, by the late Professor Jenkins).

the midship upper-deck beam exceeds 36 feet, the bracket knees to each tier of beams must not be less than three times the depth of the beam, and the depth at the throat not less than one and three-quarters the depth of the beam.[1]

A ship, when docked, especially if she has on board heavy weights, as armour or coals, is subjected to severe strains tending to change the transverse form. If the ship is supported wholly at the keel, no shores being supposed placed in position, the weight either side the middle line tends to make the sides drop, and bring the beams into tension. A ship when docked,

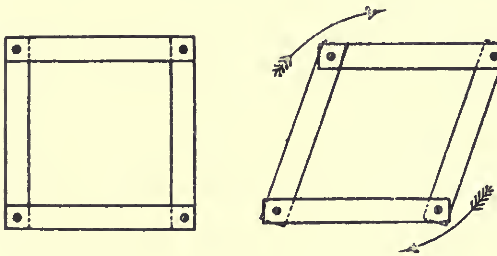

FIG. 94.

however, is partially supported by shores as well as at the keel as the water leaves, so that this case is an extreme one.

Panting.—This term is used to describe the working in and out of the plating, and it is usually found at the fore and after ends of the ship, where the surface is comparatively flat. The forward end especially is subject to severe blows from the sea, and special attention is paid to this part by working special beams and stringers to succour the plating. In vessels built to the rules of Lloyd's Register, the following rules have to be carried out to provide sufficient local strength against panting:—

All stringers, where practicable, to extend fore and aft, and to be efficiently connected at their ends with plates forming hooks and crutches of the same thickness as the floor-plates amidships, and those below the hold beams should be spaced about 4 feet apart. In vessels whose plating number[2] is 24,000,

[1] There is also in sailing ships a couple due to the sails, tending to distort the sections.

[2] See p. 194.

or above, an additional hook or crutch should be fitted at the ends of the vessel, between each tier of beams, to the satisfaction of the Surveyors.

The depth for regulating the number of tiers of beams to alternate frames in the fore peak to be taken at the collision bulkhead. All vessels to have, in addition, provision made to prevent panting by extra beams, bracket knees and stringer plates being fitted before and abaft the collision bulkheads. Panting beams and stringers to be fitted at the after end where considered necessary by the Surveyors.

The stringer plates on the panting beams to be attached to the outside plating when fitted in continuation of intercostal stringers. These plates are to extend abaft the collision bulkhead for a length of not less than one-fourth the midship breadth of the vessel, and be efficiently supported by brackets at alternate frames. Panting beams and stringers to be fitted at the after end where considered necessary by the Surveyors.

The other local strains mentioned on pp. 205, 206 have to be provided for by special local strengthening.

For a full discussion of the whole subject of the strains experienced by ships, and the stresses on the material composing the structure, the student is referred to the "Manual of Naval Architecture," by Sir W. H. White.

Examples to Chapter VI.

1. The area of the outer bottom plating of a ship, over which the plating is worked 25 lbs. per square foot, is 23,904 square feet, lapped edges and butt straps, both double-riveted. Estimate the difference in weight due to working the plating with average-sized plates $20' \times 4\frac{1}{2}'$, or with the average size $12' \times 3'$.

Ans. About 20 tons.

2. Steel angle bars $3\frac{1}{2}'' \times 3''$ are specified to be $8\frac{1}{2}$ lbs. per lineal foot instead of $\frac{7}{16}$ inch thick. Determine the saving of weight per 100 lineal feet.

Ans. 52 lbs.

3. Determine the weight per lineal foot of a steel T-bar $5'' \times 4'' \times \frac{1}{2}''$.

Ans. 14·45 lbs.

4. For a given purpose, angle bars of iron $5'' \times 3'' \times \frac{8}{16}''$ or of steel $5'' \times 3'' \times \frac{9}{20}''$ can be used. Find the saving of weight per 100 feet if steel is adopted.

Ans. 95 lbs.

5. A mast 96 feet in length, if made of iron, is at its greatest diameter,

viz. 32 inches, $\frac{9}{16}$ inch thick, and has three angle stiffeners $5'' \times 3'' \times \frac{8}{16}''$. For the same diameter, if made of steel, the thickness is $\frac{10}{20}$ inch, with three angle stiffeners $5'' \times 3'' \times \frac{9}{20}''$. Estimate the difference in weight.

Ans. About 1 ton.

6. At a given section of a ship the following is the form : The lengths of ordinates 3 feet apart are 19·6, 18·85, 17·8, 16·4, 14·5, 11·8, 7·35, and 1·0 feet respectively. Estimate the vertical position of the centre of gravity of the curve forming the section, supposing it is required to find the vertical position of the centre of gravity of the bottom plating of uniform thickness.

Ans. About $12\frac{1}{2}$ feet from the top.

7. The half-girths of the inner bottom of a vessel at intervals of 51 feet are 26·6, 29·8, 32·0, 32·8, and 31·2 feet respectively, and the centres of gravity of these half-girths are 18·6, 20·6, 21·2, 20·0, 17·4 feet respectively below the L.W.L. Determine the area of the inner bottom and the position of its centre of gravity both longitudinally and vertically. If the plating is of 15 lbs. to the square foot, what would be the weight, allowing $14\frac{1}{2}$ per cent. for butts, laps, and rivet-heads.

Ans. 12,655 square feet ; 105 feet from finer end, 20 feet below the L.W.L. ; 97 tons.

8. The whole ordinates of the boundary of a ship's deck are 6·5, 24, 29, 32, 33·5, 33·5, 33·5, 32, 30, 27, and 6·5 feet respectively, and the common interval between them is 21 feet.

The deck, with the exception of 350 square feet, is covered with $\frac{3}{8}$ inch steel plating worked flush jointed, with single riveted edges and butts. Find the weight of the plating, including straps and fastenings.

Ans. 45 tons.

9. A teak deck, $2\frac{1}{2}$ inches thick, is supported on beams spaced 4 feet apart, and weighing 15 pounds per foot run. Calculate the weight of a middle-line portion of this deck (including fastenings and beams) 24 feet long and 10 feet wide.

Ans. 1·55 tons.

CHAPTER VII.

HORSE-POWER, EFFECTIVE AND INDICATED—RESISTANCE OF SHIPS—COEFFICIENTS OF SPEED—LAW OF CORRESPONDING SPEEDS.

Horse-power.—We have in Chapter V. defined the "work" done by a force as being the product of the force and the distance through which the force acts. Into the conception of work the question of time does not enter at all, whereas "power" involves not only work, but also the time in which the work is done. The unit of power is a "*horse-power*," which is taken as "33,000 *foot-lbs. of work performed in* 1 *minute*," or "550 *foot-lbs. of work performed in* 1 *second*." Thus, if during 1 minute a force of 1 lb. acts through 33,000 feet, the same power will be exerted as if a force of 33 lbs. acts through 1000 feet during 1 minute, or if 50 lbs. acts through 11 feet during 1 second. Each of these will be equivalent to 1 horse-power. The power of a locomotive is a familiar instance. In this case the work performed by the locomotive —if the train is moving at a uniform speed—is employed in overcoming the various resistances, such as the friction of the wheels on the track, the resistance of the air, etc. If we know the amount of this resistance, and also the speed of the train, we can determine the horse-power exerted by the locomotive. The following example will illustrate this point :—

If the mass of a train is 150 tons, and the resistance to its motion arising from the air, friction, etc., amount to 16 lbs. weight per ton when the train is going at the rate of 60 miles per hour on a level plain, find the horse-power of the engine which can just keep it going at that rate.

$$\begin{aligned} \text{Resistance to onward motion} &= 150 \times 16 \\ &= 2400 \text{ lbs.} \\ \text{Speed in feet per minute} &= 5280 \\ \text{Work done per minute} &= 2400 \times 5280 \text{ foot-lbs.} \\ \text{Horse-power} &= \frac{2400 \times 5280}{33000} \\ &= 384 \end{aligned}$$

In any general case, if—

R = resistance to motion in pounds;

v = velocity in feet per minute;

V = velocity in knots (a velocity of 1 knot is 6080 feet per hour);

then—

$$\text{Horse-power} = \frac{R \times v}{33000}$$
$$= \frac{R \times V \times 101}{33000} \text{ nearly}$$

The case of the propulsion of a vessel by her own engines is much more complicated than the question considered above of a train being drawn along a level plain by a locomotive. We must first take the case of a vessel being towed through the water by another vessel. Here we have the resistances offered by the water to the towed vessel overcome by the strain in the tow-rope. In some experiments on H.M.S. *Greyhound* by the late Mr. Froude, which will be described later, the tow-rope strain was actually measured, the speed being recorded at the same time. Knowing these, the horse-power necessary to overcome the resistance can be at once determined. For example—

At a speed of 1017 feet per minute, the tow-rope strain was 10,770 lbs. Find the horse-power necessary to overcome the resistance.

$$\text{Work done per minute} = 10{,}770 \times 1017 \text{ foot-lbs.}$$
$$\text{Horse-power} = \frac{10770 \times 1017}{33000}$$
$$= 332$$

Effective Horse-power.—The effective horse-power of a vessel at a given speed is the horse-power required to overcome the various resistances to the vessel's progress at that speed. It may be described as the horse-power usefully employed, and is sometimes termed the "tow-rope" or "tug" horse-power, because this is the power that would have to be transmitted through the tow-rope if the vessel were towed through the water at the given speed. Effective horse-power is often written E.H.P. We shall see later that the E.H.P. is entirely different to the Indicated Horse-power (written I.H.P.), which is the horse-power actually measured at the vessel's engines.

Example.—Find the horse-power which must be transmitted through a tow-rope in order to tow a vessel at the rate of 16 knots, the resistance to the ship's motion at that speed being equal to a weight of 50 tons.

Ans. 5503 H.P.

Experiments with H.M.S. "Greyhound," by the late Mr. William Froude, F.R.S.—These experiments took place at Portsmouth as long ago as 1871, and they settled a number of points in connection with the resistance and propulsion of ships, about which, up to that time, little was known. The thoroughness with which the experiments were carried out, and the complete analysis of the results that was given, make them very valuable; and students of the subject would do well to consult the original paper in the *Transactions of the Institution of Naval Architects for* 1874. A summary of the experiments, including a comparison with Rankine's "Augmented Surface Theory of Resistance," will be found in vol. iii. of *Naval Science*. Mr. Froude's report to the Admiralty was published in *Engineering*, May 1, 1874.

The *Greyhound* was a ship 172′ 6″ in length between perpendiculars, and 33′ 2″ extreme breadth, the deepest draught during the experiments being 13′ 9″ mean. The displacement

Fig. 95.

corresponding to this mean draught being 1161 tons; area of midship section, 339 square feet; area of immersed surface, 7540 square feet. The *Greyhound* was towed by H.M.S. *Active*. It was essential to the accuracy of the experiments that the *Greyhound* should proceed through undisturbed water, and to avoid using an exceedingly long tow-rope a boom was rigged out from the side of the *Active* to take the tow-rope (see Fig. 95). By this means the *Greyhound* proceeded through water that had not been influenced by the wake of the *Active.* The length of the boom on the *Active* was 45 feet, and the length of the tow-rope was such that the *Greyhound's* bow was 190 feet clear of the *Active's* stern. The actual stress on the tow-rope at its extremity was not required, but the "horizontal component." This would be the stress that was overcoming the resistance, the "vertical component" being due to the weight of the tow-rope. The horizontal stress on the tow-rope and the speed were automatically recorded on a sheet of paper carried on a revolving cylinder. For details of the methods employed and the apparatus used, the student is referred to

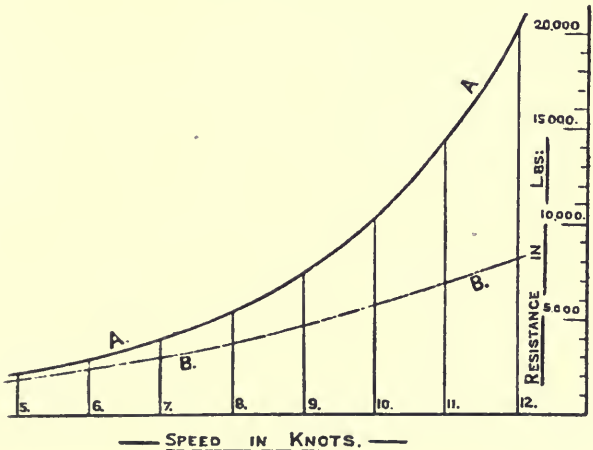

Fig. 96.

the sources mentioned above. The horizontal stress on the tow-rope was equal to the nett resistance of the *Greyhound.* The results can be represented graphically by a curve, abscissæ representing speed, and ordinates representing the resistance in pounds. Such a curve is given by A in Fig. 96.

It will be seen that the resistance increases much more rapidly at the higher than at the lower speeds; thus, on increasing the speed from 7 to 8 knots, an extra resistance of 1500 lbs. has to be overcome, while to increase the speed from 11 to 12 knots, an extra resistance of 6000 lbs. must be overcome. Beyond 12 knots the shape of the curve indicates that the resistance increases very rapidly indeed. Now, the *rate* at which the resistance increases as the speed increases is a very important matter. (We are only concerned now with the total resistance.) Up to 8 knots it was found that the resistance was proportional to the square of the speed; that is to say, if R_1, R_2 represent the resistances at speeds V_1, V_2 respectively, then, if the resistance is proportional to the square of the speed—

$$R_1 : R_2 :: V_1^2 : V_2^2$$
$$\text{or } \frac{R_1}{R_2} = \frac{V_1^2}{V_2^2}$$

By measuring ordinates of the curve in Fig. 96, say at 5 and 6 knots, this will be found to be very nearly the case. As the speed increases above 8 knots, the resistance increases much more rapidly than would be given by the above; and between 11 and 12 knots, the resistance is very nearly proportional to the fourth power of the speed.

The experiments were also conducted at two displacements less than 1161 tons, viz. at 1050 tons and 938 tons. It was found that differences in resistance, due to differences of immersion, depended, not on changes of area of midship section or on changes of displacement, but rather on changes in the area of wetted surface. Thus for a reduction of $19\frac{1}{4}$ per cent. in the displacement, corresponding to a reduction of area of midship section of $16\frac{1}{4}$ per cent., and area of immersed surface of 8 per cent., the reduction in resistance was about $10\frac{1}{2}$ per cent., this being for speeds between 8 and 12 knots.

Ratio between Effective Horse-power and Indicated Horse-power.—We have already seen that, the resistance of the *Greyhound* at certain speeds being determined, it is possible to determine at once the E.H.P. at those speeds. Now, the horse-power actually developed by the *Greyhound's* own engines, or the "indicated horse-power" (I.H.P.), when proceeding on the measured mile, was observed on a separate series of trials, and tabulated. The ratio of the

E.H.P. to the I.H.P. was then calculated for different speeds, and it was found that E.H.P. ÷ I.H.P. in the best case was only 0·42; that is to say, as much as 58 per cent. of the power was employed in doing work other than overcoming the actual resistance of the ship. This was a very important result, and led Mr. Froude to make further investigations in order to determine the cause of this waste of power, and to see whether it was possible to lessen it.

The ratio $\frac{\text{E.H.P.}}{\text{I.H.P.}}$ at any given speed is termed the "*propulsive coefficient*" at that speed. As we saw above, in the most efficient case, in the trials of the "*Greyhound*," this coefficient was 42 per cent. For modern vessels with fine lines a propulsive coefficient of 50 per cent. may be expected, if the engines are working efficiently and the propeller is suitable. In special cases, with extremely fine forms and fast-running engines, the coefficient rises higher than this. These values only hold good for the maximum speed for which the vessel is designed; for lower speeds the coefficient becomes smaller. The following table gives some results as given by Mr. Froude. The *Mutine* was a sister-ship to the *Greyhound*, and she had also been run upon the measured mile at the same draught and trim as the *Greyhound*.

Ship.	Speed on measured mile in feet per minute.	Resistance due to speed deduced from the towing experiments with *Greyhound*, including an estimate of air-resistance of masts and rigging.	Effective horse-power = resistance $\times \frac{\text{velocity}}{33{,}000}$	Actual indicated horse-power on trial.	Effective horse-power ÷ indicated horse-power.
Greyhound ...	1017	10,770	332·1	786	0·422
	845	6,200	158·7	453	0·350
Mutine	977	9,440	279·5	770	0·363
	757	4,770	109·4	328	0·334

Resistance.—We now have to inquire into the various resistances which go to make up the total resistance which a ship experiences in being towed through the water. These resistances are of three kinds—

1. Resistance due to friction of the water upon the surface of the ship.

2. Resistance due to the formation of eddies.

3. Resistance due to the formation of waves.

1. "*Frictional resistance*," or the resistance due to the friction of the water upon the surface of the ship. This is similar to the resistance offered to the motion of a train on a level line owing to the friction of the rails, although it follows different laws. It is evident that this resistance must depend largely upon the state of the bottom. A vessel, on becoming covered with barnacles, etc., while lying in a port, loses speed very considerably, owing to the greatly increased resistance caused. This frictional resistance forms a large proportion of the total at low speeds, and forms a good proportion at higher speeds.

2. *Resistance due to eddy-making.*—Take a block of wood, and imagine it placed a good distance below the surface of a current of water moving at a uniform speed V. Then the particles of water will run as approximately indicated in Fig. 97. At A we shall have a mass of water in a state of

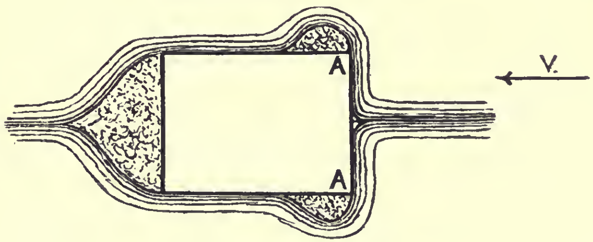

Fig. 97.

violent agitation, and a much larger mass of water at the rear of the block. Such masses of confused water are termed "*eddies*," and sometimes "*dead water*." If now we imagine that the water is at rest, and the block of wood is being towed through the water at a uniform speed V, the same eddies will be produced, and the eddying water causes a very considerable resistance to the onward motion. Abrupt terminations which are likely to cause such eddies should always be avoided in vessels where practicable, in order to keep the resistance as low as possible. This kind of resistance forms a very small proportion of the total in well-formed vessels, but in the older vessels with full forms aft and thick stern-posts, it amounted to a very considerable item.

3. *Resistance due to the formation of waves.*—For low speeds this form of resistance is not experienced to any sensible extent, but for every ship there is a certain speed above which the resistance increases more rapidly than would be the case if surface friction and eddy-making alone caused the resistance. This extra resistance is caused by the formation of waves upon the surface of the water.

We must now deal with these three forms of resistance in detail, and indicate as far as possible the laws which govern them.

1. *Frictional Resistance.*—The data we have to work upon when considering this form of resistance were obtained by the late Mr. Froude. He conducted an extensive series of experiments on boards of different lengths and various conditions of surface towed through water contained in a tank, the speed and resistance being simultaneously recorded. The following table represents the resistances in pounds per square foot due to various lengths of surface of various qualities when moving at a uniform speed of 600 feet per minute, or very nearly 6 knots. There is also given the powers of the speed to which the resistances are approximately proportional.

We can sum up the results of these experiments as follows: The resistance due to the friction of the water upon the surface depends upon—

(1) The area of the surface.

(2) The nature of the surface.

(3) The length of the surface.

and (4) The resistance varies approximately as the square of the speed.

Nature of surface.	Length of Surface in Feet.							
	2		8		20		50	
	Power of speed to which resistance is proportional.	Resistance in pounds per square foot.	Power of speed to which resistance is proportional.	Resistance in pounds per square foot.	Power of speed to which resistance is proportional.	Resistance in pounds per square foot.	Power of speed to which resistance is proportional.	Resistance in pounds per square foot.
Varnish	2·00	0·41	1·85	0·325	1·85	0·278	1·83	0·250
Tinfoil	2·16	0·30	1·99	0·278	1·90	0·262	1·83	0·246
Calico	1·93	0·87	1·92	0·626	1·89	0·531	1·87	0·474
Fine sand ...	2·00	0·81	2·00	0·583	2·00	0·480	2·06	0·405
Medium sand ...	2·00	0·90	2·00	0·625	2·00	0·534	2·00	0·488

And thus we can write—

$$R = f . S\left(\frac{V}{6}\right)^2$$

where R = resistance in pounds;

S = area of surface in square feet;

V = speed in knots relative to still water;

f = a coefficient depending upon the nature and length of the surface.

This coefficient f will be the resistance per square foot given in the above table, as is at once seen by making $S = 1$ square foot and $V = 6$ knots. It is very noticeable how the resistance per square foot decreases as the length increases. Mr. Froude explained this by pointing out that the leading portion of the plane must communicate an onward motion to the water which rubs against it, and "consequently the portion of the surface which succeeds the first will be rubbing, not against stationary water, but against water partially moving in its own direction, and cannot therefore experience as much resistance from it."

Experiments were not made on boards over 50 feet in length. Mr. Froude remarked, in his report, "It is highly desirable to extend these experiments, and the law they elucidate, to greater lengths of surface than 50 feet; but this is the greatest length which the experiment-tank and its apparatus admit, and I shall endeavour to organize some arrangement by which greater lengths may be successfully tried in open water."

Mr. Froude was never able to complete these experiments as he anticipated. It has long been felt that experiments with longer boards would be very valuable, so that the results could be applied to the case of actual ships. It is probable that in the new American experimental tank now under construction, which is to be of much greater length than any at present in existence, experiments with planes some hundreds of feet in length may be carried out.

These experiments show very clearly how important the condition of the surface is as affecting resistance. The varnished surface may be taken as typical of a surface coated with smooth paint, or the surface of a ship sheathed with bright copper, the medium sand surface being typical of the surface of a vessel sheathed with copper which has become foul. If the surface has become fouled with large barnacles, the resistance must rise very high.

In applying the results of these experiments to the case of actual ships, it is usual to estimate the wetted surface, and to take the length of the ship in the direction of motion to determine what the coefficient f shall be. For greater lengths than 50 feet, it is assumed that the resistance per square foot is the same as for the plate 50 feet long.

Take the following as an example:—

The wetted surface of a vessel is estimated at 7540 square feet, the length being 172 feet. Find the resistance due to surface friction at a speed of 12 knots, assuming a coefficient of 0·25, and that the resistance varies (*a*) as the square of the speed, and (*b*) as the 1·83 power of the speed.

$$(a)\ \text{Resistance} = 0{\cdot}25 \times 7540 \times \left(\tfrac{12}{6}\right)^2$$
$$= 7540 \text{ lbs.}$$
$$(b)\ \text{Resistance} = 0{\cdot}25 \times 7540 \times \left(\tfrac{12}{6}\right)^{1{\cdot}83}$$
$$= 6702 \text{ lbs.}$$ [1]

[1] This has to be obtained by the aid of logarithms.

$$\log (2^{1{\cdot}83}) = 1{\cdot}83 \log 2$$
$$= 0{\cdot}5508849$$
$$\therefore 2^{1{\cdot}83} = 3{\cdot}5554$$

It is worth remembering that for a smooth painted surface the frictional resistance per square foot of surface is about $\frac{1}{4}$ lb. at a speed of 6 knots.

It is useful, in estimating the wetted surface for use in the above formula, to have some method of readily approximating to its value. Several methods of doing this have been already given in Chapter II., the one known as "Kirk's Analysis" having been largely employed. There are also several approximate formulæ which are reproduced—

(1) Based on Kirk's analysis—

$$\text{Surface} = 2\text{LD} + \frac{\text{V}}{\text{D}}$$

(2) Given by Mr. Denny—

$$\text{Surface} = 1{\cdot}7\text{LD} + \frac{\text{V}}{\text{D}}$$

(3) Given by Mr. Taylor—

$$\text{Surface} = 15{\cdot}6\sqrt{\text{W.L.}}$$

L being the length of the ship in feet;
D being the mean moulded draught;
V being the displacement in cubic feet;
W being the displacement in tons.

There is also given, in Chapter II., a formula for the mean wetted girth, and this multiplied by the length will give the wetted surface. The formula is as follows:—

$$\text{Mean wetted girth} = 0{\cdot}95\,c\text{M} + 2(1 - c)\text{D}$$

where c = prismatic coefficient of fineness;
M = wetted girth on the midship section;
D = mean moulded draught.

2. *Eddy-making Resistance.*—We have already seen the general character of this form of resistance. It may be assumed to vary as the square of the speed, but it will vary in amount according to the shape of the ship and the appendages. Thus a ship with a full stern and thick stern-posts will experience this form of resistance to a much greater

extent than a vessel with a fine stern and with stern-post and rudder of moderate thickness. Eddy-making resistance can be allowed for by putting on a percentage to the frictional resistance. Mr. Froude estimated that in well-formed ships this form of resistance usually amounted to about 8 per cent. of the frictional resistance. It is possible to reduce eddy-making to a minimum by paying careful attention to the appendages and endings of a vessel. Thus shaft brackets in twin-screw ships are often made of pear-shaped section, as shown in Fig. 98.

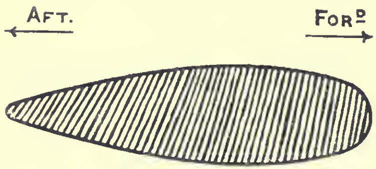

FIG. 98.

3. *Resistance due to the Formation of Waves.*—A completely submerged body moving at any given speed will only experience resistance due to surface friction and eddy-making provided it is immersed sufficiently; but with a body moving at the

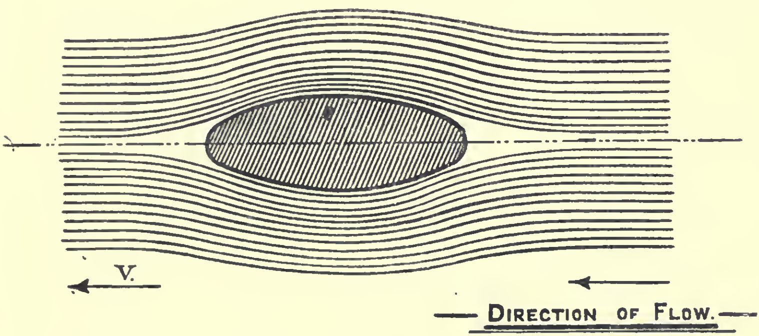

FIG. 99.

surface, such as we have to deal with, the resistance due to the formation of waves becomes very important, especially at high speeds. This subject is of considerable difficulty, and it is not possible to give in this work more than a general outline of the principles involved.

Consider a body shaped as in Fig. 99 placed a long way below the surface in water (regarded as frictionless), and suppose the water is made to move past the body with a uniform speed V. The particles of water must move past the body in certain lines, which are termed *stream-lines*. These stream-lines are straight and parallel before they reach the body, but owing to the obstruction caused, the particles of water are locally diverted, and follow curved paths instead of straight ones. The straight paths are again resumed some distance at the rear of the body. We can imagine these stream-lines making up the boundaries of a series of stream-tubes, in each of which the same particles of water will flow throughout the operation. Now, as these streams approach the body they broaden, and consequently the particles of water slacken in speed. Abreast the body the streams are constricted in area, and there is a consequent increase in speed; and at the rear of the body the streams again broaden, with a slackening in speed. Now, in water flowing in the way described, any *increase in speed* is accompanied by a *decrease in pressure*, and conversely any *decrease in speed* is accompanied by an *increase in pressure*. We may therefore say—

(1) There is a broadening of all the streams, and attendant decrease of speed and consequent excess of pressure, near both ends of the body; and—

(2) There is a narrowing of the streams, with attendant excess of speed and consequent decrease of pressure, along the middle of the body.

This relation between the velocity and pressure is seen in the draught of a fire under a chimney when there is a strong wind blowing. The excess of the speed of the wind is accompanied by a decrease of pressure at the top of the chimney. It should be noticed that the variations of velocity and pressure must necessarily become less as we go further away from the side of the body. A long way off the stream-lines would be parallel. The body situated as shown, with the frictionless water moving past it, does not experience any resultant force tending to move it in the direction of motion.[1]

[1] This principle can be demonstrated by the use of advanced mathematics. "We may say it is quite evident if the body is symmetrical, that is to say, has both ends alike, for in that case all the fluid action about the after

Now we have to pass from this hypothetical case to the case of a vessel on the surface of the water. In this case the water surface is free, and the excess of pressure at the bow and stern shows itself by an elevation of the water at the bow and stern, and the decrease of pressure along the sides shows itself by a depression of the water along the sides. This system is shown by the dotted profile of the water surface in Fig. 100. The

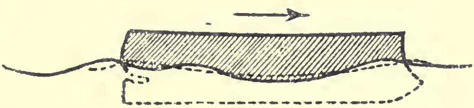

FIG. 100.

foregoing gives us the reason for the wave-crest at the stern of the ship. The crest at the bow appears quite a reasonable thing to expect, but the crest at the stern is due to the same set of causes. In actual practice the waves that are formed obscure the simple system we have described above, which has been termed the "statical wave."

Observation shows that there are two separate and distinct series of waves caused by the motion of a ship through the water—

(1) Waves caused by the bow;

(2) Waves caused by the excess pressure at the stern due to the expansion of the streams.

Each of these series of waves consists of (1) a series of diverging waves, the crests of which slope aft, and (2) a series of transverse waves, whose crests are nearly perpendicular to the middle line of the ship.

First, as to the diverging waves at the bow. "The inevitably widening form of the ship at her entrance throws off on each side a local oblique wave of greater or less size according to the speed and obtuseness of the wedge, and these waves form themselves into a series of diverging crests. These waves

body must be the precise counterpart of that about the fore body; all the stream-lines, directions, speed of flow, and pressures at every point must be symmetrical, as is the body itself, and all the forces must be equal and opposite" (see a paper by Mr. R. E. Froude, on "Ship Resistance," read before the Greenock Philosophical Society in 1894).

have peculiar properties. They retain their identical size for a very great distance, with but little reduction in magnitude. But the main point is, that they become at once disassociated with the vessel, and after becoming fully formed at the bow, they pass clear away into the distant water, and produce no further effect on the vessel's resistance." These oblique waves are not long in the line of the crest BZ, Fig. 101, and the

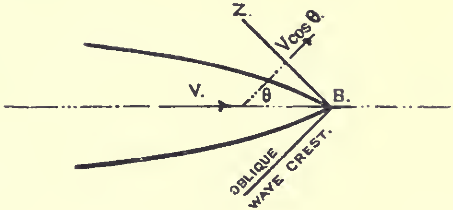

FIG. 101.

waves travel perpendicular to the crest-line with a speed of V cos θ, where V is the speed of the ship. As the speed of the ship increases the diverging waves become larger, and consequently represent a greater amount of resistance.

Besides these diverging waves, however, "there is produced by the motion of the vessel another notable series of waves, which carry their crests transversely to her line of motion." It is this transverse series of waves that becomes of the greatest importance in producing resistance as the speed is pushed to values which are high for the ship. These transverse waves show themselves along the sides of the ship by the crests and troughs, as indicated roughly in Fig. 100. The lengths of these waves (*i.e.* the distance from one crest to the other) bears a definite relation to the speed of the ship. This relation is that the length of the wave varies as the *square of the speed* at which the ship is travelling, and thus as the speed of the ship increases the length from crest to crest of the accompanying series of transverse waves increases very rapidly.

The waves produced by the stern of the ship are not of such great importance as those formed by the bow, which we have been considering. They are, however, similar in character, there being an oblique series and a transverse series.

Interference between the Bow and Stern Transverse Series of Waves.—In a paper read by the late Mr. Froude at the Institution of Naval Architects in 1877, some very important experiments were described, showing how the residuary resistance[1] varied in a ship which always had the same fore and after bodies, but had varying lengths of parallel middle body inserted, thus varying the total length. A strange variation in the resistance at the same speed, due to the varying lengths of parallel middle body was observed. The results were set out as roughly shown in Fig. 102, the resistance being set up on a

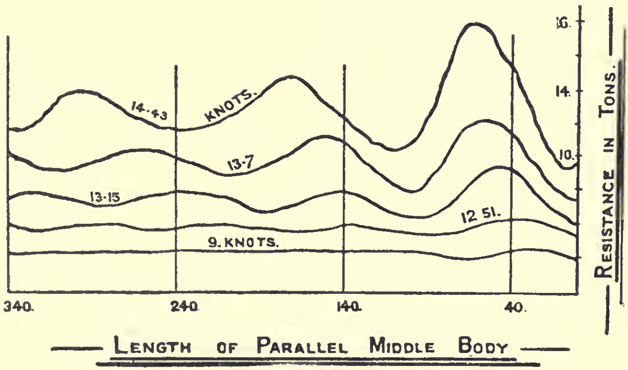

Fig. 102.

base of length of ship for certain constant speeds. At the low speed of 9 knots very little variation was found, and this was taken to show that at this speed the residuary resistance was caused by the diverging waves only.

The curves show the following characteristics :—

(1) The spacing or length of undulation appears uniform throughout each curve, and this is explained by the fact that waves of a given speed have always the same length.

(2) The spacing is more open in the curves of higher speed, the length apparently varying as the square of the speed. This is so because the length of the waves are proportionate to the square of the speed.

[1] Residuary resistance is the resistance other than frictional.

(3) The amplitude or heights of the undulations are greater in the curves of higher speeds, and this is so, because the waves made by the ship are larger for higher speeds.

(4) The amplitude in each curve diminishes as the length of parallel middle body increases, because the wave system, by diffusing transversely, loses its height.

These variations in residuary resistance for varying lengths are attributed to the interference of the bow and stern transverse series of waves. When the crests of the bow-wave series coincide with the crests of the stern wave series, the residuary resistance is at a *maximum*. When the crests of the bow-wave series coincides with the trough of the stern-wave series, the residuary resistance is at a *minimum*.

These experiments show very clearly that it is not possible to construct a formula which shall give the resistance of a ship at speeds when the wave-making resistance forms the important feature. We must either compare with the known performances of similar ships or models by using Froude's "law of comparison" (see p. 237).

The following extracts from a lecture[1] by Lord Kelvin (Sir William Thomson) are of interest as giving the relative influence of frictional and wave-making resistance :—

"For a ship A, 300 feet long, 31½ feet beam, and 2634 tons displacement, a ship of the ocean mail-steamer type, going at 13 knots, the skin resistance is 5·8 tons, and the wave resistance is 3·2 tons, making a total of 9 tons. At 14 knots the skin resistance is but little increased, viz. 6·6 tons, while the wave resistance is 6·15 tons.

"For a vessel B, 300 feet long, 46·3 feet beam, and 3626 tons, no parallel middle body, with fine lines swelling out gradually, the wave resistance is much more favourable. At 13 knots the skin resistance is rather more than A, being 6·95 tons as against 5·8 tons, while the wave resistance is only 2·45 tons as against 3·2 tons. At 14 knots there is a very remarkable result in the broader ship with its fine lines, all entrance and run, and no parallel middle body. At 14 knots the skin resistance is 8 tons as against 6·6 tons in A,

[1] Third volume "Popular Lectures and Addresses," 1887.

while the wave resistance is only 3·15 tons as against 6·15 tons in A.

"For a torpedo boat, 125 feet long and 51 tons displacement, at 20 knots the skin resistance was 1·2 tons, and the wave resistance 1·1 tons."

Resistance of a Completely Submerged Body.—The conditions in this case are completely different from those which have to be considered in the case of a vessel moving on the surface. In this latter case waves are produced on the surface, as we have seen, but with a completely submerged body this is not so, provided the vessel is immersed sufficiently. We get the clue to the form of least resistance in the shape of fishes, in which the head or forward end is comparatively blunt, while the rear portion tapers off very fine. The reason for the small resistances of forms of this sort is seen when we consider the paths the particles of water follow when flowing past. These paths are termed the stream-lines for the particular form. It will be seen that no eddies are produced for a fish-shaped form, and, as we saw on p. 225, it is the rear end which must be fined off in order to reduce eddy-making to a minimum. This was always insisted on very strongly by the late Mr. Froude, who said, "It is blunt tails rather than blunt noses that cause eddies." A very good illustration of the above is seen in the form that is given to the section of shaft brackets in twin-screw vessels. Such a section is given in Fig. 98. It will be noticed that the forward end is comparatively blunt, while the after end is fined off to a small radius.

Speed Coefficients.—The method which is most largely employed for determining the I.H.P. required to drive a vessel at a certain speed is by using coefficients obtained from the results of trials of existing vessels. They are based upon assumptions which should always be carefully borne in mind when applying them in actual practice.

1. *Displacement Coefficient.*—We have seen that for speeds at which wave-making resistance is not experienced, the resistance may be taken as varying—

(*a*) With the area of wetted surface;

(*b*) As the square of the speed;

so that we may write for the resistance in pounds—

$$R = K_1 S V^2$$

V being the speed in knots, S the area of wetted surface in square feet, and K_1 being a coefficient depending on a number of conditions which we have already discussed in dealing with resistance.

Now, $E.H.P = \frac{R \times V \times 101}{33000}$, as we have already seen (p. 215). Therefore we may say—

$$E.H.P. = K_2 S V^3$$

where K_2 is another coefficient, which may be readily obtained from the previous one. If now we assume that the total I.H.P. bears a constant ratio to the E.H.P., or, in other words, the propulsive coefficient remains the same, we may write—

$$I.H.P. = K_3 S V^3$$

K_3 being another new coefficient. S, the area of the wetted surface, is proportional to the product of the length and girth to the water-line; W, the displacement, is proportional to the product of the length, breadth, and draught. Thus W may be said to be proportional to the *cube* of the linear dimensions, while S is proportional to the *square* of the linear dimensions. Take a vessel A, of twice the length, breadth, and draught, of another vessel B, with every linear dimension twice that of the corresponding measurement in B. Then the forms of the two vessels are precisely similar, and the area of the wetted surface of A will be $2^2 = 4$ times the area of the wetted surface of B, and the displacement of A will be $2^3 = 8$ times the displacement of B. The ratio of the linear dimensions will be the cube root of the ratio of the displacements, in the above case $\sqrt[3]{8} = 2$. The ratio of corresponding areas will be the square of the cube root of the ratio of the displacements, in the above case $(\sqrt[3]{8})^2 = 4$. This may also be written $8^{\frac{2}{3}}$. We may accordingly say that for similar ships the area of the wetted surface will be proportional to the two-thirds power of the displacement, or $W^{\frac{2}{3}}$. We can now write our formula for the indicated horse-power—

$$\text{I.H.P.} = \frac{W^{\frac{2}{3}} \times V^3}{C}$$

where W = the displacement in tons;
V = the speed in knots;
C = a coefficient termed the *displacement coefficient*.[1]

If a ship is tried on the measured mile at a known displacement, and the I.H.P. and speed are measured, the value of the coefficient C can be determined, for $C = \frac{W^{\frac{2}{3}} \times V^3}{\text{I.H.P.}}$. It is usual to calculate this coefficient for every ship that goes on trial, and to record it for future reference, together with all the particulars of the ship and the conditions under which she was tried. It is a very tedious calculation to work out the term $W^{\frac{2}{3}}$, which means that the square of the displacement in tons is calculated, and the cube root of the result found. It is usual to perform the work by the aid of logarithms. A specimen calculation is given here:—

The *Himalaya* on trial displaced 4375 tons, and an I.H.P. of 2338 was recorded, giving a speed of 12·93 knots. Find the "displacement coefficient" of speed.

Here we have—

$$W = 4375$$
$$V = 12{\cdot}93$$
$$\text{I.H.P.} = 2338$$

By reference to a table of logarithms, we find—

$$\log 4375 = 3{\cdot}6410$$
$$\log 12{\cdot}93 = 1{\cdot}1116$$
$$\log 2338 = 3{\cdot}3689$$

$$\text{so that } \log (4375)^{\frac{2}{3}} = \tfrac{2}{3} \log 4375 = 2{\cdot}4273$$
$$\log (12{\cdot}93)^3 = 3 \log 12{\cdot}93 = 3{\cdot}3348$$

$$\therefore \log\left(\frac{4375^{\frac{2}{3}} \times 12{\cdot}93^3}{2338}\right) = 2{\cdot}4273 + 3{\cdot}3348 - 3{\cdot}3689$$
$$= 2{\cdot}3932$$

The number of which this is the logarithm is 247·3, and accordingly this is the value of the coefficient required.

[1] The coefficients are often termed "Admiralty constants," but it will be seen on p. 235 that they are not at all constant for different speeds of the same vessel.

2. The other coefficient employed is the "*midship-section coefficient.*"[1] If M is the area of the immersed midship section in square feet, the value of this coefficient is—

$$\frac{M \times V^3}{I.H.P.}$$

This was originally based on the assumption that the resistance of the ship might be regarded as due to the forcing away of a volume of water whose section is that of the immersed midship section of the ship. This assumption is not compatible with the modern theories of resistance of ships, and the formula can only be true in so far as the immersed midship section is proportional to the wetted surface.

In obtaining the $W^{\frac{2}{3}}$ coefficient, we have assumed that the wetted surface of the ships we are comparing will vary as the two-thirds power of the displacement; but this will not be true if the ships are not similar in all respects. However, it is found that the proportion to the area of the wetted surface is much more nearly obtained by using $W^{\frac{2}{3}}$ than by using the area of the immersed midship section. We can easily imagine two ships of the same breadth and mean draught and similar form of midship section whose displacement and area of wetted surface are very different, owing to different lengths. We therefore see that, in applying these formulæ, we must take care that the forms and proportions of the ships are at any rate somewhat similar. There is one other point about these formulæ, and that is, that the performances of two ships can only be fairly compared at "corresponding speeds."[2]

Summing up the conditions under which these two formulæ should be employed, we have—

(1) The resistance is proportional to the square of the speed.

(2) The resistance is proportional to the area of wetted surface, and this area is assumed to vary as the two-thirds power of the displacement, or as the area of the immersed midship section. Consequently, the ships we compare should be of somewhat similar type and form.

(3) The coefficient of performance of the machinery is

[1] See note on p. 233. [2] See p. 236.

assumed to be the same. The ships we compare are supposed to be fitted with the same type of engine, working with the same efficiency. Accordingly we cannot fairly compare a screw steamer with a paddle steamer, since the efficiency of working may be very different.

(4) The conditions of the surfaces must be the same in the two ships. It is evident that a greater I.H.P. would be required for a given speed if the ship's bottom were foul than if it had been newly painted, and consequently the coefficient would have smaller values.

(5) Strictly speaking, the coefficients should only be compared for "corresponding speeds."[1]

With proper care these formulæ may be made to give valuable assistance in determining power or speed for a new design, but they must be carefully used, and their limitations thoroughly appreciated.

We have seen that it is only for moderate speeds that the resistance can be said to be proportional to the square of the speed, the resistance varying at a higher power as the speed increases. Also that the propulsive coefficient is higher at the maximum speed than at the lower speeds. So if we try a vessel at various speeds, we cannot expect the speed coefficients to remain constant, because the suppositions on which they are based are not fulfilled at all speeds. This is found to be the case, as is seen by the following particulars of the trials of the *Iris*. The displacement being 3290 tons, the measured-mile trials gave the following results :—

I.H.P.	Speed in knots.
7556	18·6
3958	15·75
1765	12·5
596	8·3

The values of the speed coefficients calculated from the above are—

	Displacement coefficients.	Mid. sec. coefficients.
18·6 knots	188	595
15·75 ,,	218	690
12·5 ,,	243	770
8·3 ,,	214	677

[1] See p. 236.

It will be noticed that both these coefficients attain their maximum values at about 12 knots for this ship, their value being less for higher and lower speeds. We may explain this by pointing out—

(1) At high speeds, although the "propulsive coefficient" is high, yet the resistance varies at a greater rate than the square of the speed, and—

(2) At low speeds, although the resistance varies nearly as the square of the speed, yet the efficiency of the mechanism is not at its highest value.

Corresponding Speeds.—We have frequently had to use the terms "low speeds" and "high speeds" as applied to certain ships, but these terms are strictly relative. What would be a high speed for one vessel might very well be a low speed for another. The first general idea that we have is that the speed depends in some way on the length. Fifteen knots would be a high speed for a ship 150 feet long, but it would be quite a moderate speed for a ship 500 feet long. In trying a model of a ship in order to determine its resistance, it is obvious that we cannot run the model at the same speed as the ship; but there must be a speed of the model "*corresponding*" to the speed of the ship. The law that we must employ is as follows: "*In comparing similar ships with one another, or ships with models, the speeds must be proportional to the square root of their linear dimensions.*" Thus, suppose a ship is 300 feet long, and has to be driven at a speed of 20 knots; we make a model of this ship which is 6′ 3″ long. Then the ratio of their linear dimensions is—

$$\frac{300}{6\cdot25} = 48$$

and the speed of the model corresponding to 20 knots of the ship is—

$$20 \div \sqrt{48} = 2\cdot88 \text{ knots}$$

Speeds obtained in this way are termed "*corresponding speeds.*"

Example.—A model of a ship of 2000 tons displacement is constructed on the $\frac{1}{4}$ inch = 1 foot scale, and is towed at a speed of 3 knots. What speed of the ship does this correspond to?

Although here the actual dimensions are not given, yet the ratio of the linear dimensions is given, viz. 1 : 48. Therefore the speed of the ship corresponding to 3 knots of the model is—

$$3\sqrt{48} = 20\tfrac{3}{4} \text{ knots}$$

Expressing this law in a formula, we may say—

$$V = c\sqrt{L}$$

where V = speed in knots;
L = the length in feet;
c = a coefficient expressing the ratio $V : \sqrt{L}$, and consequently giving a measure of the speed.

We may take the following as average values of the coefficient "c" in full-sized ships:—

When $c = 0{\cdot}5$ to $0{\cdot}65$, the ship is being driven at a moderate economical speed;
$c = 0{\cdot}7$ to $1{\cdot}0$, gives the speed of mail steamers and modern battleships;
$c = 1{\cdot}0$ to $1{\cdot}3$, gives the speed of cruisers.

Beyond this we cannot go in full-sized vessels, since it is not possible to get in enough engine-power. This can, however, be done in torpedo-boats and torpedo-boat destroyers, and here we have $c = 1{\cdot}9$ to $2{\cdot}3$. These may be termed excessive speeds.

We have already seen that the $W^{\frac{2}{3}}$ coefficient of performance has a maximum value at a certain speed for a given ship. In the case of the *Iris*, we saw that this was at a speed of 12 knots. This maximum value of the coefficient is usually found to be obtained in full-sized ships at a speed corresponding to the value $c = 0{\cdot}7$. The *Iris* was 300 feet long, and the value of c at 12 knots would be $\dfrac{12}{\sqrt{300}} = 0{\cdot}69$.

Froude's Law of Comparison.—This law enables us to compare the resistance of a ship with that of her model, or the resistances of two ships of different size but of the same form. It is as follows:—

If the linear dimensions of a vessel be l times the dimensions of the model, and the resistance of the latter at speeds V_1, V_2, V_3, *etc., are* R_1, R_2, R_3, *etc., then at the "corresponding speeds" of the ship*, $V_1\sqrt{l}$, $V_2\sqrt{l}$, $V_3\sqrt{l}$, *etc., the resistance of the ship will be* R_1l^3, R_2l^3, R_3l^3, etc.

In passing from a model to a full-sized ship there is a correction to be made, because of the different effect of the friction of the water on the longer surface. The law of comparison strictly applies to the resistances other than frictional. The law can be used in comparing the resistance of two ships of similar form, and is found of great value when model experiments are not available.

In the earlier portion of this chapter we referred to the experiments of the *Greyhound* by the late Mr. Froude. A curve of resistance of the ship in pounds on a base of speed is given by A, in Fig. 96. In connection with these experiments, a model of the *Greyhound* was made and tried in the experimental tank under similar conditions of draught as the ship, and between speeds *corresponding* to those at which the ship herself had been towed. The resistance of the model having been found at a number of speeds, it was possible to construct a curve of resistance on a base of speed as shown by C in Fig. 103. The scale of the model was $\frac{1}{16}$ full size, and therefore the corresponding speeds of the ship were $\sqrt{16}$, or four times the speed of the model. If the law of comparison held good for the total resistance, the resistance of the ship should have been $16^3 = 4096$ times the resistance of the model at corresponding speeds; but this was not the case, owing to the different effect of surface friction on the long and short surfaces. The necessary correction was made as follows: The wetted surface of the model was calculated, and by employing a coefficient suitable to the length of the model and the condition of its surface, the resistance due to surface friction was calculated for various speeds as explained (p. 223), and a curve drawn through all the spots thus obtained. This is shown by the dotted curve DD in Fig. 103. Thus at 250 feet per minute the total resistance of the model is given by *ac*, and the resistance due to surface friction by *ad*. The portion of the ordinate between the curves CC and DD will give at any speed the resistance due to other causes than that of surface friction. Thus at 250 feet per minute, these other resistances are given by *cd*. This figure shows very clearly how the resistance at low speeds is almost wholly due to surface friction, and this forms at high speeds a large proportion of the total. The wave-making resistance, as we have already seen, is the chief cause of the difference between the

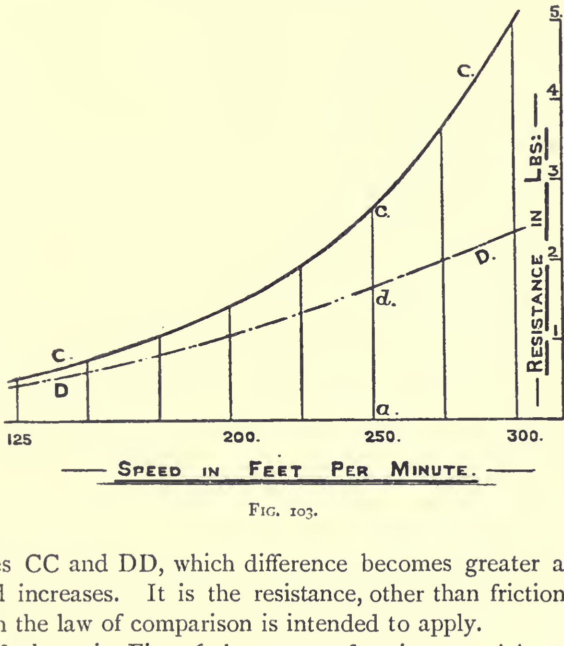

FIG. 103.

curves CC and DD, which difference becomes greater as the speed increases. It is the resistance, other than frictional, to which the law of comparison is intended to apply.

We have in Fig. 96 the curve of resistance, AA, of the *Greyhound* on a base of speed, and in precisely the same way as for the model a curve of frictional resistance was drawn in for the ship, taking the coefficient proper for the state of the surface of the ship and its length. Such a curve is given by BB, Fig. 96. Then it was found that the ordinates between the curves AA and BB, Fig. 96, giving the resistance for the ship other than frictional, were in practical agreement with the ordinates between the curves CC and DD, Fig. 103, giving the resistance of the model other than frictional, allowing for the "*law of* *comparison*" above stated. That is, at speeds of the ship $\sqrt{16}$, or four times the speeds of the model, the resistance of the ship other than frictional was practically 16^3, or 4096 times the resistance of the model.

These experiments of the *Greyhound* and her model form the first experimental verification of the law of comparison. In 1883 some towing trials were made on a torpedo-boat by Mr. Yarrow, and a model of the boat was tried at the experimental tank belonging to the British Admiralty. In this case also there was virtual agreement between the boat and the model according to the law of comparison. It is now the practice of the British Admiralty and others to have models made and run in a tank. The data obtained are of great value in determining the power and speed of new designs. For further particulars the student is referred to tbe sources of information mentioned at the end of the book.

Having the resistance of a ship at any given speed, we can at once determine the E.H.P. at that speed (see p. 215), and then by using a suitable propulsive coefficient, we may determine the I.H.P. at that speed. Thus, if at 10 knots the resistance of a ship is 10,700 lbs., we can obtain the E.H.P. as follows:—

$$\text{Speed in feet per minute} = 10 \times \frac{6080}{60}$$

$$\text{Work done per minute} = 10{,}700 \times \left(10 \times \frac{6080}{60}\right) \text{ foot-lbs.}$$

$$\text{E.H.P.} = \frac{10700 \times \frac{6080}{6}}{33000}$$

$$= 328$$

and if we assume a propulsive coefficient of 45 per cent.—

$$\text{I.H.P.} = \frac{328 \times 100}{45}$$

$$= 729$$

By the use of the law of comparison, we can pass from one ship whose trials have been recorded to another ship of the same form, whose I.H.P at a certain speed is required. It is found very useful when data as to I.H.P. and speed of existing ships are available. In using the law we make the following assumptions, which are all reasonable ones to make.

(1) The correction for surface friction in passing from one ship to another of different length is unnecessary.

(2) The condition of the surfaces of the two vessels are assumed to be the same.

(3) The efficiency of the machinery, propellers, etc., is assumed the same in both cases, so that we can use I.H.P. instead of E.H.P.

The method of using the law will be best illustrated by the following example :—

A vessel of 3290 tons has an I.H.P. of 2500 on trial at 14 knots. What would be the probable I.H.P. of a vessel of the same form, but of three times the displacement, at the corresponding speed?

$$\begin{aligned} \text{The ratio of the displacement} &= 3 \\ \therefore \text{the ratio of the linear dimensions } l &= \sqrt[3]{3} \\ &= 1{\cdot}44 \\ \therefore \text{the corresponding speed} &= 14 \times \sqrt{1{\cdot}44} \\ &= 16{\cdot}8 \text{ knots} \end{aligned}$$

The resistance of the new ship will be l^3 times that of the original, and accordingly the E.H.P., and therefore the I.H.P., will be that of the original ship multiplied by $l^{\frac{7}{2}} = (1{\cdot}44)^{\frac{7}{2}} = 3{\cdot}6$, and—

$$\begin{aligned} \text{I.H.P. for new ship} &= 2500 \times 3{\cdot}6 \\ &= 9000 \end{aligned}$$

When ships have been run on the measured mile at progressive speeds, the information obtained is found to be extremely valuable, since we can draw for the ship thus tried a curve of I.H.P. on a base of speed, and thus at intermediate speeds we can determine the I.H.P. necessary. The following example will show how such a curve is found useful in estimating I.H.P. for a new design.

A vessel of 9000 tons is being designed, and it is desired to obtain a speed of 21 knots. A ship of 7390 tons of similar form has been tried, and a curve of I.H.P. to a base of speed drawn. At speeds of 10, 14, 18, and 20 knots the I.H.P. is 1000, 3000, 7500, 11,000 respectively.

Now, the corresponding speeds of the ships will vary as the square root of the ratio of linear dimension, l.

We have—

$$\begin{aligned} l^3 &= \tfrac{9000}{7390} \\ \text{and } l &= 1{\cdot}07 \\ \sqrt{l} &= 1{\cdot}035 \end{aligned}$$

therefore the corresponding speed of the 7390-ton ship is—

$$21 \div 1 \cdot 035 = 20 \cdot 3$$

By drawing in the curve of I.H.P. and continuing it beyond the 20 knots, we find that the I.H.P. corresponding to a speed of 20·3 knots is about 11,700. The I.H.P. for the 9000-ton ship at 21 knots is accordingly—

$$\begin{aligned} 11,700 \times l^{\frac{7}{2}} &= 11,700 \times 1 \cdot 26 \\ &= 14,750 \text{ I.H.P. about} \end{aligned}$$

Examples to Chapter VII.

1. The *Greyhound* was towed at the rate of 845 feet per minute, and the horizontal strain on the tow-rope, including an estimate of the air resistance of masts and rigging, was 6200 lbs. Find the effective horse-power at that speed.

Ans. 159 E.H.P. nearly.

2. A vessel of 5500 tons displacement is being towed at a speed of 8 knots, and her resistance at that speed is estimated at 18,740 lbs. What horse-power is being transmitted through the tow-rope?

Ans. 460.

3. A steam-yacht has the following particulars given :—

Displacement on trial	176·5 tons
I.H.P on trial	364
Speed ,, ...	13·3 knots

Find the "displacement coefficient of speed."

Ans. 203.

4. A steam-yacht has a displacement of 143·5 tons, and 250 I.H.P. is expected on trial. What should the speed in knots be, assuming a displacement coefficient of speed of 196?

Ans. 12·2 knots.

5. The *Warrior* developed 5267 indicated horse-power, with a speed of 14·08 knots on a displacement of 9231 tons. Find the displacement coefficient of speed.

Ans. 233.

6. In a set of progressive speed trials, very different values of the "displacement coefficient" are obtained at different speeds. Explain the reason of this. A ship is 225 feet long, at what speed would you expect the coefficient to have its maximum value?

Ans. About 10½ knots.

7. Suppose we took a torpedo boat destroyer of 250 tons displacement and 27 knots speed as a model, and designed a vessel of 10,000 tons displacement of similar form. At what speed of this vessel could we compare her resistance with that of the model at 27 knots?

Ans. 50 knots.

8. A ship of 5000 tons displacement has to be driven at 21 knots. A model of the ship displaces 101 lbs. At what speed should it be tried?

Ans. 3 knots.

9. A ship of 5000 tons displacement is driven at a speed of 12 knots. A ship of 6500 tons of similar form is being designed. At what speed of the larger ship can we compare its performance with the 5000-ton ship?

Ans. 12·53 knots.

10. A vessel 300 feet long is driven at a speed of 15 knots. At what

speed must a similar vessel 350 feet long be driven in order that their performances may be compared?

Ans. 16·2 knots.

11. A vessel 300 feet long has a displacement on the measured-mile trial of 3330 tons, and steams at 14, 18, and 20 knots with 2400, 6000, and 9000 I.H.P. respectively. What would be the I.H.P. required to drive a vessel of similar type, but of double the displacement, at 20 knots?

Ans. 13,000 I.H.P. about.

12. A vessel of 3100 tons displacement is 270 feet long, 42 feet beam, and 17 feet draught. Her I.H.P. at speeds of 6, 9, 12, and 15 knots are 270, 600, 1350, and 3060 respectively. What will be the dimensions of a similar vessel of 7000 tons displacement, and what I.H.P. will be required to drive this vessel at 18 knots?

Ans. 354 × 55 × 22·3, about 9600 I.H.P.

13. A vessel of 4470 tons displacement is tried on the measured mile at progressive speeds, with the following results:—

Speed.	I.H.P.
8·47	485
10·43	881
12·23	1573
12·93	2117

A vessel of similar form of 5600 tons displacement is being designed. Estimate the I.H.P. required for a speed of 13 knots.

Ans. 2290 I.H.P.

APPENDIX A

Proof of Simpson's First Rule.—Let the equation of the curve referred to the axes Ox, Oy, as Fig. 31, p. 51, be—

$$y = a_0 + a_1x + a_2x^2$$

a_0, a_1, a_2 being constants ; then the area of a narrow strip length y and breadth Δx is—

$$y \times \Delta x$$

and the area required between $x = 0$ and $x = 2h$ is the sum of all such strips between these limits. Considering the strips as being a small breadth Δx, we still do not take account of the small triangular pieces as BDE (see Fig. 12), but on proceeding to the limit, *i.e.* making the strips indefinitely narrow, these triangular areas disappear, and the expression for the area becomes, using the formulæ of the calculus—

$$\int_0^{2h} y \, . \, dx$$

or, putting in the value for y given by the equation to the curve—

$$\int_0^{2h} (a_0 + a_1x + a_2x^2)dx$$

which equals—

$$\left[a_0x + \tfrac{1}{2}a_2x^2 + \tfrac{1}{3}a_3x^3\right]_0^{2h}$$

which has to be evaluated between the limits $x = 2h$ and $x =$ The expression then becomes—

$$a_02h + \tfrac{1}{2}a_14h^2 + \tfrac{1}{3}a_28h^3$$

Now, from the equation to the curve, when—

$$x = 0, y = a_0$$
$$x = h, y = a_0 + a_1h + a_2h^2$$
$$x = 2h, y = a_0 + 2a_1h + 4a_2h^2$$

But calling the ordinates in the ordinary way, y_1, y_2, and y_3—

$$\begin{aligned} \text{when } x &= 0, y = y_1 \\ x &= h, y = y_2 \\ x &= 2h, y = y_3 \end{aligned}$$

therefore we have—

$$\begin{aligned} a_0 &= y_1 \\ a_0 + a_1h + a_2h^2 &= y_2 \\ a_0 + 2a_1h + 4a_2h^2 &= y_3 \end{aligned}$$

from which may be obtained the following values for the constants a_0, a_1, a_2 :—

$$\begin{aligned} a_0 &= y_1 \\ a_1 &= \frac{1}{2h}(4y_2 - 3y_1 - y_3) \\ a_2 &= \frac{1}{2h^2}(y_3 - 2y_2 + y_1) \end{aligned}$$

or substituting above—

$$\begin{aligned} \text{Area} &= y_1 2h + \frac{4h^2}{2} \cdot \frac{1}{2h}(4y_2 - 3y_1 - y_3) + \frac{8h^3}{3} \cdot \frac{1}{2h^2}(y_3 - 2y_2 + y_1) \\ &= \tfrac{1}{3}h(y_1 + 4y_2 + y_3) \end{aligned}$$

which is the expression known as Simpson's First Rule.

Proof of Simpson's Second Rule.—Let the equation to the curve be—

$$y = a_0 + a_1x + a_2x^2 + a_3x$$

then the area will be given by—

$$\int_0^{3h} (a_0 + a_1x + a_2x^2 + a_3x^3)dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The area given by the rule is—

$$\tfrac{3}{8}h(y_1 + 3y_2 + 3y_3 + y_4) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) we find that the area is—

$$3a_0h + 9a_1\frac{h^2}{2} + 9a_2h^3 + \tfrac{81}{4}a_3h^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now—

$$\begin{aligned} y_1 &= a_0 \\ y_2 &= a_0 + a_1h + a_2h^2 + a_3h^3 \\ y_3 &= a_0 + 2a_1h + 4a_2h^2 + 8a_3h^3 \\ y_4 &= a_0 + 3a_1h + 9a_2h^2 + 27a_3h^3 \end{aligned}$$

These are four equations for determining a_0, a_1, a_2, a_3.

If we substitute the values given above for y_1, y_2, etc., in the expression for the area as given by Simpson's second rule in (2), we find the result is the same as the expression (3). This shows that Simpson's second rule will correctly integrate a curve whose equation is that given above.

The truth of the five-eight rule given on p. 12 can be proved in a similar manner.

The dynamical stability of a ship at any given angle of heel is equal to the area of the curve of statical stability up to that angle.

Referring to Fig. 67 showing a ship heeled over to a certain angle θ, imagine the vessel still further heeled through a very small additional angle, which we may call $d\theta$. The centre of buoyancy will move to B″ (the student should here draw his own figure to follow the argument). B′B″ will be parallel to the water-line W′L′, and consequently the centre of buoyancy will not change level during the small inclination. Drawing a vertical B″Z through B″, we draw GZ′, the new righting arm, perpendicular to it. Now, the angle ZGZ′ $= d\theta$, and the vertical separation of Z and Z′ $=$ GZ $\times\ d\theta$. Therefore the work done in inclining the ship from the angle θ to the angle $\theta + d\theta$ is—

$$W \times (GZ \, . \, d\theta)$$

Take now the curve of statical stability for this vessel. At the angle θ the ordinate is W $\times$ GZ. Take a consecutive ordinate at the angle $\theta + d\theta$. Then the area of such a strip $=$ W $\times$ GZ $\times\ d\theta$; but this is the same as the above expression for the work done in inclining the vessel through the angle $d\theta$, and this, being true for any small angle $d\theta$, is true for all the small angles up to the angle θ. But the addition of the work done for each successive increment of inclination up to a given angle is the dynamical stability at that angle, and the sum of the areas of such strips of the curve of statical stability as we have dealt with above is the area of that curve up to the angle θ. Therefore we have the dynamical stability of a ship at any given angle of heel is equal to the area of the curve of statical stability up to that angle, where the ordinates of this curve represent the righting moments.

The area of a curve of displacement divided by the load displacement gives the distance of the centre of buoyancy below the L.W.L.

Let OBL, Fig. 104, be the curve of displacement of a vessel constructed in the ordinary way, OW being the load mean draught, and WL being the load displacement.

Take two level lines, AB, A′B′, a small distance Δz apart.

Call the area of the water-plane at the level of AB, A square feet, and the distance of this water-plane below the L.W.L., z. The volume between the water-lines AB, A′B′ will be $A \times \Delta z$, or, sup-

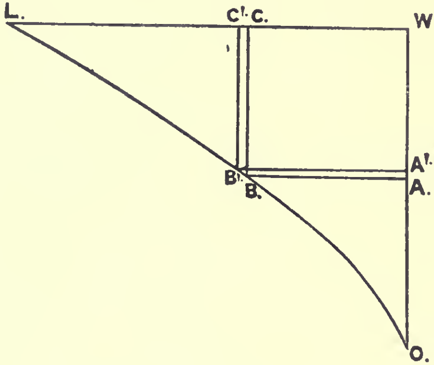

FIG. 104.

posing they are indefinitely close together, $A \times dz$. The moment of this layer about the L.W.L. will be $A . z . dz$.

The difference between the lengths of A′B′ and AB will evidently be the weight of the volume of water between those two level lines, or $\frac{A \times dz}{35}$. Draw B′C′, BC vertically as shown. Then the breadth of the strip B′C is $\frac{A \times dz}{35}$, and the area of this strip is $\frac{A \times z \times dz}{35}$. The area of the curve will be the area of all such strips, or—

$$\int \frac{A . z . dz}{35}$$

The moment of the volume of displacement about the L.W.L. is given by—

$$\int A . z . dz$$

and the distance of the C.B. below the L.W.L. is found by dividing this moment by the load displacement in cubic feet, or—

$$\int \frac{A . z . dz}{WL \times 35}$$

The area of the curve divided by the load displacement in tons is—

$$\frac{\int \frac{A . z . dz}{35}}{WL}$$

which is the same thing.

Normand's Approximate Formula for the Position of the Centre of Buoyancy.—This formula was given on p. 63.

$\frac{V}{A}$ is known as "Rankine's mean depth," and we may for convenience say—

$$\frac{V}{A} = D$$

The formula then becomes—

$$\left.\begin{array}{c}\text{Distance of CB below}\\ \text{the LWL in feet}\end{array}\right\} = \frac{1}{3}\left(\frac{d}{2} + D\right)$$

In Fig. 105, let ABC be the curve of water-plane areas, DC being the mean draught *d*. Draw the rectangle AFCD. Make DE

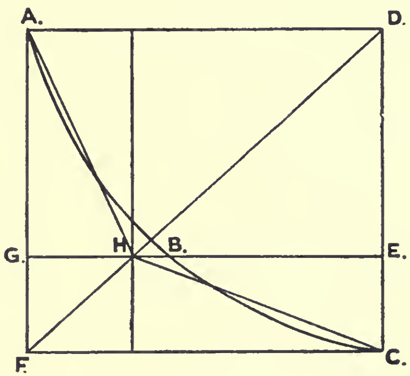

FIG. 105.

= D. Draw EG parallel to DA, cutting the diagonal FD in H. Finish the figure as indicated. The assumption made is that the C.G. of the area DABC, which will give the vertical position of the centre of buoyancy, is in the same position as the C.G. of the area DAHC. This is seen, on inspection, to be very nearly the case. These two figures have the same area, as we now proceed to show.

By using the principle of similar triangles, we have—

$$\frac{AF}{AD} = \frac{GF}{GH} = \frac{ED}{HE}$$

and therefore—

$$GF \times HE = ED \times GH$$
$$\text{or } EC \times HE = AG \times GH$$

or the triangles AGH, HEC are equal in area ; therefore—

$$\text{Area AHCD} = \text{area AGED}$$

and this latter area equals the volume of displacement, being a rectangle having its adjacent sides equal to A and $\frac{V}{A}$ respectively.

The area of the curved figure DABC also evidently represents the volume of displacement. Therefore the figures DABC and DAHC are equal in area.

We now have to determine the position of the C.G. of DAHC in relation to the L.W.L.

$$\frac{\text{Area AGH}}{\text{area AGED}} = \frac{\frac{1}{2} \times \text{AG} \times \text{GH}}{\text{AG} \times \text{AD}} = \frac{1}{2} \cdot \frac{\text{GH}}{\text{AD}}$$

$$= \frac{1}{2} \cdot \frac{\text{GF}}{\text{AF}} = \tfrac{1}{2}\left(\frac{d - \text{D}}{d}\right)$$

$$\text{or the triangle AGH} = \tfrac{1}{2}\left(\frac{d - \text{D}}{d}\right) \times \text{rectangle AGED}$$

We may regard the figure DAHC as made by taking AGH away from the rectangle AE, and putting it in the position HEC. The shift of its C.G. downwards during this operation is $\frac{d}{3}$, therefore the C.G. of the whole figure will shift downwards, using the principle explained in p. 96, an amount equal to x say, x being given by—

$$\text{AGED} \times x = \text{AGH} \times \frac{d}{3}$$

or putting in the value found above, for the triangle AGH—

$$x = \tfrac{1}{2}\left(\frac{d - \text{D}}{d}\right)\frac{d}{3} = \tfrac{1}{6}(d - \text{D})$$

The C.G. of AGED is at present a distance $\frac{\text{D}}{2}$ below the L.W.L. ; therefore the C.G. of DAHC is below the L.W.L. a distance—

$$\frac{\text{D}}{2} + \tfrac{1}{6}(d - \text{D}) = \tfrac{1}{3}\left(\frac{d}{2} + \text{D}\right)$$

$$= \tfrac{1}{3}\left(\frac{d}{2} + \frac{\text{V}}{\text{A}}\right)$$

and taking the assumption given above, this is the distance of the C.B. below the L.W.L. very nearly.

Strength of Rudder-heads.—For vessels built to Lloyd's rules, the diameter of the rudder-head is determined by the *longitudinal number* (see p. 194) and is given in the tables. For steam-vessels the diameters of rudder-heads are calculated by the following formula—

$$d = \frac{1}{32}\sqrt[3]{D \times B^2 \times S^2}$$

where D = feet draught, B = breadth of rudder in inches, and S = speed in knots; but in no case is the diameter to be less than that given in the table.

The following is the method adopted for determining the size of rudder-head, having given particulars of size, speed of ship, and maximum angle of rudder. To obtain the *twisting moment* acting on the rudder, we must know (1) the pressure of the water on the rudder; (2) the position of the centre of effort of this pressure.

As regards (1), the pressure in pounds on a plane moving uniformly broadside on (see Fig. 106), by the formula—

$$P = 1{\cdot}12Av^2$$

where A is the area of the plane in square feet;
v is the velocity in feet per second.

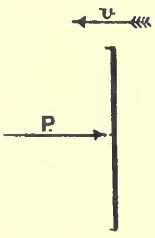

Fig. 106.

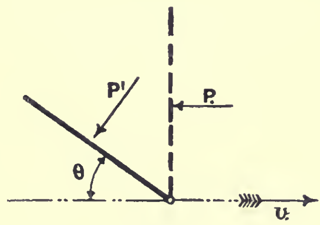

Fig. 107.

This supposes the rudder to be round to an angle of 90°. For an angle θ, the normal pressure is given by—

$$P' = P \times \sin\theta \text{ (see Fig. 107)}$$

With regard to the position on the rudder at which this normal pressure may be regarded as acting, it is found that for values of θ between 30° and 40°, the centre of effort is about four-tenths the breadth of the rudder from the leading edge, the rudder being hinged at the fore side.

By this means we are able to determine the *twisting moment* on the rudder-head. If a screw steamship is being propelled through the water at a certain speed the sternward velocity of the water from the propeller past the rudder, is greater than the speed of the ship. The velocity of water past the rudder can be taken as 10 per cent. greater than the velocity of the ship. Take the following as an example. A rectangular rudder 13 feet broad, hinged on the fore side, 180 square feet in area; maximum angle, 35°; speed of ship, 16 knots.

$$16 \text{ knots} = \frac{16 \times 101}{60} = 27 \text{ feet per second}$$

$$\text{Velocity of water past rudder} = 27 + 2{\cdot}7$$
$$= 29{\cdot}7, \text{ say } 30 \text{ feet per second}$$

$$\text{The pressure on the rudder} = 1{\cdot}12 \times \frac{180 \times 30 \times 30}{2240} \times \sin 35°$$
$$= 46{\cdot}5 \text{ tons}$$

Taking the centre of effort as $\frac{4}{10} \times 13 = 5{\cdot}2$ feet, or 62·4 inches from the leading edge—

$$\text{The twisting moment} = 46{\cdot}5 \times 62{\cdot}4$$
$$= 2901 \text{ inch-tons}$$

To determine the size of rudder-head for a twisting moment of T inch-tons, we use the following formula :—

$$T = \tfrac{1}{16}\pi f d^3$$

where d = diameter in inches,

f = strength of material per square inch, allowing a factor of safety.

For wrought iron, f is taken as 4 tons
cast steel, f " " 5 "
phosphor bronze, f " " 3 "

For a twisting moment of 2901 inch-tons, the diameter of a rudder-head, if of wrought iron, will be given by—

$$d^3 = \frac{2901}{\frac{1}{16} \times \frac{22}{7} \times 4}$$
$$\text{or } d = 15\tfrac{1}{2} \text{ inches}$$

Example.—A rudder is $243\frac{1}{2}$ square feet in area, and the centre of pressure is estimated to be 6·12 feet abaft the centre of rudder-head, at 35°. If the speed of the ship is 19 knots, estimate the diameter of the rudder-head, taking the stress at 4 tons and 5 tons.

Ans. 20·1", 18·7."

Launching Calculations.—Before starting on these calculations, it is necessary to estimate as closely as possible the launching weight of the ship, and also the position of the centre of gravity, both vertically and longitudinally. The case of the *Daphne*, which capsized on the Clyde[1] on being launched, drew special

[1] See *Engineering*, 1883, for a report on the *Daphne*, by Sir E. J. Reed.

attention to the necessity of providing sufficient stability in the launching condition. A ship in the launching condition has a light draught, great freeboard, and high position of the C.G. It is possible, by the use of the principles we have discussed at length, to approximate to the metacentric height, and if this is not considered sufficient, the ship should be ballasted to lower the centre of gravity. It has been suggested that a minimum G.M. of 1 foot should be provided in the launching condition. If the cross-curves of stability of the vessel have been made, it is possible very quickly to draw in the curve of stability in the launching condition, and in case of any doubt as to the stability, this should be done.

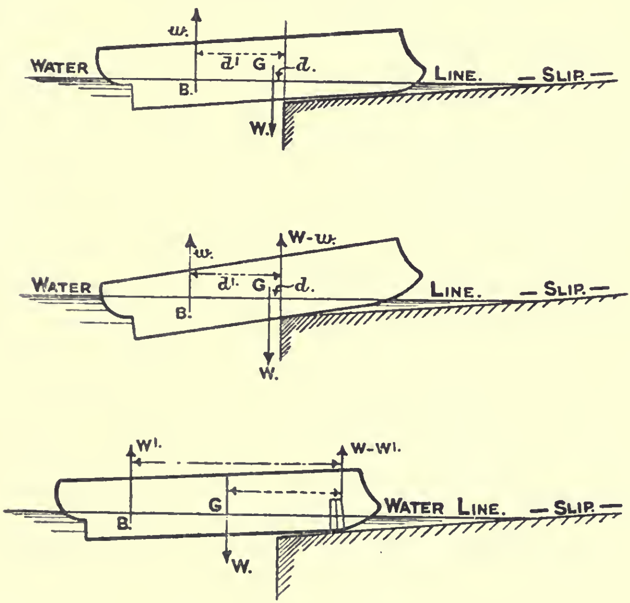

FIG. 108.

We now have to consider the longitudinal stability of a vessel as she goes down the ways, and describe the calculations that are made to see if the vessel will "*tip*"* (*i.e.* the stern drops over the end of the ways, and the forward end lifts up off the ways), and to ascertain the pressure that comes on the fore poppets when the vessel lifts. After the ship has run a certain distance (see Fig. 108)

* Tipping is shown by the second diagram in Fig. 108.

the C.G. will pass beyond the end of the ways, and will give rise to a "tipping moment;" but by this time a portion of the ship is in the water, and the upward support of the buoyancy will give rise to a moment in the opposite direction. Call W the weight of the ship, and at the position shown in Fig. 108, let d be the distance of the C.G. beyond the end of the ways, w the support of the buoyancy, and d' the distance of the centre of buoyancy beyond the end of the ways. Then the support of the ways at this particular position of the ship's travel is $W - w$, and if the vessel tipped, *i.e.* if the moment of the weight $W \times d$ were greater than the moment of the buoyancy $w \times d'$, this upward support would all be concentrated at the end of the ways, and very possibly the bottom of the vessel would be forced in, or the end of the ways give way, and so cause a stoppage of the ship, or at any rate make the ship slide down the rest of the distance on her keel. If the vessel stopped on the ways, she would be in a very critical condition as the tide fell. We therefore see that at all points the moment of the buoyancy should exceed the moment of the weight about the end of the ways. In order to see if this is so, we proceed as follows: Assume a height of tide that may safely be expected for launching, and take a series of positions as the ship goes down the ways, and determine the buoyancy and the distances of the C.G. and C.B. from the end of the ways, viz. w, d, and d' as above.

Then on a base-line (Fig. 109) representing the distance the ship has travelled, we draw two curves: first, a curve AA, giving the moment of buoyancy about the end of the ways; and second, BB, giving the moment of weight about the end of the ways (this latter being a straight line, since the weight is constant). The distance between these curves will give us the "*moment against tipping*" at any particular place, and the minimum distance between them gives the margin of safety. If a curve of moment of buoyancy is obtained, as the dotted curve A'A' crossing BB, then there would be a "*tipping moment.*" This might be the case if the ways were not long enough, or the tide might not rise so high as expected, or the position of the centre of gravity might be wrongly estimated. The remedy would be either to lengthen the ways or to place some ballast right forward. Either of these would push the point G, at which the centre of gravity of the ship comes over the end of the ways, further along. Ships have been launched successfully which had an adverse tipping moment; but the velocity was very great, and owing to this the ships were safely launched.

There is another point that has to be considered. As the ship goes further down the ways, there comes a certain position in which the moment of the buoyancy about the fore poppet equals the moment of the weight about the fore poppet. At this point the stern of the ship will begin to lift; and if W is the weight of the ship, and W′ the buoyancy, the ship will be partially supported by the fore poppets, and the amount of this support is W − W′. Thus we have a great strain, coming both on the slip and on the vessel, concentrated over a small distance. The amount of this can be determined as follows: In a similar manner in which we

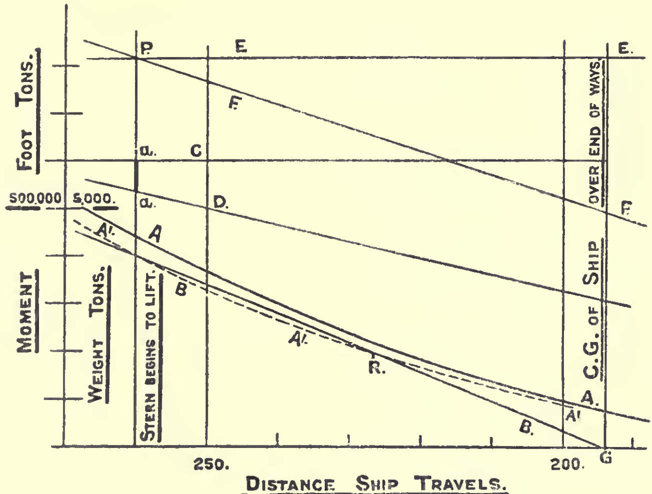

Fig. 109.

constructed the curves A and B we can construct E and F; E gives the moment of the weight of the ship about the fore poppet, which moment is a constant quantity, and F gives the moment of the buoyancy about the fore poppet. E and F intersect at the point P; and this is the place where the moments of weight and buoyancy are equal, and therefore the place where the stern will begin to lift. Now construct also C and D, C being the weight of the ship, in this case 6000 tons, and D being the buoyancy as the ship moves down the ways. Then the intercept between these, at the distance given by the point P, viz. *aa*, will give the weight borne by the fore poppets when the stern begins to lift. In the case for which the curves are given in Fig. 109, this weight was 675 tons. The launching curves for H.M.S. *Sanspareil* are given in Mr. Mackrow's

"Pocket Book," and in that case the weight of the ship was 5746 tons, and the weight on the fore poppets 870 tons.

The internal shoring of the ship must be specially arranged for in the neighbourhood of the fore poppets, and the portion of the slip under them at the time the stern lifts must be made of sufficient strength to bear the concentrated weight

APPENDIX B

SYLLABUS OF THE EXAMINATIONS IN NAVAL ARCHITECTURE CONDUCTED BY THE DEPARTMENT OF SCIENCE AND ART.

(From the Directory of the Department of Science and Art, by permission of the Controller of Her Majesty's Stationery Office.)

First Stage or Elementary Course.

No candidate will be permitted to pass who fails to obtain marks in any one of the three sections of this stage.

I. Practical Shipbuilding.—Students should be able to describe the methods usually adopted by the workmen in forming and combining the several parts of a steel or iron ship's hull, including the transverse and longitudinal framing, stems and sternposts, inner and outer bottom plating, beams, pillars (both fixed and portable), deck plating and planking, and the wood and copper sheathing of sheathed ships: also the methods adopted in forming and combining the framing and bottom planking of wood and composite ships. Also description of tools used in plating, planking, and caulking ships.

II. Ship Calculations.—Fundamental conditions which must be fulfilled by bodies when floating freely and at rest in still water. Calculations relating to the computation of the areas of plane surfaces and the displacement of ships by applications of Simpson's and the five-eight-minus-one (or one-twelfth) rules: "tons per inch immersion;" and the sinkage of a vessel in passing from sea into river water. Also a knowledge of the specific gravities of materials used in shipbuilding, and simple calculations based thereon.

III. Drawing.—Students will be required to make sketches to scale.

It is intended that the examination in Practical Shipbuilding shall be one principally relating to steel and iron ships, but one or two questions may be set on the important parts of wood and composite vessels mentioned above.

Second Stage or Advanced Course.

In addition to the subjects for the Elementary Stage, students presenting themselves for examination in the Advanced Course will be expected to have received instruction in the following—

Practical Shipbuilding.—The structural details of water-tight bulkheads; methods of testing water-tight work; the longitudinal and transverse stresses to which ships are liable in still water and amongst waves, and the structural arrangements which give the necessary strength to resist those stresses: also the various local stresses to which a ship is liable, and the special arrangements worked to meet them; the qualities of the various materials used in shipbuilding, and the tests to which these materials are subjected; precautions to be observed when working steel plates and angles hot; effect of annealing.

Laying Off.—Comprising a knowledge of the work carried on in the Mould Loft and at the Scrive Board in connection with ordering materials and laying off the several parts of an iron or steel mercantile vessel; also the similar work relating to warships, both sheathed with wood and unsheathed.

Ship Calculations.—Curves of "displacement" and "tons per inch immersion;" definitions of centre of flotation and centre of buoyancy; use of Simpson's and other rules for finding the position of the centre of gravity of plane areas, and for calculating the position of the centre of buoyancy; graphic or geometrical method of calculating displacement and position of centre of buoyancy; definitions of the terms "metacentre" and "metacentric height;" rules for calculating positions of transverse and longitudinal metacentres; metacentric diagrams, their construction and use; tensile strength of material between widely and closely spaced holes punched and drilled in steel plates; shearing strength of iron and steel rivets, and spacing of rivets; strength of butt fastenings; more advanced weight calculations, such as those for the weight of a deck or bulkhead.

Honours.

The examination in Honours will be divided into two parts, which cannot both be taken in the same year, and no candidate who has not been successful in Part I. can be examined in Part II. A certificate or medal will only be given when a success in Part II. has been obtained.

PART I.

In addition to the subjects prescribed for the preceding stages, this examination will embrace questions upon some or all of the subjects specified below :—

PRACTICAL SHIPBUILDING.—Important fittings of ships, including ventilating and coaling arrangements, anchor and capstan gear, masts and mastwork, etc. ; methods adopted for preventing deterioration of hull, both when being built and whilst on service ; launching arrangements ; principles of water-tight subdivision of war and merchant ships.

SHIP CALCULATIONS.—Definition of "change of trim ;" moment to change trim one inch ; change of trim due to moving weights already on board, and that due to the addition or removal of weights of moderate amount ; displacement sheet, general arrangement of calculations usually made thereon ; approximate and detailed calculations relating to the weight and position of centre of gravity of hull ; inclining experiment made to ascertain position of centre of gravity of a vessel and precautions necessary to be observed to ensure accuracy ; calculations for strength of bottom plating.

PART II.

Those candidates for Honours who successfully pass the above-mentioned examination may sit for examination in the following subjects, relating to the higher branches of Theoretical Naval Architecture, including those enumerated below :—

Proofs of Simpson's and the five-eight-minus-one rules.

Heeling produced by the pressure of wind on sails.

Calculation of the shearing forces and bending moments set up in a ship in still water, and also when floating amongst waves.

Construction of equivalent girder.

Statical and dynamical stability : Atwood's and Moseley's formulæ, and methods of calculating stability based thereon. Experimental methods of obtaining the stability at any angle. Use of Amsler's integrator for calculating stability. Ordinary curves of stability ; their construction and uses. Cross-curves of stability.

Reech's method of obtaining the co-ordinates of the centre of buoyancy corresponding to any given angle of heel.

Effect of free liquid in hold on stability; stability of petroleum-carrying vessels.

Rate of inflow of water through hole in bottom, and calculation of alteration of trim and heel due to admission of water through damage to bottom.

Construction of curves of buoyancy, curves of flotation, surfaces of buoyancy, surfaces of flotation. The "metacentric" or "locus of pro-metacentres."

Radius of curvature $\left(R = \frac{I}{V}\right)$ at any point of the curve of buoyancy, and Leclert's formula $\left(r = \frac{dI}{dV} \text{ or } = R + V\frac{dR}{dV}\right)$ for radius of curvature at any point of the curve of flotation.

Loss of initial stability due to grounding.

Launching curves made to ensure that the length of groundways proposed is sufficient, etc.

Froude's experiments on frictional resistance of water.

The resistances experienced by ships in their passage through the water. Methods of calculating the indicated horse-power required to drive a vessel at any given speed. Froude's law of "corresponding speeds." Effective horse-power. Propulsive coefficient; definition of, and values in typical ships.

Methods of measuring speed of ships on their trial trips; precautions necessary to ensure accuracy. Progressive trials.

Calculations relating to the steering of ships. Methods of determining necessary size of rudder-head.

Time of complete oscillation $\left(\tau = 2\pi\sqrt{\frac{\rho^2}{gm}} \text{ or } = 1{\cdot}108\frac{\rho}{\sqrt{m}}\right)$ for a vessel rolling unresistedly in still water. Effect on time of oscillation of raising or "winging" weights. Curves of "extinction" (or "declining angles"). Causes operating to reduce the amplitude of oscillation of a vessel set rolling in still water or amongst waves. Usefulness of bilge keels; formula for calculating resistance due to bilge keels when vessel is rolling. Effect of synchronism between motion of ship and that of waves amongst which she is rolling. Definitions of "effective wave-slope" and "virtual upright." Methods of observing the rolling motions of ships.

Definitions of "stiff" and "steady" vessel; elements of design affecting these qualities.

Methods of calculating wetted surfaces.

The vibration of steamships: how caused, and methods of minimizing.

Those candidates who answer the questions in the written papers in a sufficiently satisfactory way may be called upon to sit for a practical examination at South Kensington. This examination will be held on two consecutive days. The time allowed for work on each day will be seven hours.

Candidates will be required to make a sheer draught of a vessel from particulars to be furnished.

Candidates must themselves provide all drawing instruments, moulds, battens, straight-edges, squares, etc., and all other necessaries, except drawing boards, drawing paper, batten weights, and drawing battens and straight-edges over two feet in length, which will be furnished by the Department.

Neatness and accuracy in drawing will be insisted on.

APPENDIX C

Questions set on Calculations in Examination Papers of the Science and Art Department.[1]

THIS selection of questions is reproduced by permission of the Controller of Her Majesty's Stationery Office.

The complete examination set in 1898 is given, together with the general instructions and rules. This was the first examination set on the new syllabus given on p. 257.

ELEMENTARY.

The total area of the deck plan of a vessel is 4500 square feet. What would be the surface of deck plank to be worked if there are—

4 hatchways, each $4' \times 2\frac{1}{2}'$
2 „ „ $10' \times 6'$

and two circular skylights, each 4 feet in diameter, over which no plank is to be laid?

Write down and explain Simpson's second rule for finding the area of a plane surface.

The half-ordinates of the water-plane of a vessel in feet are respectively, commencing from abaft, 2, 6·5, 9·3, 10·7, 11, 11, 10, 7·4, 3·6, and 0·2, and the common interval between them is 15 feet. Find the area of the plane in square feet.

A piece of African oak keelson is $24' \times 12'' \times 10''$. What is its weight?

[1] The complete examination papers set by the examiners of the Science and Art Department are published yearly, price 6*d*. Questions in Naval Architecture set at the Science and Art Examinations, with answers, are issued for Elementary, Advanced, and Honours by Mr. T. H. Watson, 10, Neville Street, Newcastle-on-Tyne.

A steel plate is of the form and dimensions shown. What is its weight?

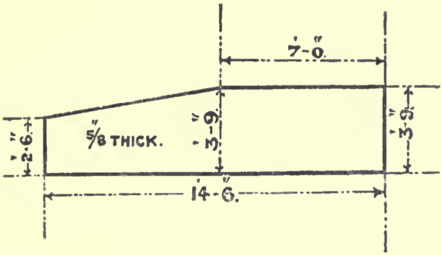

FIG. 110.

Write down and explain Simpson's first rule for finding the area of a plane surface.

The half-ordinates of a deck plan in feet are respectively $1\frac{1}{4}$, $5\frac{1}{2}$, $10\frac{1}{4}$, $13\frac{1}{2}$, $14\frac{3}{4}$, $14\frac{1}{2}$, $12\frac{1}{2}$, 9, and $3\frac{1}{2}$, and the length of the plan is 128 feet. Find the area of the deck plan in square yards.

Referring to the previous question, find the area in square feet of the portion of the plan between the ordinates $1\frac{1}{4}$ and $5\frac{1}{2}$.

The areas of the water-line sections of a vessel in square feet are respectively 2000, 2000, 1600, 1250, and 300. The common interval between them is $1\frac{1}{2}$ feet. Find the displacement of the vessel in tons, neglecting the small portion below the lowest water-section.

A steel plate $\frac{9}{16}$ of an inch thick, is of the form and dimensions shown below. Find its weight in pounds.

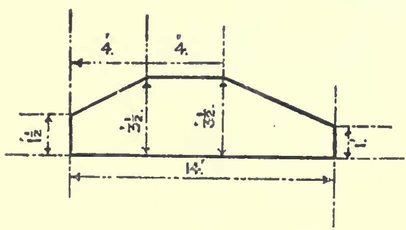

FIG. 111.

Write down—

(1) Simpson's first rule,
(2) Simpson's second rule,

for finding the areas of plane surfaces, and state under what conditions each rule is applicable.

The half-ordinates of the midship section of a vessel are 12·7, 12·8, 12·9, 12·9, 12·9, 12·8, 12·5, 11·9, 10·4, 5·9, and 1·4 feet respectively, and the common interval between them is 18 inches. Find the area of the section in square feet.

A Dantzic oak deck plank is 25′ 6″ long and $4\frac{1}{4}$″ thick. It is 8 inches wide at one end, and tapers gradually to 6 inches at the other end. What is its weight?

What is meant by "tons per inch immersion"?

The "tons per inch immersion" at the load water-plane of a ship is 30·5. What is the area of the load water-plane? and what would be the displacement in cubic feet and in tons, of a layer 4 inches thick in the vicinity of this plane?

Write down and explain Simpson's first rule for finding the area of a plane surface.

The half-ordinates of the load water-plane of a vessel are 0·1, 2·6, 5, 8·3, 10, 10·8, 11, 11, 10·5, 9·6, 7·6, 5·5, and 0·4 feet respectively, and the common interval between the ordinates is 9 feet. Find the area of the load water-plane.

What is meant by "tons per inch immersion"?

Referring to the previous question, what number of tons must be taken out of the vessel to lighten it $3\frac{1}{2}$ inches? and what weight would have to be put into the vessel to increase her draught of water by 2 inches?

Define displacement.

A cylindrical pontoon is 50 feet long and 4 feet in diameter. It floats in sea-water with its axis at the surface of the water. What is its displacement in cubic feet and in tons?

A steel plate, $\frac{5}{8}$ of an inch thick, is of the form and dimensions shown below. Find its weight in pounds.

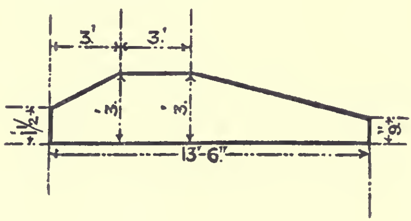

Fig. 112.

Write down—

(1) Simpson's first rule,

(2) Simpson's second rule,

for finding the areas of plane surfaces, and clearly explain in what cases each of these rules is applicable.

The semi-ordinates of the boundary of the deck of a vessel in feet are respectively 0·1, 0·5, 11·6, 15·4, 16·8, 17, 16·9, 16·4, 14·5, 9·4, and 0·1, the semi-ordinates being 11 feet apart. Find the area of the deck in square yards.

What is meant by the displacement of a vessel?

A vessel is of a rectangular section throughout; it is 80 feet long, 15 feet broad, and draws in sea-water 6 feet forward and 8 feet aft. What is its displacement in cubic feet and in tons?

A rectangular "steel" deck-plate is 14′ 3″ long, 3′ $3\frac{1}{2}$″ wide, and $\frac{1}{2}$″ thick. A circular piece 13 inches in diameter is cut out of the centre of the plate. What is the weight of the plate?

Write down and explain—

(1) Simpson's first rule,

(2) Simpson's second rule,

for finding the areas of plane surfaces, and state under what conditions each rule is applicable.

The half-ordinates of the midship section of a vessel are 12·8, 12·9, 13, 13, 13, 12·9, 12·6, 12, 10·5, 6·0, and 1·5 feet respectively, and the distance between each of them is 18 inches. Find the area of the midship section in square feet.

A Dantzic oak plank is 24 feet long and $3\frac{3}{4}$ inches thick. It is 7 inches wide at one end, and tapers gradually to $5\frac{3}{4}$ inches at the other end. What is its weight in pounds?

Having given a deck plan of a ship with ordinates thereon to find its area, how would you know which of Simpson's rules it would be necessary to use?

The semi-ordinates of the load-water plane of a vessel are 0·2, 3·6, 7·4, 10, 11, 10·7, 9·3, 6·5, and 2 feet respectively, and they are 15 feet apart. What is the area of the load water-plane?

What is meant by "tons per inch immersion"?

Referring to the previous question, what weight must be taken out of the vessel to lighten her $3\frac{1}{2}$ inches?

What additional immersion would result by placing 5 tons on board?

Give approximately the weights per cubic foot of—

(1) Teak.

(2) Dantzic oak.

(3) English elm.

(4) Iron.

(5) Steel.

A solid pillar of iron of circular section is 6′ 10″ long and $2\frac{1}{4}$″ in diameter. What is its weight?

Write down and explain Simpson's first rule for finding the area of a plane surface.

The half-ordinates of a transverse section of a vessel are 12·2, 12·2, 12·1, 11·8, 11·2, 10·0, and 7·3 feet in length respectively, and their common distance apart is 16 inches. Neglecting the portion below the lowest ordinate, find the total area of the section in square feet.

The water-planes of a vessel are 4 feet apart, and their areas, commencing with the load water-plane, are 12,000, 11,500, 8000, 3000, and 0 square feet respectively. Find the displacement of the vessel in tons.

What weight would have to be taken out of the vessel referred to in the previous question, in order to lighten her 4 inches from her load water-plane?

A wrought-iron armour plate is 15′ 3″ long, 3′ 6″ wide, and $4\frac{1}{2}$″ thick. Calculate its weight in tons.

The half-ordinates of the midship section of a vessel are 22·3, 22·2, 21·7, 20·6, 17·2, 13·2, and 8 feet in length respectively. The common interval between consecutive ordinates is 3 feet between the first and fifth ordinates, and 1′ 6″ between the fifth and seventh. Calculate the total area of the section in square feet.

Write down and explain Simpson's second rule for finding the areas of plane surfaces.

Obtain the total area included between the first and fourth ordinates of the section given in the preceding question.

The "tons per inch immersion" of a vessel when floating at a certain water-plane is 44·5. What is the area of this plane?

A Dantzic fir deck-plank is 22 feet long and 4 inches thick, and tapers in width from 9 inches at one end to 6 inches at the other. What is its weight?

ADVANCED.

The half-ordinates of the load water-plane of a vessel in feet, *commencing from abaft*, are respectively $2\frac{1}{2}$, 8, $11\frac{1}{2}$, $13\frac{1}{2}$, $13\frac{3}{4}$, $12\frac{1}{2}$, $9\frac{1}{4}$, $4\frac{1}{2}$, and $\frac{1}{4}$, the common interval between the ordinates being 16 feet. Find—

(1) the area of the plane;

(2) the longitudinal position of its C.G. *abaft the foremost* ordinate.

Write down and explain Simpson's second rule for finding the area of a plane surface.

The half-ordinates of a water-plane of a vessel in feet are respectively, commencing from abaft, 2, 6·5, 9·3, 10·7, 11, 11, 10, 7·4, 3·6, and 0·2, and the common interval is 14 feet. Find the area of the plane in square feet.

Having given the dimensions on the load water-plane, state the practical rules by which a close approximation may be made to the weight which must be added, or removed, to change the mean draught of water one inch, in three different types of vessels.

Define displacement. The areas of the vertical traverse sections of a ship up to the load water-plane in square feet are respectively 25, 100, 145, 250, 470, 290, 220, 165, and 30, and the common interval between them is 20 feet. The displacement in tons before the foremost section is 5, and abaft the aftermost section is 6. Find the load displacement of the ship in tons and in cubic feet.

Write down and explain the formula giving the height of the transverse metacentre above the centre of buoyancy.

State clearly the use that can be made of this height by the naval architect when he knows it for any particular vessel.

What principle should be followed in arranging the fastenings in a stringer plate at the beams and at the butts?

A stringer plate is $42'' \times \frac{1}{2}''$. Show the riveting in a beam and at a butt, stating size and pitches of rivets, and show that the arrangement you give is a good one.

The half-ordinates in feet of the load water-plane of a vessel are respectively 0·2, 4, 8·3, 11·3, 13·4, 13·4, 10·4, 7·2, and 2·2, the length of the plane being 130 feet. Find—

(1) the area of the plane;

(2) the distance of its C.G. from No. 5 ordinate;

(3) the decrease in draught of water by removing 25 tons from the vessel,

A ship passes from sea-water to river-water. Show how an estimate may be made for the change in the draught of water.

What will be the mean draught in *river*-water of a ship whose mean draught in *sea*-water is 25 feet, her length on the water-line being 320 feet; breadth extreme 48 feet, and displacement 7500 tons?

The areas of five equidistant water-planes of a vessel in square feet are respectively—

(1) 4100
(2) 3700
(3) 3200
(4) 2500
(5) 1400

the common interval between them is 2 feet, and the displacement below the lowest water-plane is 50 tons. Find—

(1) the tons per inch at each of the water-planes;
(2) the displacement in tons up to each of the first four water-planes.

Explain clearly, illustrating your remarks with rough sketches, how *curves of displacement* and *curves of tons per inch immersion* are constructed, and state what use is made of them.

A portion of a cylindrical steel stern shaft-tube $1\frac{1}{2}$ inches thick, is $15\frac{1}{4}$ feet long, and its external diameter is 15 inches. Find its weight.

When would you use—

(1) Simpson's first rule,
(2) Simpson's second rule,

for finding the area of a plane surface?

The half-ordinates of a water-plane of a vessel, in feet, are respectively, commencing from "forward," 0·3, 3·8, 7·6, 10·2, 11·5, 11·5, 11, 9·5, 6·7, and 2, and the common interval between them is 16 feet. Find—

(1) the area of the plane in square feet;
(2) the distance of the centre of gravity of the plane "abaft" the "foremost" ordinate.

Explain briefly the method of finding the displacement of a ship from her drawings.

What is the "centre of buoyancy"?

The load displacement of a ship is 5000 tons, and the centre of buoyancy is 10 feet below the load water-line. In the light condition the displacement of the ship is 2000 tons, and the centre of gravity of the layer between the load and light lines is 6 feet below the load-line. Find the vertical position of the centre of buoyancy below the light line in the light condition.

The ordinates of the boundary of the deck of a ship are 6·5, 24, 29, 32, 33·5, 33·5, 33·5, 32, 30, 27, and 6·5 feet respectively, and the common interval between them is 21 feet.

The deck, with the exception of a space of 350 square feet, is covered with $\frac{3}{8}$-inch steel plating, worked flush-jointed with single riveted edges and butts. Find the weight of the plating, including straps and fastenings.

In arranging the butt fastenings of the bottom plates of a ship what principles would guide you in determining the rows of rivets and the spacing of the rivets, in the butt straps?

The half-ordinates of a vessel's load water-plane are 0·1, 2·6, 5, 8·3, 10, 10·8, 11, 11, 10·5, 9·6, 7·6, 5·5, and 0·4 feet respectively; the common interval between the ordinates is 9 feet. Find—

(1) the area of the load water-plane—

(2) the distance of its centre of gravity "abaft" the "foremost" ordinate;

(3) the "tons per inch immersion" at the load water-plane.

Define "displacement" and "centre of buoyancy."

The transverse sections of a vessel are 25 feet apart, and their "half"-areas below the L.W.L. are 1, 37, 81, 104, 107, 105, 88, 48, and 6 square feet respectively. Find—

(1) the displacement in tons up to the L.W.L.;

(2) the longitudinal position of the centre of buoyancy "abaft" the "foremost" section.

How is a "curve of displacement" constructed? What are its uses?

A transverse bunker is filled with coal stowed in the ordinary way. Find the weight of the coal from the following particulars:—

The tranverse section is of the same form throughout the length of the bunker, which is 12 feet long.

The semi-ordinates in feet of the transverse section are respectively 6, 9, $10\frac{1}{2}$, $11\frac{1}{2}$, $12\frac{1}{2}$, $12\frac{1}{2}$, and 12 feet, the common interval being 2 feet.

44 cubic feet of coal as ordinarily stowed weighs 1 ton.

What principle should be followed in arranging the fastenings in a stringer plate at the beams and at the butts?

A stringer plate is 40″ × $\frac{5}{8}$″. Show the riveting in a beam and at a butt, stating sizes and pitches of rivets, and show that the arrangement you give is a good one.

The semi-ordinates of the load water-plane of a vessel in feet are respectively 0·1, 5, 11·6, 15·4, 16·8, 17, 16·9, 16·4, 14·5, 9·4, and 0·1, and the common interval is 11 feet. Find—

(1) the area of the plane in square feet;

(2) the distance of its G.C. from the 17-feet ordinate, stating whether the G.C. is before or abaft that ordinate;

(3) the tons per inch.

How is a curve of "tons per inch" constructed? What use is made of such a curve?

The tons per inch at the successive water-planes of a vessel, which are $1\frac{1}{2}$ feet apart, are respectively 6·5, 6·2, 5·6, 4·5, and 0. Construct the curve of tons per inch on a scale of 1 inch to 1 foot of draught and 1 inch to 1 ton.

Define displacement.

Having given the length at L.W.L., breadth extreme, and the mean draught of water, give approximate rules for finding the displacement in tons of—

(1) gun vessels of the Royal Navy;

(2) mercantile steam-ships having high speeds.

A rectangular pontoon 100 feet long, 50 feet wide, 20 feet deep, is empty and floating in sea-water at a draught of 10 feet. What alteration will take place in the floating condition of the pontoon, if the centre compartment is breached and in free communication with the sea, if—

(1) the pontoon were divided into five equal water-tight compartments by transverse bulkheads of the full depth of the pontoon;

(2) the water-tight bulkheads referred to in (1) ran up to and stopped at a deck which is "not" water-tight, 12 feet from the bottom of the pontoon?

Referring to the fourth question back, if a deck surface of equal area to that of the load water-plane therein mentioned were covered with $\frac{3}{8}$-inch steel plating worked flush jointed with single riveted edges and butts, what would be the weight of the plating, including straps and fastenings?

When would you use—

(1) Simpson's first rule,

(2) Simpson's second rule,

for finding the area of a plane surface?

The half-ordinates of a water-plane are 15 feet apart, and their lengths, "commencing from forward," are respectively 1·9, 6·6, 11, 14·5, 17·4, 19·4, 20·5, 20·8, 20·3, 18·8, 15·8, 10·6, and 2·6. Find—

(1) the area of the plane in square feet;

(2) the distance of the centre of gravity of the plane abaft the foremost ordinate.

Define displacement and centre of buoyancy.

The areas of the vertical transverse sections of a vessel in square feet up to the load water-plane, "commencing from forward," are respectively 25, 100, 145, 250, 470, 290, 220, 165, and 30, and the common interval between the sections is 20 feet. Neglecting the appendages before and abaft the end sections, find—

(1) the displacement of the vessel in tons;

(2) the longitudinal position of the centre of buoyancy abaft the foremost section.

A portion of a cylindrical steel stern shaft-casing is $12\frac{3}{4}$ feet long, $1\frac{1}{4}$ inches thick, and its external diameter is 14 inches. Find its weight in pounds.

State the conditions under which a ship floats freely and at rest at a given water-line in still water, and describe what calculations have to be made in order to ascertain that the conditions will be fulfilled.

The semi-ordinates in feet of the load water-plane of a vessel are, commencing from forward, 0, 0·7, 3, 7, 8·5, 8, 6·5, 5, 2·5, and 1 respectively, and the total length is 126 feet. Find—

(1) the area of the plane;

(2) the longitudinal position of its centre of gravity "abaft" the foremost ordinate;

(3) the increase in draught caused by placing 20 tons on board.

If a deck surface of equal area to the load water-plane, referred to in the previous question, were covered with $\frac{5}{16}$-inch steel plating, worked flush, jointed with single riveted edges and butts, what would be the weight of the plating, including straps and fastenings?

How is a curve of tons per inch immersion constructed? What are its uses? Draw roughly the ordinary form of such curves.

A stringer plate is 40 inches wide and $\frac{1}{2}$ inch thick. Show the rivets in a beam and at a butt, and prove by calculation that the arrangement is a good one.

The transverse sections of a vessel are 20 feet apart, and their areas up to the load water-line, commencing from forward, are 3, 35, 83, 136, 175, 190, 179, 146, 98, 50, and 11 square feet respectively. Find—

(1) the displacement of the vessel in tons;

(2) the distance of the centre of buoyancy from the foremost section.

A teak deck, $2\frac{1}{2}$ inches thick, is supported upon beams spaced 4 feet apart and weighing 15 lbs. per foot run. Calculate the weight of a middle-line portion of this deck (including fastenings and beams), 24 feet long and 10 feet wide.

Show by sketch and description how a curve of displacement is constructed, and state its uses.

If it were required to so join two plates as to make the strength at the butt as nearly as possible equal to that of the unpierced plates, what kind of butt strap would you adopt?

Supposing the plates to be of mild steel 36 inches wide and $\frac{1}{2}$ inch thick, give the diameter, disposition, and pitch of rivets necessary in the strap.

The half-ordinates of the load water-plane of a vessel are 12 feet apart, and their lengths are 0·5, 3·8, 7·7, 11·5, 14·6, 16·6, 17·8, 18·3, 18·5, 18·4, 18·2, 17·9, 17·2, 15·9, 13·4, 9·2, and 0·5 feet respectively. Calculate—

(1) the total area of the plane in square feet;

(2) the longitudinal position of its centre of gravity with reference to the middle ordinate; and

(3) the tons per inch immersion at this water-plane.

The beams of a deck are 3 feet apart, and weigh 22 lbs. per foot run; the deck plating weighs 10 lbs. per square foot, and this is covered by teak planking 3 inches thick. Calculate the weight of a part 54 feet long × 10 feet wide of this structure, including fastenings.

A vessel has the "tons per inch" specified below at the several water-planes, viz. 17, 16, 14·6, 12·7, 9·7, 4·5, and 0, and the planes are 3 feet apart. Calculate the displacement of the vessel in tons.

State—

(1) the shearing stress of a $\frac{3}{4}$-inch steel rivet;

(2) the ultimate tensile strength of mild steel plates.

What reduction is allowed for in calculating the strength of the material left between closely spaced punched holes in mild steel plates?

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The "tons per inch" of five equidistant water-planes of a ship are respectively 9·8, 8·8, 7·6, 5·9, and 3·4, the water-planes being $2\frac{1}{4}$ feet apart. Below the lowest of the planes mentioned is an appendage of 60 tons. Calculate the displacement in tons up to each of the water-planes.

Referring to the previous question, construct the curve of displacement on a scale of $\frac{3}{4}$ inch per foot of draught, and $\frac{3}{4}$ inch per 100 tons of displacement, the lowest water-plane mentioned being 3 feet above the keel.

Obtain the expression for the height of the transverse metacentre above the centre of buoyancy.

The displacement of a vessel is 400 tons, and the transverse metacentre is $5\frac{3}{4}$ feet above the centre of buoyancy. A weight of 12 tons, already on board, is moved 8 feet across the deck: find the inclination of the vessel to the upright, the C.G. of the vessel being 3 feet above the C.B.

$$\tan 4^\circ = 0{\cdot}0699$$
$$\tan 5^\circ = 0{\cdot}0875$$
$$\tan 6^\circ = 0{\cdot}1051$$

What is meant by "moment to change trim"? Write down and explain the expression which gives the moment to alter trim one inch.

Suppose a weight of moderate amount to be put on board a ship, where must it be placed so that the ship shall be bodily deeper in the water without change of trim? Give reasons for your answer.

A transverse iron water-tight bulkhead is worked in a ship at a station whose semi-ordinates are (commencing from below) 6, 9, $10\frac{1}{2}$, $11\frac{1}{2}$, $12\frac{1}{2}$, $12\frac{1}{2}$, and 12 feet respectively, the common interval being 2 feet. Find the weight of the bulkhead, the following particulars being given:—

Plates, lap jointed, lap butted, single riveted $\begin{cases}\frac{3}{8} \text{ inch for lower 5 feet.} \\ \frac{5}{16} \text{ inch for upper 7 feet.}\end{cases}$

Angle bar stiffeners 24 inches apart on one side of bulkhead $\Big\}$ $2\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{5}{16}''$.

Distinguish between hogging and sagging strains. A vessel has an excess of weight amidships: to what conclusion would you generally arrive as to the strains produced?

Point out, illustrating your remarks by a simple example, that your general conclusion might not be correct in some cases.

What are the causes which influence the *forms* of curves of stability?

Give an example of such curves for—

(1) a low freeboard mastless armourclad;

(2) a high freeboard cruiser with large sail spread.

Explain how lifeboats are designed to automatically right themselves when capsized, and to free themselves of water when swamped.

The half-ordinates of a portion of a deck plan of a vessel, commencing from abaft, are 2, 8, and 11½ feet respectively, and the common interval is 16 feet. On the beams between the two aftermost ordinates, steel plating ½ inch thick is to be worked. What is the weight of the plating?

From the particulars given below, find the displacement in tons of the vessel up to L.W.L.

	Half-ordinates in feet at stations.				
	1	2	3	4	5
L.W.L.	0·2	7·4	11·0	9·3	2·0
2 W.L.	0·2	5·3	10·5	6·5	0·3
3 W.L.	0·2	2·0	7·6	2·6	0·3

Stations apart, 30 feet; water-lines apart, 3 feet. (Appendages before and abaft end ordinates and below No. 3 W.L. being neglected.)

Describe clearly how you would proceed to obtain from the drawings of a vessel, a close approximation to the area of "wetted surface" for a given draught of water forward and aft.

A pontoon raft is formed by two cylindrical pontoons, each 50 feet long and 4 feet in diameter. The distance between the centres of the pontoons is 8 feet throughout their lengths. In sea-water the raft floats with the axes of the pontoons at the surface of the water. Find the height of the transverse metacentre above the centre of buoyancy of the raft.

What is meant by "change of trim"?

It is desired that the draught of water *aft* in a steam-ship shall be constant whether the coals are in or out of the ship. Show how the approximate position of the C.G. of the coals may be found, in order that the desired condition may be fulfilled.

Briefly describe an *experimental* method of obtaining a curve of stability for a ship. Compare the method with the calculation method.

How would you proceed in arranging the fastenings in a stringer plate at the beams and at the butts?

A stringer plate is 38 inches wide and $\frac{7}{16}$ inch thick. Sketch the riveting in a beam and at a butt, and show that the arrangement is a good one.

Show how a comparison may be made between the turning effects on a ship of—

(1) A narrow rudder held at a certain angle by a given force at the end of a tiller; and—

(2) A broader rudder of equal depth held by an equal force at a smaller angle.

From the particulars given below find—

(1) the displacement in tons of the vessel up to L.W.L.;

(2) the distance of the centre of buoyancy abaft the foremost station;

(3) the depth of the centre of buoyancy below L.W.L.

	Half-ordinates in feet at stations.				
	1	2	3	4	5
L.W.L.	0·3	8·9	13·2	11·2	2·4
2 W.L.	0·3	6·4	12·6	7·8	0·4
3 W.L.	0·3	2·4	9·0	3·1	0·4

Stations apart, 27 feet; water-lines apart, 3 feet. (Appendages before and abaft end ordinates and below No. 3 W.L. being neglected.)

A pontoon raft is formed by three cylindrical pontoons, each 50 feet long and 4 feet in diameter. The outer pontoons are each 8 feet from the middle one (centre to centre) throughout their lengths. Find the displacement of the raft, and the height of the transverse metacentre above the centre of buoyancy, the pontoons floating with their axes at the surface of the water.

Explain briefly how the vertical position of the C.G. of a ship and its equipment is accurately obtained.

How is a metacentric diagram constructed, and what are its uses?

In what classes of ships would you expect the metacentre to fall quickly as the draught lightens, and after reaching a minimum height to gradually rise again?

A steel ship is found, on her first voyage at sea, to be structurally weak longitudinally. How would you attempt to effectually strengthen the ship with the least additional weight of material, giving your reasons?

What is meant by "curves of weight" and "curves of buoyancy" as applied to the longitudinal distribution of weight and buoyancy in ships?

Show how these curves are obtained; and draw, approximately, on the same scale and in one diagram, such curves for any type of vessel with which you are acquainted, mentioning the type you have taken.

The half-ordinates of a vessel's load water-plane are 0·1, 2·6, 5, 8·3, 10, 10·8, 11, 11, 10·5, 9·6, 7·6, 5·5, and 0·4 feet respectively, the common interval being 9 feet. The water-planes of this vessel are $1\frac{3}{4}$ feet apart, and the "tons per inch" for those below the load water-plane are 3·5, 3·0, 2·4, and 0·9 respectively. The keel appendage is of 5 tons displacement, and at 7·5 feet below the load water-plane. Find—

(1) the total displacement in tons;

(2) the vertical position of the centre of buoyancy below the load water-plane.

Explain how the height of the transverse metacentre above the centre of buoyancy of a ship may be found—

(1) accurately;

(2) approximately and quickly;

and state clearly what use is made of the result by the naval architect.

What is meant by change of trim?

Write down the expression which gives the "moment to change trim," and explain by means of a diagram how the expression is obtained.

A ship is 375 feet long, has a longitudinal metacentric height (G.M.) of 400 feet, and a displacement of 9200 tons. If a weight of 50 tons, already on board, be shifted longitudinally through 90 feet, what will be the change in trim?

Under what circumstances may it be expected that the cargoes of vessels will shift?

In a cargo-carrying vessel, the position of whose C.G. is known, show how the new position of the C.G. due to a portion of the cargo shifting may be found.

A ship of 4000 tons displacement, when fully laden with coals, has a metacentric height of $2\frac{1}{2}$ feet. Suppose 100 tons of coal to be shifted so that its C.G. moves 18 feet transversely and $4\frac{1}{2}$ feet vertically; what would be the angle of heel of the vessel, if she were upright before the coal shifted?

$$\tan 10^\circ = 0{\cdot}1763$$
$$\tan 11^\circ = 0{\cdot}1944$$
$$\tan 12^\circ = 0{\cdot}2126$$
$$\tan 13^\circ = 0{\cdot}2309$$

What portions of a properly constructed steel ship are most effectual in resisting longitudinal bending? Why?

What principle should be followed in arranging the fastenings in a stringer plate at the beams and at the butts?

A stringer plate is $40'' \times \frac{5}{8}''$. Show the riveting in a beam and at a butt, stating size and pitches of rivets, and show that the arrangement you give is a good one.

Enumerate the strains to which ships are subjected which tend to produce changes in their "transverse" forms.

When are the most severe transverse strains likely to be experienced by a ship at "rest"? Point out in such a case the forces acting on the ship, and state what parts of the ship, if she be properly constructed, will effectually assist the structure in resisting change of form.

From the particulars given below, find—

(1) the displacement in tons up to the L.W.L.;

(2) the depth of the centre of buoyancy below the L.W.L.;

(3) the longitudinal position of the centre of buoyancy with respect to No. 3 station.

Forward.	Half-ordinates in feet at stations.					Aft.
	1	2	3	4	5	
L.W.L.	0·3	8·0	12·0	10·0	2·0	
2 W.L.	0·3	7·0	12·0	9·0	0·5	
3 W.L.	0·3	6·0	11·5	7·0	0·3	
4 W.L.	0·3	4·5	10·3	5·0	0·3	
5 W.L.	0·3	2·2	8·3	2·7	0·3	

Stations apart, 25 feet; water-lines apart, $1\frac{1}{2}$ feet. (Appendages before and abaft end ordinates and below No. 5 W.L. being neglected.)

Define centre of gravity. Write down and explain the rule for finding the "transverse" position of the C.G. of the "longitudinal" half of a water-plane.

The ordinates of half a water-plane in feet are respectively—0·1, 5, 11·6, 15·4, 16·8, 17, 16·9, 16·4, 14·5, 9·4, and 0·1, and the common interval is 11 feet. Find the "transverse" position of the C.G. of the half water-plane.

The semi-ordinates of the boundary of a ship's deck in feet are respectively, commencing from forward, 0·3, 9·2, 17, 22·5, 26, 28, 29, 29·5, 29·5, 29·5, 29·5, 29·5, 29·3, 29, 28·5, 27·5, 25·5, 21, and 11·5; the common interval being 18 feet.

A steel stringer plate is worked on the ends of the deck beams on each side of the ship, of the following dimensions: $54'' \times \frac{5}{8}''$ for half the length amidships, tapering gradually to $32'' \times \frac{1}{2}''$ at the fore extremity, and to $40'' \times \frac{1}{2}''$ at the after extremity. The butts are treble chain riveted. Find approximately the weight of the stringer plate, including fastenings and straps.

Define the term "metacentre." Prove the rule for finding the height of the transverse metacentre above the centre of buoyancy. State clearly what use is made of the result by the naval architect.

What is a metacentric diagram? How is such a diagram constructed? For what classes of ships are such diagrams specially useful?

Draw a typical metacentric diagram for merchant ships of deep draught in proportion to their beam when fully laden, with approximate vertical sides between the load and light lines.

Sketch the water-tight subdivision of an efficiently subdivided steam mercantile ship.

In some ships the transverse water-tight bulkheads are so badly arranged that it would be preferable, as a safeguard against rapid foundering, if the vessels were seriously damaged below the water-line, to dispense with these bulkheads. Explain, with sketches, how this conclusion is arrived at.

What portions of a ship's structure offer resistance to cross-breaking strains at any transverse section?

Explain clearly how you would proceed with the calculations, and state what assumptions you would make, in finding the strength of the midship section of an iron ship against cross-breaking strains.

Define "displacement," "centre of buoyancy," and "tons per inch immersion."

From the particulars given below, find :—

(*a*) the displacement of the vessel in tons ;

(*b*) the longitudinal position of the centre of buoyancy, with respect to No. 3 station ;

(*c*) the tons per inch at the L.W.L.

Forward.	Half-ordinates in feet at stations.					Aft.
	1	2	3	4	5	
L.W.L.	5·0	15·4	17·0	16·4	9·4	
2 W.L.	5·0	15·4	17·0	15·8	6·1	
3 W.L.	4·8	15·0	16·4	14·2	3·3	
4 W.L.	4·1	13·3	14·5	9·3	1·3	

Stations apart, 22 feet ; water-lines apart, $1\frac{1}{2}$ feet. The appendage below No. 4 W.L. is 50 tons, and has its centre of gravity $5\frac{1}{2}$ feet before No. 3 station. Appendages before and abaft end sections may be neglected.

Explain fully the work necessary to obtain the "correct" vertical position of the centre of gravity of a ship and her lading.

Show how the expression which gives the height of the transverse metacentre above the centre of buoyancy is obtained, and state clearly what use is made of this information by the naval architect when he knows it for any particular ship.

What is a metacentric diagram? Explain how such a diagram is constructed.

A vessel of box form is 20 feet wide, 10 feet deep, and the C.G. of the vessel and its lading is at the middle of its depth for all variations in draught of water. Construct to scale the metacentric diagram of the vessel.

How are curves of stability constructed, and what are their uses?

In constructing such curves for warships which are completed state—

(*a*) what facts must be known ;

(*b*) what assumptions must be made ;

(*c*) what justification there is for making some of the assumptions which may be open to objection.

What are the most important changes, from a designer's point of view, that have taken place in recent years, in—

(*a*) ships of the Mercantile Marine ;

(*b*) armoured ships of war?

From the particulars given below, find—

(1) the area of the L.W. plane, and tons per inch immersion at the L.W.L.;

(2) the displacement of the vessel in tons;

(3) the position of the centre of buoyancy in relation to the L.W.L. and No. 3 station.

Aft.	Half-ordinates in feet at stations.					Forward.
	5	4	3	2	1	
L.W.L.	2·0	9·3	11·0	7·6	0·2	
2 W.L.	0·5	8·2	11·0	6·4	0·2	
3 W.L.	0·3	6·5	10·5	5·3	0·2	
4 W.L.	0·3	4·6	9·5	4·0	0·2	
5 W.L.	0·3	2·5	7·6	2·0	0·2	

Stations apart, 25 feet; water-lines apart, $1\frac{1}{2}$ feet; all appendages being neglected.

How would you obtain a close approximation to the area of wetted surface of a ship?

What use could be made of this information?

Obtain the expression which gives the height of the transverse metacentre above the centre of buoyancy.

The displacement of a vessel is 400 tons, the transverse metacentre is $5\frac{3}{4}$ feet above the centre of buoyancy, and the centre of gravity is 3 feet above the centre of buoyancy. A weight of 12 tons is moved 8 feet across the deck. Find the inclination of the vessel.

$$\tan 4^\circ = 0·0699$$
$$\tan 5^\circ = 0·0875$$
$$\tan 6^\circ = 0·1051$$

Obtain the expression which gives the "moment to change trim one inch."

In what position would you place a moderate weight on board a ship in order that there should be no change of trim? Give your reason.

How would you rapidly determine whether any practicable alteration in the distribution of weights which can be shifted would enable a ship to go over a bar which would be an impossibility in her existing trim?

How is the information obtained for constructing curves showing the longitudinal distribution of weight and buoyancy of a ship?

How are the curves made, and what are their uses? What checks would you adopt to verify the accuracy of the curves, and what guarantee would you have that the conditions of the checks are correct?

What are the most severe strains likely to be experienced by a ship at rest? Point out the forces acting on the ship, and state what parts of the structure operate in resisting change of form.

What are equivalent girders for ships? Briefly describe how they are constructed.

The water-lines of a vessel are 5 feet apart, and the "tons per inch" at those lines, commencing from the load water-line, are 23·8, 21·9, 19·5, 16·4, and 6 tons respectively. Neglecting the appendage below the lowest water-line, calculate the displacement of the vessel, and the vertical position of the centre of buoyancy—

(1) when floating at her load water-line;

(2) when floating at the line next below her load water-line.

Obtain the formula giving the height of the longitudinal metacentre above the centre of buoyancy.

What is meant by "change of trim"?

A ship 220 feet long has a longitudinal metacentric height of 252 feet, and a displacement of 1950 tons. Calculate the change of trim due to shifting a weight of 20 tons, already on board, through a longitudinal distance of 60 feet.

Describe in detail how you would proceed to obtain the statical stability at large angles of inclination of a vessel of known form.

Show how you would calculate the weight of the outer bottom plating of a vessel, and the position of its centre of gravity.

The shearing force and bending moment operating at every transverse section of a vessel floating at rest in still water being required, how would you proceed to obtain this information?

Explain clearly how the stability of a vessel at small angles of inclination is affected by the presence of water on board, which is free to shift transversely.

A box-shaped vessel, 105 feet long and 30 feet broad, floats at a uniform draught of 10 feet. A central compartment, 20 feet long, contains water which is free to shift from side to side. Calculate by how much the metacentric height is reduced from what its value would have been had the water been *fixed*.

Write down and explain Simpson's first and second rules for obtaining the areas of plane surfaces.

The half-ordinates of the load water-plane of a vessel are 13 feet apart, and their lengths are 0·6, 4·6, 9·1, 12·8, 15·5, 17·2, 18·1, 18·4, 18·5, 18·5, 18·4, 18·2, 17·7, 16·9, 15·3, 12·5, and 7·0 feet respectively. Calculate—

(1) the total area of the plane in square feet ;

(2) the area included between the third and sixth ordinates.

The water-planes of a vessel are 3 feet apart, and the displacements up to the several planes are 2380, 1785, 1235, 740, 325, 60, 0 tons respectively. Calculate the vertical position of the centre of buoyancy, and show that the method you employ is correct.

Define the terms "metacentre" and "metacentric height."

The load displacement of a vessel is 1880 tons when floating at the water-plane given in the second question above. Calculate her longitudinal metacentric height, assuming the ship's centre of gravity to be in the load water-line, and her centre of buoyancy 6 feet below that line.

Sketch a metacentric diagram for any one type of ship, specifying the type chosen. How is such a diagram constructed?

What is meant by the *dynamical stability* of a vessel at any angle of inclination? Obtain Moseley's formula for calculating its value.

Explain how you would proceed to calculate the weight, and position of centre of gravity, of the transverse framing of a vessel.

Show how you would calculate the "wetted surface" of a vessel with considerable accuracy. Describe any method of *rapidly* calculating wetted surfaces with which you may be acquainted.

1898.

SUBJECT IV.—NAVAL ARCHITECTURE.

EXAMINER: J. J. WELCH, ESQ., R.C.N.C.

GENERAL INSTRUCTIONS.

If the rules are not attended to, the paper will be cancelled.

You may take the Elementary stage, or the Advanced stage, or Part I. of Honours, or (if eligible) Part II. of Honours, but you must confine yourself to one of them.

Put the number of the question before your answer.

You are to confine your answers *strictly* to the questions proposed.

Your name is not given to the Examiner, and you are forbidden to write to him about your answers.

The value attached to each question is shown in brackets after the question. A full and correct answer to an easy question will in all cases secure a larger number of marks than an incomplete or inexact answer to a more difficult one.

The examination in this subject lasts for four hours.

FIRST STAGE OR ELEMENTARY EXAMINATION.

Instructions.

You are permitted to answer only *ten* questions.

You must attempt No. 11. Two of the remaining questions should be selected from the Calculations; and the rest from the Practical Shipbuilding section.

Practical Shipbuilding.

1. Give a sketch of a side bar keel, and describe how the several lengths are secured together, and how the work is made water-tight. (8)

2. Describe the operation of getting an ordinary bar stem into its correct position on the blocks. (8)

3. Sketch a transverse frame, from keel to water-tight longitudinal (or to margin plate), of a vessel having a double bottom. Specify usual sizes of plates and angles for a large ship. (12)

4. Describe the usual method of bending and bevelling the frame angle-bars of a vessel. (8)

5. In some cases it is necessary to work the floor-plate of a transversely framed vessel in two lengths. Describe, with sketches, two usual methods of connecting these lengths together. (10)

6. Show how a beam of  section is secured to the framing, giving usual disposition and pitch of rivets. (8)

7. If, after a bottom plate is in position, it is found that the rivet holes do not quite correspond with those of the adjacent plates, what should be done to rectify this and to ensure sound work? (8)

8. Name, with sketches, the finished forms of rivets employed in shipwork. (8)

9. Sketch a usual shift of butts of deck plating, and show how the several plates are secured together, and to the beams. Specify

size and pitches of rivets which would be used, assuming the plating to be $\frac{5}{16}''$ thick. (12)

10. Give a rough sketch of a frame of a composite vessel, naming the several parts of which it is made up. (8)

Drawing.

11. What does sketch, Fig. 113, represent? Draw it neatly in pencil on a scale twice the size shown. (14)

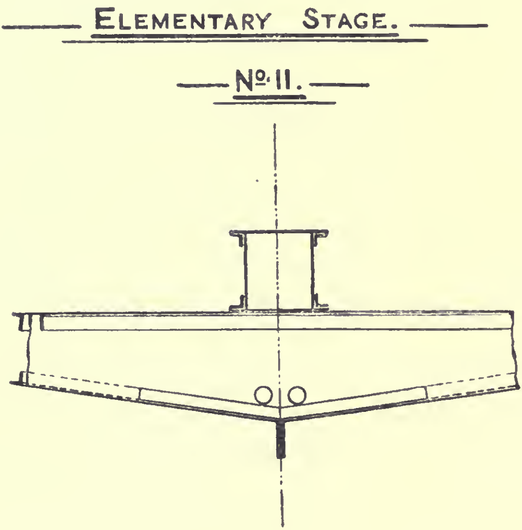


Fig. 113.

Calculations.

12. State the fundamental conditions which must be fulfilled by a vessel when floating freely and at rest in still water. (6)

13. The areas of successive water-planes of a vessel are, beginning with the load water-plane, 14,850, 14,400, 13,780, 12,950, 11,770, 10,130, and 7680 square feet respectively, and the common interval between the planes is $3\frac{1}{4}$ feet. Neglecting the part below the lowest water-plane, calculate the vessel's load displacement in tons. (12)

14. Point out why it is that a vessel sinks deeper in the water when passing from the sea into a river.

How much would you expect the vessel in the preceding example to sink from her load-line under such circumstances? (8)

15. The external diameter of a hollow steel shaft is 18 inches, and its internal diameter 10 inches. Calculate the weight of a 20-feet length of this shafting. (8)

SECOND STAGE OR ADVANCED EXAMINATION.

Instructions.

Read the General Instructions on p. 282.

You are permitted to answer only *twelve* questions.

You must attempt Nos. 29 and 33. The remaining questions may be selected from any part of the paper in this stage, provided that one or more be taken from each section, viz. Practical Shipbuilding, Laying Off, and Calculations.

Practical Shipbuilding.

21. Give sketches showing the sections of moulded steel in general use for shipbuilding purposes, and say for what parts of the structure each is used. (12)

22. For what purposes are *web frames* fitted in vessels?

How is a web frame secured where it crosses a continuous stringer plate? (12)

23. At what stage of the work are the deck-beams attached to the frames of a transversely framed ship, and when is the riveting of beam knees performed?

Give a sketch of a beam knee, showing the fastenings. (14)

24. Sketch a satisfactory shift of butts of bottom plating, and show the riveting adopted in edges and butts, and for security to the frames. Specify size and pitches of rivets, assuming the plating is $\frac{7}{16}''$ thick. (20)

25. Describe fully the operations of getting in place and riveting up a bottom plate, and the precautions necessary to ensure sound and efficient work. (14)

26. Describe how a large transverse water-tight bulkhead is constructed, and secured in position. (14)

27. What tests are applied to—

(1) mild steel plates,

(2) rivet steel and rivets,

before acceptance from the manufacturer? (16)

28. An ordinary steel deck is to be covered with planking.

State the order in which this work would be proceeded with, and show how the planking would be fastened.

Give, on a large scale, a sketch of one of the fastening bolts. (20)

29. Enumerate the several causes which tend to produce the transverse straining of ships, and point out the parts of the structure which supply the necessary transverse strength to resist these straining actions.

Laying Off.

30. How is a Scrive board constructed, and what are its uses? (10)

31. Describe briefly the system of fairing the body adopted—

(1) in the middle portion,

(2) at the extremities, of a vessel. (12)

32. How would you find the true form of the plane of flotation of a vessel which has a considerable trim by the stern, and also a list to starboard or port? (12)

Drawing.

33. What does the drawing Fig. 114 represent? Draw it neatly in pencil on a scale twice the size shown. (24)

— Advanced Stage. —

— No. 33. —

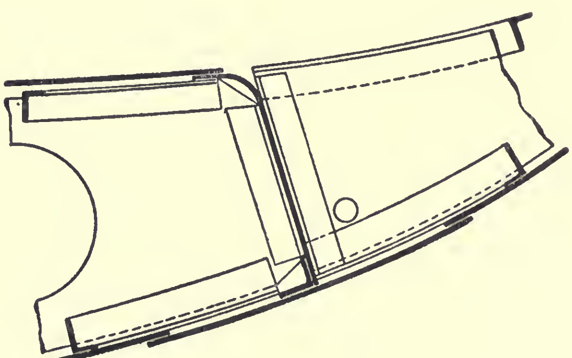


Fig. 114.

Calculations.

34. The transverse sections of a vessel are 18 feet apart, and their areas up to the load water-line, commencing from forward, are 6·5, 55·8, 132·0, 210·9, 266·3, 289·5, 280·2, 235·7, 161·2, 77·8, and 10·9 square feet respectively. Calculate the displacement of the vessel in tons, and the longitudinal position of her centre of buoyancy. (14)

35. Referring to the preceding question, calculate the volume of displacement comprised between the first and sixth sections, and the distance of the centre of buoyancy of that portion from the first section. (16)

36. What is a *curve of tons per inch immersion?* Show how such a curve is constructed, and give a sketch indicating its usual shape. (8)

37. A steel stringer plate is 48 inches wide and $\frac{7}{16}$ inch thick. Sketch the fastenings in a beam and at a butt, and show by calculations that the butt connection is a good one. (20)

HONOURS EXAMINATION.—Part I.

Instructions.

Read the General Instructions on p. 282.

You are permitted to answer only *fourteen* questions. You must attempt Nos. 70 and 74; the remainder you may select from any part of the paper in this stage, provided that one or more be taken from each section, viz. Practical Shipbuilding, Laying Off, and Calculations.

Practical Shipbuilding.

60. What is the usual spacing adopted for transverse frames in—
(1) a first-class battleship;
(2) a large merchant vessel?

Give reasons for the differences noted between the two classes in this respect, and also for the different spacing adopted in the several parts of a battleship. (20)

61. What are the characteristic qualities of the following shipbuilding materials: (1) Dantzic fir, (2) East India teak, (3) cast steel?

State where these materials are employed. (16)

62. Give a rough sketch of the midship section of a vessel having a double bottom, and point out the order in which the work of

erecting the framing of such a ship would be proceeded with in way of the double bottom. (30)

63. Roughly sketch the stern-post of a screw ship, showing how it is connected to the keel and bottom plating. (16)

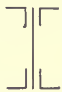
64. What considerations govern the lengths and breadths of plates used on the bottom of a ship? Describe fully the work of getting into place and riveting-up one such plate. (16)

65. Give sections of the beams commonly employed in ship-building, and say where each form is employed. (12)

66. In what vessels is straining at the butts of bottom plating specially liable to take place, and why? What method of stiffening butt straps has been designed to prevent the above action? (16)

67. Show by sketch and description how water-tight work is secured—

(1) at the upper edge of a longitudinal bulkhead;

(2) where a middle line keelson of  form, worked above the floors, passes through a transverse bulkhead. (16)

68. Describe the work of laying and fastening the planking of a deck, the beams of which are not covered with plating. (25)

69. Sketch, and describe the working of, a large sliding water-tight door as fitted to a bulkhead between machinery compartments. (30)

70. Enumerate the principal *local* stresses experienced by ships, and point out what special provision is made to meet each. (25)

Laying Off.

71. What information and drawings would you require before proceeding with the work of laying off a vessel on the mould loft floor?

Show how the extremities of a ship are usually laid off and faired. (20)

72. How is the shape of longitudinal plate frames obtained in those parts of a ship where there is not much curvature? Sketch a mould for a longitudinal plate, showing the marks which would be put upon it for the information of the workman. (16)

73. The lines of a vessel sheathed with wood having been given to the outside of sheathing, show how you would obtain the body plan to outside of framing, (1) approximately, (2) accurately. (25)

CALCULATIONS.

74. The half-ordinates of the load water-plane of a vessel are spaced 18 feet apart, and their lengths, commencing from forward, are 0·6, 3·4, 7·1, 11·4, 16·0, 20·3, 24·0, 26·8, 28·8, 30·0, 30·5, 30·5, 30·0, 28·9, 27·0, 24·3, 21·1, 17·2, 12·7, 7·7, and 3·0 feet respectively. Calculate the total area of the plane in square feet, and the longitudinal position of its centre of gravity. (25)

75. Prove the formula used for calculating the distance between the centre of buoyancy and the transverse metacentre of a vessel. (12)

76. A vessel, 200 feet long between perpendiculars and of 1080 tons displacement, floats at a draft of 11′ 3″ forward and 12′ 3″ aft, and has a longitudinal metacentric height of 235 feet. Supposing a weight of 20 tons to be moved forward through a distance of 120 feet, what would be the new drafts of water forward and aft, assuming the centre of gravity of the water-plane area is 10 feet abaft the midship section? (16)

77. The deck of a vessel is covered with $\frac{5}{16}$-inch mild steel plating, and the beams, spaced 3 feet apart, weigh 20 lbs. per foot run. The half-ordinates of the foremost 84 feet length of this deck are 0·8, 3·5, 6·5, 9·4, 12·1, 14·5, 16·6, 18·4, and 20 feet respectively. Calculate the total weight of plating and beams for this portion of the deck. (20)

78. Show how the work of estimating the weight and position of the centre of gravity of the outer bottom plating of a vessel from her drawings would be proceeded with. (16)

HONOURS EXAMINATION.—PART II.

Instructions.

Read the General Instructions on p. 282.

You are not permitted to answer more than *fourteen* questions, of which two at least must be taken from the Practical Shipbuilding and Laying Off section.

NOTE.—*No Candidate is eligible for examination in Part II. of Honours who has not already obtained a first or second class in Honours of the same subject in a previous year.*

Those students who answer the present paper sufficiently well to give them a reasonable chance of being classed in Honours, will be required to take a practical examination at South Kensington. Honours candidates admissible to this Examination will be so informed in due course.

Practical Shipbuilding and Laying Off.

84. Describe the usual method of bending and bevelling Z-bar frames by hand.

What advantages are claimed to accrue from the use of bevelling machines? (30)

85. What are the reasons for working mast partners? Sketch and describe an arrangement of mast partners for a steel ship. (20)

86. Describe, with illustrative sketches, the characteristic features of the launching arrangements adopted for a large ship. (35)

87. A raking mast, of uniform diameter at its lower end, stands upon a deck which has considerable round-up and sheer. Show how the true shape of the lowest plate of mast would be obtained. (25)

Ship Calculation and Design.

88. The tons per inch immersion at the several water-planes of a vessel are 29·1, 28·8, 28·2, 27·3, 26·0, 24·3, 21·9, 18·6, and 13·1 respectively, the common interval between the planes being $2\frac{1}{2}$ feet. The part of the ship below the lowest water-plane has a displacement of 300 tons, and its centre of buoyancy is $21\frac{1}{2}$ feet below the load water-line. Estimate (1) the total displacement of the vessel in tons; (2) the vertical position of her centre of buoyancy. (25)

89. State and prove Simpson's second rule for the calculation of plane areas, pointing out clearly the assumptions involved. (25)

90. What are *curves of displacement* and *curves of tons per inch immersion?* Give sketches showing their usual shapes. (16)

91. How would you proceed to estimate the shearing force and bending moment acting at any cross-section of a given vessel, when floating freely and at rest in still water? (30)

92. Prove Atwood's formula for the statical stability of a vessel at any angle of heel. Show how a curve of statical stability is constructed, and explain its uses. (25)

93. A weight of moderate amount is to be placed on board a given vessel in such a position that the draft of water aft will be unaffected by the addition. Explain how the necessary position of the weight can be calculated. (30)

94. Describe any method by which the statical stability of a vessel of known form and lading can be obtained experimentally. (25)

95. What are *cross-curves* of statical stability? How are these related to the ordinary stability curves? (30)

96. Point out clearly how the presence of water or other liquid having a free surface in the hold of a vessel affects her stability. (30)

97. What resistances are experienced by a vessel when being towed through water at a uniform speed? What is the relative importance of these resistances (1) at low speeds, (2) at high speeds? (16)

98. Write down and explain Froude's law of "corresponding speeds."

A certain vessel of 1000 tons displacement can be propelled by engines of 1150 I.H.P. at 14 knots. What will be the "corresponding speed" of an exactly similar vessel of 8000 tons displacement, and what indicated horse-power is likely to be required to propel the larger vessel at that corresponding speed? (35)

99. What is meant by—

(1) a *stiff* vessel;
(2) a *steady* vessel?

What features of the design affect these qualities? (25)

100. A rudder hung at its forward edge and entirely below water is rectangular in shape, 14 feet deep, and 10 feet broad. Calculate the diameter of steel rudder-head required, the maximum speed of the vessel being 14 knots, and the greatest helm angle 35°.

Note.—Sin 35° = 0·574. (30)

101. A hole 1 square foot in area is pierced in a vessel 12 feet below her load water-line in wake of an empty compartment. Calculate the capacity in tons per hour of the pumps required to just keep this leak under. (25)

BOOKS ON "THEORETICAL NAVAL ARCHITECTURE."

"Transactions of the Institution of Naval Architects."

"Transactions of the North-East Coast Institution of Engineers and Shipbuilders."

"Transactions of the Institution of Engineers and Shipbuilders in Scotland."

"Shipbuilding, Theoretical and Practical." By Prof. Rankine and Mr. F. K. Barnes, M.I.N.A.

"Naval Science." Edited by Sir E. J. Reed, K.C.B., F.R.S.

"Theoretical Naval Architecture." By Mr. Samuel J. P. Thearle, M.I.N.A.

"Yacht Architecture." By Mr. Dixon Kemp, Assoc. I.N.A.

"Manual of Naval Architecture." By Sir W. H. White, K.C.B., F.R.S.

"Stability of Ships." By Sir E. J. Reed, K.C.B., F.R.S.

"Text Book of Naval Architecture," for the use of Officers of the Royal Navy. By Mr. J. J. Welch, M.I.N.A.

"Know your own Ship," for the use of ships' officers, etc. By Mr. Thomas Walton.

"Naval Architects', Shipbuilders', and Marine Engineers' Pocket Book." By Mr. Clement Mackrow, M.I.N.A.

"Resistance and Propulsion of Ships." By Mr. D. W. Taylor, M.I.N.A.

"Applied Mechanics" (Appendix on "Resistance and Propulsion of Ships"). By Professor Cotterill, F.R.S.

"Encyclopædia Britannica," article on "Shipbuilding." By Sir Nathaniel Barnaby, K.C.B.

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